STUDIES OF A STRUCTURAL FORM FOR UNDERWATER STRUCTURES

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DOCTOR OF PHILOSOPHY

UNIVERSITY OF EDINBURGH

1980

DECLARATION

I hereby declare that this thesis was composed by me. The works and results reported were carried out by me under the supervision of Dr. Rodney Royles, unless otherwise stated.

Edinburgh, November 1980

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CONTENTS

ACKNOWLEDGEMENTS

A number of people have contributed towards the completion of this work. I seize this opportunity to express my utmost gratitude to all. In particular I am indebted to Prof. A.W. Hendry (the Head of Civil Engineering Department) for granting the opportunity and providing the facility that enables me to carry out the research in the Department, Dr.R. Royles (my supervisor) for introduction to the problem, continuous guidance, encouragement and kindness throughout the research, the technical staff of Civil Engineering Department for their cooperation, the computing staff of E.R.C.C. for their excellent computing facilities, the Wave Power Project staff led by Mr. S. Salter for their help and generosity in providing some of the equipment used in the experiment, Mr. John Mistry of Liverpool University for his invaluable help and permission to use the finite element program developed by him, Strain and North Holland Publishing Company for the figures in pages 192 and 134, the people of my country (Nigeria) who sacrificed and provided the fund of the research, numerous friends, colleagues - in and out of the department - , family and relatives without whom the entire endeavour would have been worthless and the typist, Mrs. J. Hollingdale who did an excellent job of typing.

NOTATION

A list of the symbols used in this work is given below:

A	parameter, $\frac{N}{\gamma}$
CONST	variable in computer programs indicating variation in pressure head (Chapter Four)
DS	design stress of shell
DX	step-length
DX1	x coordinate value of new initial condition of differential equations system (2.1)
DX1	step-length value (2.3)
DX2	step-length value
DX3	step-length value
DXX	step-length
DZ_A	depth of end A of shell from water level
DZ_B	depth of end B of shell from water level
ER_1, ER_2, ER_3	different error values
G	unit weight of fluid
L	Lipschitz constant
M_ϕ, M_θ	bending moment
$M_{\phi\theta}$	twisting moment
N	stress resultant at design head
N_ϕ	Normal force per unit length in direction tangential to the meridian

N_{θ}	Normal force per unit length in direction tangential to the parallel circle
$N_{\phi\theta}$	shear force
$N_{\phi'}$	stress resultant in the meridional direction at level defined by $\phi = \phi'$
$N_{\theta'}$	stress resultant in the parallel circle direction at level defined by $\phi = \phi'$
$N_{\phi(DESIGN)}$	design head value of N_{ϕ}
P	acting pressure
P_A	atmospheric pressure
P_o	pressure at apex of tank
R	resultant of the external load acting vertically downwards on shell
$R_{\phi'}$	R value at level defined by $\phi = \phi'$
T	shell thickness
T_o	design stress
T_1, T_2	(from Novozhilov) correspond to N_{ϕ}, N_{θ}
Z	radial load intensity
ZO, d	design head of shell
ds	(length of) small arc of shell
$\frac{d(\ )}{dx}$	differentiation or first derivative of ( ) with respect to x
$\frac{d^2(\ )}{dx^2}$	second derivative of ( ) with respect to x
$\left.\frac{dz}{dx}\right\|_1$	computed $\frac{dz}{dx}$ value at point 1
$f(x,y)$	function of x and y

g	acceleration due to gravity
h	step-length (2.4)
h	increase in design head (4.2)
k	gauge factor
r_o	radius of curvature of parallel circle
r_1, r_2	radii of curvature in the meridional plane and in the normal plane perpendicular to meridian
s	meridian distance
t	time
u	$\sin \phi$ (2.3)
u	meridian displacement (5.3)
v	circumferential displacement
w	normal displacement
x, z	coordinates in an orthogonal coordinate system
x', z'	x, z coordinates at point given by $\phi = \phi'$
x_A, x_B	distances of ends A and B of shell from origin of coordinate system
$\dot{x}$	$\frac{dx}{d\phi}$
$x(1), z(1), u(1)$	x, z and u coordinates of point next to apex of shell
$z_1(1), u_1(1), z_2(1), \ldots$	z, u values corresponding to different step-lengths
y'_n	$\left.\frac{dy}{dx}\right\rvert_n$
$y(x)$	function of x

β	rotation
Δv	out of balance voltage
δR	small change in R
δs	small arc of meridian (length)
δx	small change in x
$\delta \phi$	small change in ϕ
ε	a small positive number (2.3)
ε	parameter, $\frac{\gamma}{N}$ (2.5)
ε	strain (6.2.4)
γ	unit weight of fluid
γd	pressure head at apex of shell
λ	parameter, $\dfrac{P_o}{\sqrt{T_o \rho}}$
ρ	specific gravity
ρ_L	liquid mass density
σ_ϕ, σ_θ	axisymmetric stresses
ϕ, θ	angles defining position on meridian
θ	circumferential coordinate of element (5.3)
ϕ'	a particular value of angle ϕ defining a parallel circle (4.2)
σ_{EQ}	equivalent stresses

LIST OF FIGURES

LIST OF FIGURES (contd.)

ABSTRACT

This thesis concerns itself with studies of the drop shaped tank (i.e., shell of revolution having uniform or constant strength) and the possibility of using it in underwater constructions. Beginning with the nonlinear ordinary differential equations system of its meridian possible methods of shape prediction are considered. After comparing these a reliable method is suggested. Following this, a simple procedure for selecting a particular shell, if such shells are to be built in reality, is given. An approximate theoretical method of analysis based on membrane shell theory is considered in evaluating the forces developed by the drop shaped shell when the pressure head at its apex is different from the one for which it is designed. The finite element method is then used to improve on this and also serves as a theoretical basis for predicting the response of the shell due to a varying hydrostatic pressure head. An experiment carried out to investigate this response is reported and the results obtained are discussed with the theoretical ones. The overall outcome of the investigation is encouraging. Recommendations are made for possible ways of improvement, alternative approaches and further future work.

FOREWORD

The author would like to bring the following to the notice of the reader:

(i) the words "shell" and "tank" are used interchangeably,

(ii) the Figures in each chapter are placed at the end of the chapter,

(iii) the number of computer print-outs actually listed in the Appendices is reduced due to excessive cost and the need to reduce the size of the thesis (but adequate information is supplied at appropriate places indicating where the omissions take place),

(iv) the heads whose results are listed in Tables 5.4.1 and 5.4.2 are the specific design heads and 10 × design heads. Other intermediate heads are omitted for the same reasons as in (iii), and

(v) the papers published during the period of this research are given in Appendices A1.0.1 and A7.0.1.

CHAPTER ONE: GENERAL INTRODUCTION

Section (1.0) The Problem

Section (1.1) The Drop Shaped Tank

Section (1.2) An Overview of Thesis

CHAPTER ONE: GENERAL INTRODUCTION

Section (1.0) The Problem

Providing alternative habitations for various species on Earth is a problem that continues to receive attention as world population continues growing. In this respect, some scientists exploring other planets in the outer space are investigating the possibilities of inhabiting these planets while others are looking with great interest at the areas of land covered by water on Earth.[1,2] Another problem of significance, not unrelated to the previous concerns storage in offshore oil exploration. Ferrying crude oil by means of tankers and/or expensive pipelines from off-shore locations to land before refining and distribution is expensive. In some instances, it may be more economical to provide an adequate storage at the exploration site. Towards this end, some oil companies are using gravity platforms in various forms when constructing storage tanks. These and other possibilities need looking into especially as very little information is made available by oil companies mostly dealing with this problem. It is needless to state that the energy shortage has brought about an increasing interest in renewable energy sources. Wave energy devices being prominent among the numerous alternatives that are being investigated. It should be interesting to ascertain the role of underwater enclosures in this growing field of human endeavour, especially as the designers of some of these devices, e.g. Lockheed [3] are still to provide adequate information regarding the structural safety of their proposed forms.

The air-sea interface is in most cases a more demanding and hostile environment when compared with the air or sea medium alone.[4] Floating or partly submerged structures are therefore exposed to more hazards than fully submerged ones. Consequently, interest will be restricted in this thesis to fully submerged enclosures or chambers. Before the realisation of an underwater chamber a number of difficulties would have to be surmounted. Some of these are outlined in (5, 6, 7) and briefly as stated in the remainder of this paragraph. The environment in which the structure is to operate must be fully studied and understood. This environment is a harsh one and its effect on a structure can only be evaluated after an understanding of the mechanisms of wind, waves, tides, seismic and chemical reaction which may act on the structure. The materials of construction need to be tested and evaluated. It will appear that the most likely main materials of construction are concrete and steel. Both materials have their advantages and drawbacks. As for methods of construction, installation and maintenance one may have to consider new along with some old ones. In this respect the various problems would be more capable of solution if there were related precedents, natural or otherwise, to guide the investigators.

When faced with a problem, man sometimes finds it convenient and useful to study the way nature has dealt with the same or similar problems[8]. As the object of interest in this thesis is underwater enclosures it is worth paying some attention to the marine animal kingdom in general and the shape of the shell of the sea-urchin (phylum Echinodermata) in particular. This shape has a close resemblance to that of the drop shaped tank or shell, i.e. the shell of revolution of constant or uniform strength. It has been suggested that this shape may provide a designer enclosures of optimum strength[9,10]. This shape is described in section (1.1) but it is to be noted that other shapes have been and are capable of being utilized in the construction of underwater enclosures. Some of these can be found in [11 - 15]. What then is the drop shaped tank or shell?

Section (1.1) The Drop Shaped Tank/Shell

The drop shaped tank or the shell of revolution of constant or uniform strength is a shell of uniform or constant thickness in which the membrane forces at all points are equal. If there is uniform thickness then it means that under elastic conditions, the shell has uniform stressing or strain if other design conditions of the shell are satisfied. Using membrane shell theory, it has been shown[16,17,18,19] that to contain a liquid of unit weight γ in a tank such that the equivalent internal pressure head at the apex is γd the form necessary for uniform stressing to exist in all parts of the tank is the shape taken up by a drop of liquid lying on a plane surface (see Fig. 1.1.1). This shape is dependent on the pressure head at the apex, the stress in the tank and its thickness.

The drop shape had been used for constructing storage tanks on land primarily by Chicago Bridge and Iron Company (refer to Fig. 2 of Ref.[9] - see Appendix A1.0.1). This company built and tested the first one in 1928[20]. Their design was based on such skin acting in tension. Some tests conducted by this company gave some indication that such a skin of suitable material might be able to withstand external pressure. It was further reported that a tensile designed tank was accidentally subjected to an external pressure without suffering any damage. An experiment carried out to determine whether membrane theory is applicable to a compression structure of constant strength seems to confirm the possibility of subjecting such tank to external pressure[9]. Prior to employing such shell shape in the design and construction of underwater enclosures or chambers a number of experiments and theoretical investigations must be carried out. To the author's knowledge, very little has been done and/or reported in this respect. The work of this thesis is hopefully a small contribution towards the possible realisation of the drop shape being employed in underwater activities.

Section (1.2) An Overview of Thesis

Before reporting the research carried out by the writer an overview of the thesis is given.

The mathematical equation predicting the meridional shape of the drop shaped tank is complicated. It does not lend itself to any known closed form or analytical solution. The quantitative and graphical methods of solution suggested by[16,17,18,19] are unreliable and inefficient, especially when several shapes have to be generated. This of course will be the case in reality. Therefore, before this shape can be considered for underwater enclosures other more reliable methods of solution would have to be found for this equation. This problem of shape prediction is tackled in chapter two. After the consideration of shape prediction, establishing a "library of shapes" from which a designer can choose an appropriate one becomes imperative. A procedure for doing this is suggested in chapter three. Next, if this shape is to be used underwater, its response to hydrostatic, hydrodynamic and nondeterministic loadings must be evaluated. This may be done analytically and experimentally. The response of the tank due to varying hydrostatic pressure head is theoretically investigated in chapters four and five. In chapter six an experiment to collaborate the theoretical work of the previous chapter is reported. Chapter seven summarises the work of this thesis, discusses possible improvements and mentions some extensions.

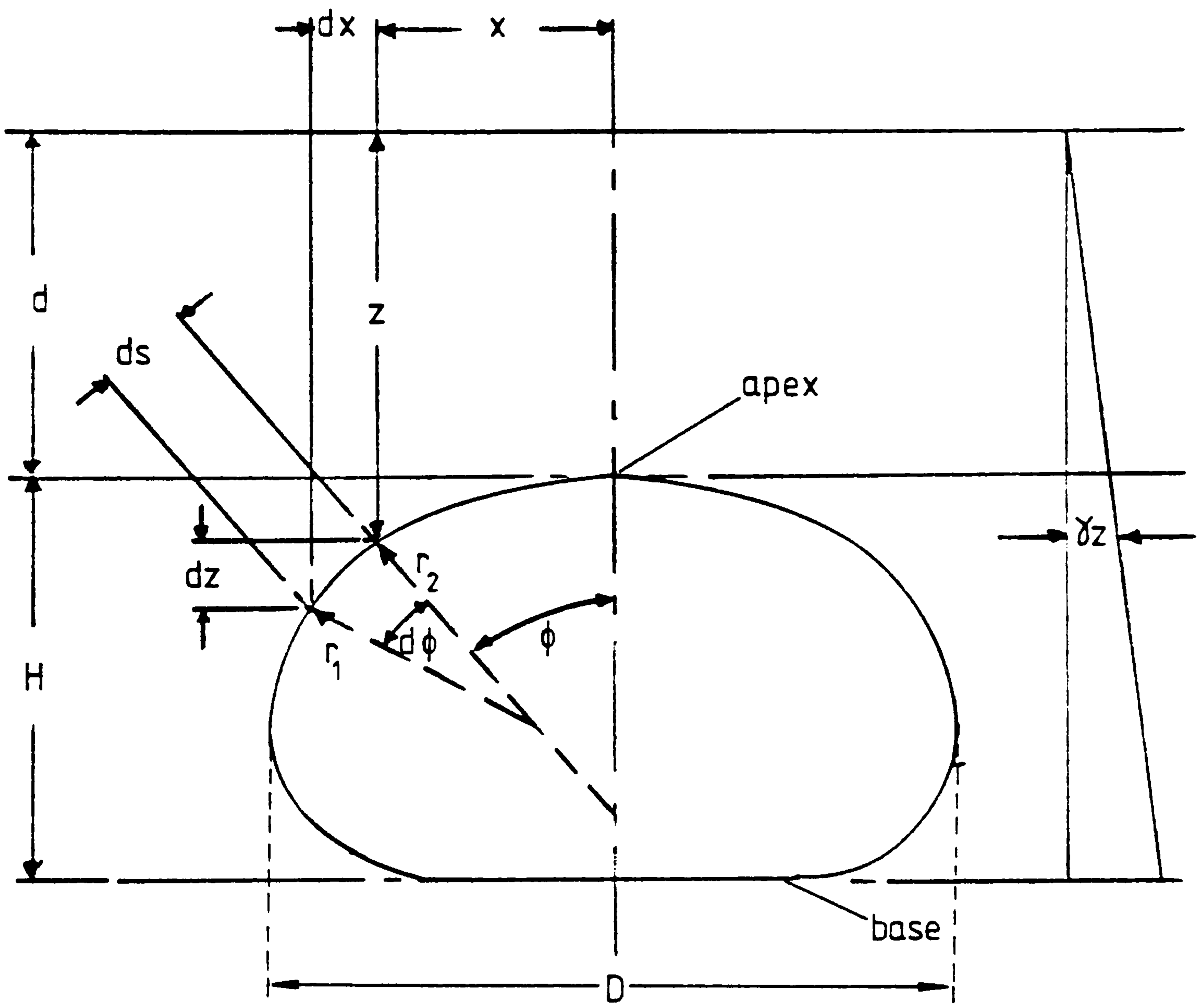

Fig. 1.1.1 the drop shape

CHAPTER TWO: ON THE DIFFERENTIAL EQUATIONS OF THE DROP SHAPED SHELL

CHAPTER TWO: ON THE DIFFERENTIAL EQUATIONS OF THE DROP SHAPED SHELL

Section (2.0) Introduction

An equilibrium equation in membrane analysis of shells of revolution is

$$\frac{N_\phi}{r_1} + \frac{N_\theta}{r_2} = - Z \qquad (2.0.1)$$

where, for a typical small element of the shell, N_ϕ, N_θ are the normal forces per unit length in directions tangential to the meridian and parallel circle; r_1, r_2 are radii of curvature in the meridional plane and in the normal plane perpendicular to meridian and Z is the radial load intensity, positive radially inwards[17]. By using equation (2.0.1), the ordinary differential equations system of the meridian of the drop shaped shell can be derived (see appendix A 2.0.1).

These equations are

$$\left.\begin{aligned} &\frac{d(\sin\phi)}{dx} + \frac{\sin\phi}{x} = \frac{\gamma z}{N} && (2.0.2a) \\ &\frac{dz}{dx} = \frac{\sin\phi}{\sqrt{(1-\sin^2\phi)}} && (2.0.2b) \\ &\text{with the initial condition} && \\ &z = d, \quad \sin\phi = 0 \quad \text{when} \quad x = 0. && (2.0.2c) \end{aligned}\right\} \qquad (2.0.2)$$

Before discussing the nature and some properties of this system it is beneficial to state another form of the equations.

By differentiating (2.0.2a) with respect to the independent variable, x, and substituting (2.0.2b) in the resulting equation,

$$\frac{d^2(\sin\phi)}{dx^2} + \frac{1}{x}\frac{d(\sin\phi)}{dx} - \sin\phi\left[\frac{1}{x^2} + \frac{\gamma}{N}\cdot\frac{1}{\sqrt{(1-\sin^2\phi)}}\right] = 0 \qquad (2.0.3a)$$

is obtained.

This equation with its initial condition

$$z = d,\ \sin\phi = 0 \text{ and } \frac{d(\sin\phi)}{dx} = \frac{\gamma d}{N} \text{ when } x = 0 \qquad (2.0.3b)$$

is the equivalent second order form of (2.0.2). Having stated the differential equations system of the drop shaped shell, some properties of the system are discussed in the next section.

Section (2.1) Some Properties of the Differential Equations System

In this section, some properties of the differential equations system of the drop shaped shell are stated and discussed. First, it is useful to note that the system (2.0.2 and/or 2.0.3) is non-linear and does not have any known analytical or closed form of solution. Due to this, various methods are to be tried out in a quest for its solution. It is noticed that the initial condition (2.0.2c) of (2.0.2) renders the $\frac{\sin \phi}{x}$ term of (2.0.2a) indeterminate. To overcome this difficulty a new initial condition

$$z = d, \quad \sin \phi = u_o \neq 0 \quad \text{when } x = DX1 \neq 0 \tag{2.1.1}$$

close to the old initial condition is considered. The way this is derived is explained in section (2.3). The corresponding new initial condition of (2.0.3) can be derived similarly. From the statement of the system, it is observed that the known condition is of the initial nature. Therefore, the problem of solving this system, i.e. shape generation, constitutes an initial value problem rather than a boundary value or eigenvalue problem. One then tries to ascertain whether the classical existence and uniqueness theorem of the general theory of ordinary differential equations is satisfied by the system. By this theorem, one is saying that the first-order ordinary differential equation

$$\frac{dy}{dx} = F(x,y), \quad y(x_0) = y_0 \tag{2.1.2}$$

has a unique continuous differentiable solution $y(x)$ on

$x_0 \leq x \leq b$ satisfying (2.1.2) if

1) $F(x,y)$ is defined and continuous in the strip $x_0 \leq x \leq b$, $-\infty < y < \infty$ with x_0 and b finite and
2) there exists a constant L such that for any x in $x_0 \leq x \leq b$ and any two numbers y and z

$$|F(x,y) - F(x,z)| \leq L|y-z| \ .$$ (21)

One observes immediately for the system (2.0.2) that the above theorem is not satisfied, since for example, in the domain of its investigation, equation (2.0.2b) becomes singular when angle $\phi = 90^{\circ}$. This difficulty is overcome numerically by changing the independent variable of integration before the singular point is reached. The point of change is stipulated by a value assigned to angle ϕ. For different values of ϕ in the same problem, one will obtain different shapes of tank. This peculiarity requires further attention.

One should mention very briefly that there are also some physical limitations to the derived system of equations. For example, in employing membrane theory in its derivation, the effects of bending moments and radial shears are assumed small relative to the membrane forces and therefore are negligible. This is not always the case in reality where for an actual structure the conditions of membrane theory are violated. These violations do indicate a need for other analyses and experimentations.

Finally, it should be noticed that in deriving the system of equations, the acting load intensity is internal and thus radially outwards. In the present investigation when the tank is to be loaded from the outside, this load is radially inwards. The effect of this is a change in the direction and nature of the stresses in the tank - stresses becoming compressive. Ignoring this change in sign the problems involved with the method of shape generation for the shell of constant strength can be examined.

Section (2.2) Solving the Differential Equations System of the Drop Shaped Shell

Firstly in this section, the methods of solution proposed by some previous workers for solving the system of equations (2.0.2) are discussed. Following this, other new methods of solution considered in this work will be mentioned. These methods fall broadly speaking into the following categories: (a) Graphical, (b) Numerical, (c) Analogue Computing and (d) a combination of some of (a), (b) and (c). As the ultimate objective in this chapter is to compare the various available methods before deciding on one as the most suitable a number of possible methods are investigated.

Generally graphical methods tend to be slow and inaccurate. Numerical methods when implemented on the digital computer are usually fast and economical in most cases if the method in a particular circumstance is chosen judiciously. It is observed that in solving an initial value problem of this nature numerically an attempt is made to replace a continuous solution curve by discrete values at some specified points within the domain of interest. After deciding on a numerical method, one has to tackle the general and important questions of (a) convergence, i.e., do the computed values of the dependent variable tend to their "true" or "actual" values over a range of step-length values of the independent variable, as the step-length is reduced?,

(b) rate of convergence, i.e., how fast is (a) being achieved?, (c) stability, i.e., is the method applied to the problem such that when some of the problem parameters are perturbed slightly the corresponding solution is slightly perturbed and (d) errors, i.e., with what confidence can one state that the solution as obtained by using the method differs very little from the "actual" solution of the problem?

On the other hand, the simulation of an engineering and/or a mathematical problem on the analogue computer is quite popular.[22-27] This approach tends to replace effectively the slow graphical methods by a faster visual procedure. An advantage of this approach is in problems which require spontaneous parameter-value changes. Occasionally its disadvantage is in calibration of results. It is observed that among many other things, results from the analogue computer may be useful for the confirmation of results obtained by other means.

In some cases mathematical analysis (analytical and/or semi-analytical methods) can supply meaningful answers to some problems and be useful in evaluating a method. However, mathematical results for different methods may be incomplete or difficult to compare since a rigorous mathematical analysis usually requires some simplifying assumptions for any nontrivial problem. Suffice it to state here that such discussion will be pursued whenever necessary later in this work. Attention is directed now to methods of solution suggested by some previous workers.

Novozhilov[18] suggested a graphical method for solving the system of equations. This method is slow, tedious, cumbersome and does not lend itself to an electronic digital manipulation. This is a big drawback as it may be necessary to generate many shapes before choosing a suitable one in practice. It is difficult also to assess the accuracy of a shape drawn by this method. The method of solution suggested by Den Hartog[16] is also graphical. As accuracy is very important in this work and the labour of graphical plotting is overwhelming, graphical methods are not pursued. The numerical method suggested by Timoshenko and Woinowsky-Krieger[17] for integrating the system of equations is the explicit Euler method. This method is elementary and requires in most cases very small step-lengths before any reasonable and accurate results can be obtained. The method of solution put forward by Flugge[19] is the same as that of Timoshenko and Woinowsky-Krieger[17] It is noticed that except by Flugge, very little is said about approximate analytical solutions of the problem. It is worth exploring this area as it might cast some light on the solutions. So far, no one seems to have considered employing the analogue computer in the solution of this problem. This is worth investigating.

The above methods, namely, graphical and explicit Euler, are not very reliable and since from a practical viewpoint it is important to have available reliable and simple methods for solving the system of differential equations and thus be able to generate various shapes of the tank when necessary, some other methods of solution need to be considered. It should be noted also that through this process of looking for a reliable method of solution one may develop an insight into the computational and analytical processes involved in solving mathematical problems of this nature.

In this work, equation (2.0.2) rather than (2.0.3) is investigated as it is easier to examine a system of equations of a lower order rather than the equivalent single equation of a higher order. This approach is also in agreement with that of the previous mentioned workers. Finally one should remark that in this work emphasis will be placed more on numerical methods. This may be supplemented by analytical and/or other methods, no matter how crude these may be, as in the next section.

Section (2.3) Treatment of the Initial Condition

The way the original initial condition, equation (2.0.2c) of the system of equations (2.0.2) is dealt with is described here. As remarked earlier, this condition renders the $\frac{\sin \phi}{x}$ term of equation (2.0.2a) indeterminate and is therefore not suitable for any intended numerical exercise. However, by slightly perturbing this point, the difficulty is overcome. The important question is how much numerically is a slight perturbance?. Using different values for the perturbance, one ends up with different shapes for the same problem (see appendix A 2.3.1). In this section, what is attempted is to try and overcome this difficulty by establishing a bound on the first step-length, DX1, which the independent variable of integration can have. This fixes the "new" initial condition (i.e., $x = x(1) = DX1$, $z = z(1)$ and $u = u(1)$ where $u = \sin \phi$), which is used subsequently in the various numerical methods of shape generation. Two approaches are considered below for dealing with the initial condition.

Section (2.3.1) First approach

In order to solve the system of equations

$$\frac{du}{dx} + \frac{u}{x} = \frac{\gamma z}{N} \tag{2.3.1a}$$

$$\frac{dz}{dx} = \frac{u}{\sqrt{(1-u^2)}} \tag{2.3.1b}$$

with the initial condition

$$z = d, \quad u = 0 \quad \text{when} \quad x = 0 \tag{2.3.1c}$$

and where, $u = \sin \phi$,

the following expressions are used to compute the coordinates of the point next to the apex of the shell:

$x(1) = DX$ (2.3.2a) where DX is a step-length,

$$z(1) = d\left[1 + \frac{(DX)^2}{4A}\right] \tag{2.3.2b}$$

$$u(1) = d\left(\frac{DX}{2A}\right) \tag{2.3.2c}$$

where $A = \frac{N}{\gamma}$ (see Timoshenko and W-Kreiger[17]).

If for the same problem one considers different values $DX1$, $DX2 = \frac{DX1}{2}$ of the step-length DX, one obtains different z and u values which determine the point next to the apex as follows,

$$z_1(1) = d\left[1 + \frac{(DX1)^2}{4A}\right] \tag{2.3.3a}$$

$$u_1(1) = d\left(\frac{DX1}{2A}\right) \tag{2.3.3b}$$

$$z_2(1) = d\left[1 + \frac{(DX2)^2}{4A}\right] \tag{2.3.4a}$$

$$u_2(1) = d\left(\frac{DX2}{2A}\right) . \tag{2.3.4b}$$

If in particular one considers the obtained z-values, i.e., equations (2.3.3a) and (2.3.4a), the absolute-value of their difference is

$$|z_1(1) - z_2(1)| = \frac{d}{4}\left|\frac{(DX1)^2 - (DX2)^2}{A}\right| . \tag{2.3.5}$$

By considering various DX1 and consequently DX2 values one tries now to make this difference as small as practicable. Mathematically this is equivalent to choosing a small positive number ε, such that

$$\frac{d}{4}\left|\frac{(DX1)^2 - (DX2)^2}{A}\right| < \varepsilon \quad . \tag{2.3.6}$$

Once this is done DX = DX1 can be used to start off the numerical integration process.

Section (2.3.2) Second approach

Here, one considers again different step-length values say, DX1, DX2, DX3, etc. In this case, DX2 need not be $\frac{DX1}{2}$. Following Timoshenko and W-Krieger[17] the point next to the apex of the shell is evaluated. In particular the z-coordinate will have for these step-lengths the following values:

$$\left.\begin{aligned} z_1(1) &= d\left[1 + \frac{(DX1)^2}{4A}\right] , \quad (2.3.7a)\\ z_2(1) &= d\left[1 + \frac{(DX2)^2}{4A}\right] , \quad (2.3.7b)\\ z_3(1) &= d\left[1 + \frac{(DX3)^2}{4A}\right] . \quad (2.3.7c) \end{aligned}\right\} \tag{2.3.7}$$

Working backwards by re-evaluating corresponding values given by these points for the original starting point using the relation,

$$z(0) = z(1) - \left\{\left.\frac{dz}{dx}\right|_1\right\}(DX)$$

where $\left.\frac{dz}{dx}\right|_1$ is the computed value of $\frac{dz}{dx}$ at the point $z(1)$ (see equation (2.3.1b), one obtains

$$z_1(0) = z_1(1) - \left\{\frac{dz}{dx}\Big|_1\right\}(DX1) \quad , \tag{2.3.8a}$$

$$z_2(0) = z_2(1) - \left\{\frac{dz}{dx}\Big|_1\right\}(DX2) \quad , \tag{2.3.8b}$$

and

$$z_3(0) = z_3(1) - \left\{\frac{dz}{dx}\Big|_1\right\}(DX3) \quad . \tag{2.3.8c}$$

The value of the errors obtained by this process are evaluated and given by

$$ER_1 = (z_1(0) - d) \quad , \tag{2.3.9a}$$

$$ER_2 = (z_2(0) - d) \quad , \tag{2.3.9b}$$

and

$$ER_3 = (z_3(0) - d) \quad . \tag{2.3.9c}$$

The graph of the error, ER against the step-length, DX can be drawn as shown in Fig. 2.3.1. From this graph, a point A at which theoretically the error is zero is extrapolated. The value of the length of segment OA is then used as the first step-length, DX1, of the numerical integration.

It should be observed that some questions need to be answered in trying to utilise either of these approaches. Their answers should be interesting and may justify the approaches. In subsequent work approach 2 is employed. This is due to the fact that except by a trial and error procedure, the way to choose the value of ε in approach 1 is still to be established.

In the section that follows several numerical methods are used in evaluating the differential equations system of the drop shaped shell. It should be noted that the list of available methods is fairly long but for the sake of economy and time only five of these are considered. The reasons for choosing these methods are given and some results discussed.

Section (2.4) Numerical Methods

In this section the numerical methods used in integrating the ordinary differential equations of the drop shaped shell are described. These methods are:-

a) explicit Euler,

b) explicit improved or modified Euler,

c) implicit Euler,

d) Runge-Kutta, and

e) Adams-Bashforth predictor formula and Adams-Moulton corrector formula.

Explicit Euler is considered for replicative purposes. Explicit improved or modified Euler is a natural extension of explicit Euler. This method is referred to by some as improved Euler and by others as modified Euler. Once the form of the method is stated as below there will be no confusion. The reason for considering this method in this work is that the results given by it are usually an improvement on the original explicit Euler. The implicit Euler method is another variant of (a). It does not appear quite as popular maybe due to the fact that for a non-linear problem, like the problem of the drop shaped system, one ends up with a non-linear system of algebraic equations. The resulting system has to be solved by some other approximate methods. The Runge-Kutta method is considered here since its theoretical and practical advantages outweigh its disadvantages. It is also a method that is recommended by various authors possibly due to the ease with which it can be implemented on the computer. The Adams-Bashforth/Adams-Moulton method is one of the easiest in the class of multi-step methods. It is considered here so as to have some indication of how a problem of this nature will respond to a multi-step treatment. It is realised that in using a multi-step method a big drawback is that methods of this nature are not self starting. To overcome this difficulty, Runge-Kutta was used to start the process. The implementation of these methods on the available computer is worthy of discussion. For a general discussion of the application of numerical methods to ordinary differential equations the interested reader is referred to[28,29,30,31,32].

Fortran computer programs written for these methods are used in evaluating the coordinates of these tanks on one side of the axis of symmetry. This is achieved in three stages starting from the apex after the slight perturbance of the original initial conditions[9,17]. The stages are controlled by the values of the angle ϕ. In the first stage, ϕ varies from 0 to 45 degrees. In the second stage it varies from 45 to 135 degrees and for the third stage from 135 degrees up until when the tank-profile becomes horizontal.

Section (2.4.1) Explicit Euler Method

In solving the initial value problem

$$\frac{dy}{dx} = F(x,y) \;, \quad y(x_0) = y_0 \qquad (2.4.1)$$

explicit Euler method takes the form

$$y_{n+1} = y_n + hF(x_n,y_n) \,, \quad n = 0,1,2,3, \ldots \qquad (2.4.2)$$

where $h = x_{n+1} - x_n$ is the step-length of integration[33,34]. This method is the simplest amongst the class of methods belonging to a Taylor series expansion. It is referred to as a first order method because one takes only the constant term and the term containing the first power of h in its approximate Taylor series expansion. The omission of further expansion terms causes an error referred to as the truncation error of the method. For small h, i.e. $h < 1$, these neglected terms will be small compared with h^2 , the first neglected term and one can say that the truncation error per step is of order h^2 [33,35]. In addition to these errors there are rounding errors and accumulated errors which may affect the accuracy of the computed values as the process of integration goes on. The practical value of higher order methods based on Taylor series is limited as they do require successive differentiation of the function, $F(x,y)$ (see equation 2.4.1). This becomes too complicated for non-trivial problems. In some cases, the resulting series converges too slowly to be of practical value.

In Currie's work [36] explicit Euler method was used in generating the various considered shell shapes and the Fortran computer program written then is given in appendix A 2.4.1. An up-dated and modified form of this program is presented in appendix A 2.4.2. The result of a computational exercise using the program in appendix A 2.4.2 for a typical set of parameter values:

Design Head = 1000.0 mm,

Thickness = 4.0 mm,

Design stress = 0.15 MN/m^2,

Unit weight of fluid = 11.61 KN/m^3

and different step lengths, DX = DZ = (8.0, 4.0, 2.0, 1.0, 0.5, 0.25 mm) is given in Table 2.4.1. From this table it is observed that results obtained by this method for the typical set of parameter values are unreliable even when the step-lengths are small. More will be said in Section (2.7) about this. Consider next the explicit modified or improved Euler method.

x-values	z-values for step-length					
	8.0	4.0	2.0	1.0	0.5	0.25
8	0.0001	0.1550	0.2325	0.2713	0.2908	0.3005
16	0.6212	0.9332	1.0897	1.1680	1.2072	1.2268
24	1.8748	2.3496	2.5881	2.7076	2.7674	2.7974
32	3.7855	4.4327	4.7584	4.9218	5.0036	5.0446
40	6.3937	7.2281	7.6489	7.8602	7.9661	8.0191
48	9.7601	10.8037	11.3313	11.5965	11.7295	11.7961
56	13.9745	15.2599	15.9118	16.2400	16.4048	16.4873
64	19.1708	20.7483	21.5517	21.9571	22.1608	22.2629
72	25.5568	27.5088	28.5091	29.0155	29.2703	29.3982

(x and z values in mm)

Problem parameters:

Design head (z0) = d = 1000.0 mm

Thickness (T) = 4.0 mm

Design stress (DS) = 0.15 MN/m^2

Unit weight of fluid (G) = 11.61 KN/m^3

Step-lengths (DX, DZ) varying from 8.0 mm to 0.25 mm.

Computer program used is in appendix A 2.4.2.

TABLE 2.4.1

Coordinates of the first part of a shell obtained by explicit Euler method

Section (2.4.2) Explicit Improved or Modified Euler Method

This method which is referred to as improved Euler by some and modified Euler by others is an extension of the explicit Euler method. Its form for the initial value problem, equation (2.4.1) is

$$y_{n+1} = y_n + \frac{h}{2}\left[F(x_n,y_n)+F(x_{n+1},y_{n+1})\right], n = 0,1,2,3,... \quad (2.4.3)$$

where $h = x_{n+1} - x_n$ is the step-length. The local truncation error of the method for small $h < 1$ is h^3 (35). This is an improvement on that of the explicit Euler method. The computed value at each new point can be improved upon successively until satisfied by introducing into the scheme an iterative process. This approach is recommendable but time consuming. A Fortran computer program using this method in solving the differential equations of the drop shaped shell is given in appendix A 2.4.3. It is obtained by slightly modifying the previous program for explicit Euler (see appendix A 2.4.2). Using the same parameter values as before the computed values using the new program are obtained as in Table 2.4.2. From Table 2.4.2 it is observed that the results obtained by this method are in agreement and thus encouraging. This will be discussed further in Section (2.7). Next consider the implicit Euler method.

x-values	z-values for step-length					
	8.0	4.0	2.0	1.0	0.5	0.25
8	0.3106	0.3103	0.3102	0.3102	0.3102	0.3102
16	1.2482	1.2469	1.2466	1.2465	1.2465	1.2465
24	2.8313	2.8283	2.8276	2.8274	2.8273	2.8273
32	5.0930	5.0874	5.0860	5.0857	5.0856	5.0856
40	8.0849	8.0753	8.0729	8.0723	8.0722	8.0721
48	11.8834	11.8680	11.8641	11.8631	11.8629	11.8628
56	16.6024	16.5781	16.5720	16.5704	16.5700	16.5699
64	22.4170	22.3782	22.3684	22.3660	22.3653	22.3652
72	29.6123	29.5480	29.5317	29.5276	29.5266	29.5263

(x and z values in mm)

Problem parameters:

Design head (ZO) = α = 1000.0 mm

Thickness (T) = 4.0 mm

Design stress (DS) = 0.15 MN/m^2

Unit weight of fluid (G) = 11.61 KN/m^3

Step-lengths (DX,DZ) varying from 8.0 mm to 0.25 mm.

Computer program used is in Appendix A2.4.3.

TABLE 2.4.2

Coordinates of the first part of a shell obtained by explicit improved or modified Euler method

Section (2.4.3) Implicit Euler Method

The implicit Euler method is unpopular probably due to its implicit form. Using this method the differential equations system is transformed to an equivalent algebraic system. If the system of equations is non-linear, the resulting algebraic system is also non-linear. The solution of the non-linear algebraic system is then obtained using an approximate method, e.g., Newton-Raphson's.[37,38] To implement the method in this work requires little effort. For this reason, it is included in this work.

The form of the method for solving the initial value problem, equation (2.4.1) is simply

$$y_{n+1} = y_n + hF(x_{n+1}, y_{n+1}), \quad n = 0,1,2,3, \ldots \qquad (2.4.4)$$

where $h = x_{n+1} - x_n$ is the step-length. This form is similar to explicit Euler's. The difference is the implicit nature of the function $F(x,y)$ in this method rather than its explicit form in the other case. The Fortran computer program written for this method is given in appendix A 2.4.4 and the coordinate-values given by the method with the set of parameter-values that have been used with earlier methods are as in Table 2.4.3.

The results of the computation exercise given in Table 2.4.3. are unreliable like those obtained by explicit Euler method. The results rounded up to one place of decimal agree for step-lengths DX, DZ = (1.0, 0.5, 0.25 mm) and x-values lying between 8 and 24 mm (inclusive). One then observes that even for such small step-lengths, there is still a lot to be done. These results will be discussed further in Section (2.7) but for the moment attention is turned to the Runge-Kutta method.

x-values	z-values for step-length					
	8.0	4.0	2.0	1.0	0.5	0.25
8	0.6215	0.4657	0.3879	0.3490	0.3296	0.3199
16	1.8770	1.5610	1.4036	1.3250	1.2857	1.2661
24	3.7922	3.3081	3.0673	2.9472	2.8872	2.8573
32	6.4090	5.7442	5.4141	5.2496	5.1676	5.1265
40	9.7905	8.9261	8.4978	8.2847	8.1783	8.1252
48	14.0302	12.9381	12.3983	12.1300	11.9963	11.9295
56	19.2685	17.9057	17.2345	16.9014	16.7354	16.6526
64	25.7255	24.0235	23.1890	22.7758	22.5701	22.4675
72	33.7752	31.6136	30.5609	30.0414	29.7833	29.6546

(x and z values in mm)

Problem parameters:

Design head (zO) = d = 1000.0 mm

Thickness (T) = 4.0 mm

Design stress (DS) = 0.15 MN/m^2

Unit weight of fluid (G) = 11.61 KN/m^3

Step-lengths (DX, DZ) varying from 8.0 mm to 0.25 mm.

Computer program used is in appendix A 2.4.4.

TABLE 2.4.3

Coordinates of the first part of a shell obtained by implicit Euler method

Section (2.4.4) Runge-Kutta Method

This is a popular numerical method used in solving ordinary differential equations. It is a self-starting single step method and requires several function evaluations at each step. It is particularly advantageous when memory requirements are to be minimised. Various forms of the method do exist. The one considered here is a fourth order type with a local truncation error of order h^5 for small h . Its form for the initial value problem, equation (2.4.1) is

$$y_{n+1} = y_n + \frac{1}{6}(K_1+2K_2+2K_3+K_4), \quad n = 0,1,2, \ldots \qquad (2.4.5)$$

where

$$K_1 = hF(x_n, y_n)$$

$$K_2 = hF(x_n+\frac{h}{2}, y_n + \frac{K_1}{2})$$

$$K_3 = hF(x_n+\frac{h}{2}, y_n + \frac{K_2}{2})$$

$$K_4 = hF(x_n+h, y_n + K_3)$$

and $h = x_{n+1} - x_n$ is the step-length.[39]

The Fortran computer program written for this method is in appendix A 2.4.5 and the computational results for the problem having the same set of parameter-values as in previous methods of this section are in Table 2.4.4. From Table 2.4.4. it is seen that the solution-set obtained by this method agrees with that of explicit modified or improved Euler correct to three decimal places when the step-length is 1.0 mm. For smaller step-lengths the behaviour of the solution-set becomes doubtful. A shadow is therefore cast on the reliability of the Runge-Kutta method for the problem at hand by this.

Next one considers a multi-step method. The one considered utilises the Adams-Bashforth predictor formula and Adams-Moulton corrector formula and is explained below.

x-values	z-values for step-length					
	8.0	4.0	2.0	1.0	0.5	0.25
8	0.0599	0.2814	0.3044	0.3102	0.3118	0.3010
16	1.1340	1.2234	1.2403	1.2465	1.2480	1.2315
24	2.7409	2.8040	2.8205	2.8273	2.8287	2.8088
32	5.0056	5.0601	5.0778	5.0855	5.0871	5.0644
40	7.9892	8.0428	8.0631	8.0721	8.0739	8.0490
48	11.7687	11.8274	11.8519	11.8628	11.8651	11.8383
56	16.4550	16.5252	16.5562	16.5699	16.5732	16.5447
64	22.2145	22.3059	22.3471	22.3651	22.3701	22.3401
72	29.3137	29.4425	29.5098	29.5263	29.5343	29.5029

(x and z values in mm)

Problem parameters:

Design Head (ZO) = d = 1000.0 mm

Thickness (T) = 4.0 mm

Design stress (DS) = 0.15 MN/m^2

Unit weight of fluid (G) = 11.61 KN/m^3

Step lengths (DX,DZ) varying from 8.0 mm to 0.25 mm.

Computer program used is in Appendix A 2.4.5.

TABLE 2.4.4

Coordinates of the first part of a shell obtained by Runge-Kutta method

Section (2.4.5) Adams-Bashforth Predictor Formula and Adams-Moulton Corrector Formula Method

So far only one or single-step methods had been considered for solving the initial value problem, equation (2.4.1). Here, a multi-step method is considered. This method employs Adams-Bashforth predictor formula and Adams-Moulton corrector formula.

Multi-step methods are those that use previously obtained information about more than one point to generate a new point. Specifically, if y_1 at x_1, y_2 at x_2, ... , y_n at x_n are known, this information is then used to determine y_{n+1} at x_{n+1}. As these methods are not self-starting by their nature, a self-starting method having a local truncation error of the same order as the multi-step method must be used to start the procedure. For the particular method being considered here, Adams-Bashforth predictor formula is of the form,

$$y_{n+1} = y_n + \frac{h}{24}(55y'_n - 59y'_{n-1} + 37y'_{n-2} - 9y'_{n-3}) \qquad (2.4.6)$$

where $h = x_{n+1} - x_n$ is the step-length, $y'_n = \left.\frac{dy}{dx}\right|_n = F(x_n,y_n)$.

This formula has a local truncation error of order h^5.

Adams-Moulton corrector formula has the form

$$y_{n+1} = y_n + \frac{h}{24}(9y'_{n+1} + 19y'_n - 5y'_{n-1} + y'_{n-2}) \qquad (2.4.7)$$

with a local truncation error of order h^5 [40].

In order to generate the starting points for this method, the fourth order Runge-Kutta method with a local truncation error of order h^5 is used. After the starting points have been generated, the predictor formula, equation (2.4.6) is used to predict the next point. After the prediction, the corrector formula, equation (2.4.7) is used to correct this result. The corrector formula can be used iteratively until one is satisfied with the result obtained. In this work the latter approach is not adopted since the step-lengths are sufficiently small, rendering the iterative process unnecessary.

The Fortran computer program written for this method is listed in Appendix A 2.4.6, and Table 2.4.5 shows the computational results obtained by this method using the set of parameter-values of the problem considered for earlier methods of the section. From this Table it is noticed that the results obtained by this method are very poor and unreliable. This will be further considered in Section (2.7). This section is thus concluded. In this section, five computational methods had been used in evaluating the differential equations system of the drop shaped shell. The results of a computational exercise using the same set of parameter-values for each method are stated and slightly commented upon. These results will further be discussed in Section (2.7). In the next section, an attempt is made to investigate the differential equations of the shell qualitatively.

x-values	z-values for step-length					
	8.0	4.0	2.0	1.0	0.5	0.25
8	0.0599	0.2814	0.3138	0.3102	0.3049	0.2943
16	1.1340	1.2809	1.2730	1.2465	1.2383	1.2246
24	2.7409	2.9481	2.8678	2.8273	2.8174	2.8018
32	5.3044	5.2710	5.1365	5.0855	5.0748	5.0573
40	8.5570	8.3121	8.1320	8.0721	8.0597	8.0415
48	12.5816	12.1528	11.9309	11.8628	11.8493	11.8300
56	17.5222	16.9093	16.6461	16.5699	16.5554	16.5349
64	23.5644	22.7569	22.4500	22.3651	22.3495	22.3277
72	31.0119	29.9788	29.6212	29.5263	29.5093	29.4861

(x and z values in mm)

Problem parameters:

Design head (zO) = d = 1000.0 mm

Thickness (T) = 4.0 mm

Design stress (DS) = 0.15 MN/m^2

Unit weight of fluid (G) = 11.61 KN/m^3

Step-lengths (DX, DZ) varying from 8.0 mm to 0.25 mm.

Computer program used is in appendix A 2.4.6.

TABLE 2.4.5

Coordinates of the first part obtained

by Adams-Bashforth/Adams-Moulton method

Section (2.5) Qualitative Investigation of the Differential Equations System of the drop shaped shell

Consider for the moment a qualitative investigation of the differential equations system of the drop shaped shell. Then one is interested in the existence or otherwise of some important properties like, critical points, periodicity of the solution of the system without actually solving it. A very important reason for this type of analysis is that in most numerical investigations, some of the parameter-values that are used are approximations in reality. Due to this, one may be interested in the behaviour of the solution when these parameter-values are slightly perturbed.

For this investigation the form of the system of equations considered is

$$\frac{du}{dx} = -\frac{u}{x} + \varepsilon z \quad , \qquad (2.5.1a)$$

$$\frac{dz}{dx} = \frac{u}{\sqrt{(1-u^2)}} \quad , \qquad (2.5.1b)$$

with the initial condition

$$z = d, \; u = u_0 \neq 0 \quad \text{when} \quad x = x_0 = DX1 \neq 0 \; , \qquad (2.5.1c)$$

(2.5.1)

and where $u = \sin\phi$ and $\varepsilon = \frac{\gamma}{N}$.

This system of equations, i.e., equations (2.5.1) is non-autonomous since the right hand side of equation (2.5.1a) contains a term including the independent variable, x [41,42]. Consequently it is difficult to analyse the system in this form. Therefore the system is transformed into an autonomous one by letting $x = y$, where $\frac{dx}{dy} = 1$.

Now the system becomes,

$$\frac{du}{dy} = -\frac{u}{x} + \varepsilon z \quad , \tag{2.5.2a}$$

$$\frac{dz}{dy} = \frac{u}{\sqrt{(1-u^2)}} \quad , \tag{2.5.2b}$$

$$\frac{dx}{dy} = 1 \quad , \tag{2.5.2c}$$

with the initial condition

$$z = d,\ u = u_0 \neq 0 \quad \text{when} \quad y = x = x_0 = DX1 \neq 0. \tag{2.5.2d}$$

The right hand side of system (2.5.2) is independent of the new independent variable, y.

Unfortunately this new system does not lend itself to linearisation as it has an x factor, for example, in the denominator of equation (2.5.2a). As this approach does not appear to be yielding any fruitful results, an alternative one is considered.

This approach utilises the equations of the system in its original form, i.e., equations (2.5.1).

When u is numerically less than unity, $\frac{1}{\sqrt{(1-u^2)}}$ can be expanded binomially to give

$$\frac{dz}{dx} = u + \frac{1}{2}u^3 + \frac{3}{8}u^5 + \frac{5}{16}u^7 + \ldots \quad . \tag{2.5.3}$$

Then it is hoped that by taking enough terms of this expression, the system may be simplified and solved using this approximation. Towards this end, the Fortran computer programs namely BMP1, BMP2, BMP3, BMP4 and BMP5 given in appendices A 2.5.1 to A 2.5.5 are written. They are used to generate the first part of the shell using the parameter-values of previous section, with the exception that not all values of step-lengths used earlier are used here. These programs are such that they have a common $\frac{du}{dx}$ equation, i.e.,

$$\frac{du}{dx} = -\frac{u}{x} + \varepsilon z.$$

However, BMP1 utilises only the first term of the binomial expansion of equation (2.5.3), BMP2 the first two terms, BMP3 the first three terms, etc. Thus the following approximate problems are considered in the following programs:

for BMP1

$$\left.\begin{aligned} \frac{du}{dx} &= -\frac{u}{x} + \varepsilon z \\ \frac{dz}{dx} &= u \end{aligned}\right\}$$

For BMP2

$$\left.\begin{aligned} \frac{du}{dx} &= -\frac{u}{x} + \varepsilon z \\ \frac{dz}{dx} &= u + \frac{1}{2}u^3 \end{aligned}\right\}$$

For BMP3

$$\left.\begin{aligned} \frac{du}{dx} &= -\frac{u}{x} + \varepsilon z \\ \frac{dz}{dx} &= u + \frac{1}{2}u^3 + \frac{3}{8}u^5 \end{aligned}\right\}$$

For BMP4

$$\left.\begin{aligned} \frac{du}{dx} &= -\frac{u}{x} + \varepsilon z \\ \frac{dz}{dx} &= u + \frac{1}{2}u^3 + \frac{3}{8}u^5 + \frac{5}{16}u^7 \end{aligned}\right\}$$

and for BMP5

$$\left.\begin{aligned} \frac{du}{dx} &= -\frac{u}{x} + \varepsilon z \\ \frac{dz}{dx} &= u + \frac{1}{2}u^3 + \frac{3}{8}u^5 + \frac{5}{16}u^7 + \frac{35}{128}u^9 \end{aligned}\right\} .$$

The initial conditions used in all these cases are identical, namely, $z = d$, $u = u_0 \neq 0$ when $x = x_0 = DX1 \neq 0$.

The results for step-lengths 2.0, 1.0 and 0.5 mm and programs BMP1 to BMP5 are shown in Tables 2.5.1, 2.5.2, and 2.5.3. These are compared with the corresponding values given by the old approach, i.e., where the differential equations system solved is equation (2.5.1). [A slightly modified form of explicit improved Euler method is used for this purpose.]

It is evident from the obtained results that this approach, an approximation of an infinite series by a finite one is not very encouraging. Possibly many more terms of the expansion have to be considered before any useful results can be obtained. This approach is therefore abandoned. Another alternative is considered in the next section. This is the investigation of the differential equations system by using the analogue computer.

x(mm)	z (mm) Values (step-length = 2 mm) for					
	BMP1	BMP2	BMP3	BMP4	BMP5	Explicit Improved Euler
8	.3097	.3102	.3102	.3102	.3102	.3102
16	1.2389	1.2465	1.2465	1.2465	1.2465	1.2465
24	2.7885	2.8264	2.8275	2.8275	2.8275	2.8275
32	4.9599	5.0798	5.0856	5.0859	5.0859	5.0859
40	7.7552	8.0482	8.0704	8.0725	8.0728	8.0728
48	11.1769	11.7865	11.8531	11.8622	11.8636	11.8638
56	15.2282	16.3624	16.5315	16.5630	16.5697	16.5716
64	19.9129	21.8580	22.2383	22.3314	22.3569	22.3679
72	25.2353	28.3704	29.1502	29.3930	29.4778	29.5308

TABLE 2.5.1

Coordinates of the first part of shell - Binomial Expansion Approach

x(mm)	z (mm) Values (step-length = 1 mm) for					
	BMP1	BMP2	BMP3	BMP4	BMP5	Explicit Improved Euler
8	.3097	.3102	.3102	.3102	.3102	.3102
16	1.2389	1.2464	1.2465	1.2465	1.2465	1.2465
24	2.7886	2.8263	2.8273	2.8274	2.8274	2.8274
32	4.9600	5.0795	5.0853	5.0856	5.0856	5.0856
40	7.7553	8.0479	8.0700	8.0720	8.0723	8.0723
48	11.1771	11.7860	11.8523	11.8614	11.8628	11.8630
56	15.2285	16.3617	16.5304	16.5618	16.5684	16.5703
64	19.9133	21.8572	22.2368	22.3295	22.3550	22.3658
72	25.2358	28.3694	29.1481	29.3903	29.4748	29.5274

TABLE 2.5.2

Coordinates of the first part of shell - Binomial Expansion Approach

x (mm)	z (mm) Values (step-length = 0.5 mm) for					
	BMP1	BMP2	BMP3	BMP4	BMP5	Explicit Improved Euler
8	.3097	.3102	.3102	.3102	.3102	.3102
16	1.2389	1.2464	1.2465	1.2465	1.2465	1.2465
24	2.7886	2.8263	2.8273	2.8273	2.8273	2.8273
32	4.9600	5.0794	5.0852	5.0855	5.0856	5.0856
40	7.7554	8.0478	8.0698	8.0719	8.0721	8.0722
48	11.1771	11.7859	11.8522	11.8612	11.8626	11.8628
56	15.2286	16.3616	16.5302	16.5615	16.5681	16.5700
64	19.9134	21.8569	22.2364	22.3291	22.3545	22.3653
72	25.2359	28.3691	29.1476	29.3897	29.4740	29.5263

TABLE 2.5.3

Coordinates of the first part of shell - Binomial Expansion Approach

Section (2.6) Analogue Computer Investigation of the System of Differential Equations of the Drop Shaped Shell

In this section attempt is made in investigating the differential equations system of the drop shaped shell using an anologue computer. An analogue computer solves a mathematical problem by simulation. It operates by creating an electrical analogy of the problem under consideration. The analogue computer is applicable to a wide variety of problems among which is solving non-linear differential equations[(43-45)]. In some cases, the main limitation of this approach is the amount of computing elements available.

In using an analògue computer, a computer flow diagram of the problem that is to be solved must first be written. For a mathematical problem the diagram is such that each individual mathematical operation and the value of each term at inputs and outputs are derived from the mathematical form of the problem. This program is a circuit diagram which is unscaled initially. It is later scaled to represent the magnitude of the variables and parameters of the mathematical problem as a decimal of one machine unit (1 MU). The scaled computer diagram is used in patching the machine panel with the computer components arranged to agree with the problem. The computer variables are displayed using a digital voltmeter or an X-Y plotter or an oscilloscope or a multi-channel recorder.

The available analogue computer used in investigating this system is the Electronic Associated Limited (EAL) 380. It is a desk-top ten volt machine with a patch panel that is removable.

The differential equations system of the shell is first transformed into the new system:

$$\frac{dx}{d\phi} = \frac{Ax \cos \phi}{xz - A \sin \phi} , \qquad (2.6.1a)$$

$$\frac{dz}{d\phi} = \frac{Ax \sin \phi}{xz - A \sin \phi} , \qquad (2.6.1b)$$

with the initial condition

$$z = z(0) = d, \quad x = x(0) = DX1 \quad \text{when}$$

$$\phi = \sin^{-1} u(0) = \phi(0) \qquad (2.6.1c)$$

This system can be solved by normal analogue computer method procedure(46-48) as illustrated below with an example.

The example considered has the parameter-values: Design Head (ZO) = 1000.0 mm, Thickness (T) = 4.0 mm, Design Stress (DS) = 0.15 MN/m^2, Unit weight of Fluid (G) = 11.61 KN/m^3 with $A = (DS)(T)/G = 0.5167 \times 10^5$ mm^2 but for the purpose of simplicity, A is approximated to 0.5×10^5 mm^2.

Writing equations (2.6.1) in a form suitable for analogue computation, one has the equivalent integral form:

$$x = - \int_{\phi(0)}^{\phi} - \dot{x} \, d\phi + x(0) = - \int_{\phi(0)}^{\phi} \frac{-Ax \cos\phi}{xz - A\sin\phi} \, d\phi + x(0) , \qquad (2.6.2a)$$

$$z = - \int_{\phi(0)}^{\phi} - \dot{z} \, d\phi + z(0) = - \int_{\phi(0)}^{\phi} \frac{-Ax \sin\phi}{xz - A\sin\phi} \, d\phi + z(0) , \qquad (2.6.2b)$$

$$\phi = - \int_{\phi(0)}^{\phi} - \, d\phi + \phi(0) , \qquad (2.6.2c)$$

where the dot denotes differentiation with respect to ϕ .

This set of equations can be scaled using the estimated maximum values of the variables and parameters obtained from earlier digital outputs (Section 2.4) or by inspection. The scaling is done as below:

(a) Amplitude scaling:

problem variable	estimated maximum size	computer variable
ϕ	π	$\left[\frac{\phi}{\pi}\right]$
$\sin\phi$	1	$\left[\frac{\sin\phi}{1}\right]$
$\cos\phi$	1	$\left[\frac{\cos\phi}{1}\right]$
x	300	$\left[\frac{x}{300}\right]$
z	1175	$\left[\frac{z}{1175}\right]$
A	0.6×10^5	$\left[\frac{A}{0.6 \times 10^5}\right]$

(b) Time Scaling:

let $\tau = \beta\phi$ and set $\beta = 1$

(c) Amplitude and time scaled equations

$$\left[\frac{x}{300}\right] = -\int_{\phi(0)}^{\tau} \frac{-0.0236\left[\frac{x}{300}\right]\left[\frac{\cos\phi}{1}\right]}{0.1667\left[\frac{x}{300}\right]\left[\frac{z}{1175}\right]-0.0237\left[\frac{\sin\phi}{1}\right]}\,d\tau + \left[\frac{x(0)}{300}\right], \qquad (2.6.3a)$$

$$\left[\frac{z}{1175}\right] = -\int_{\phi(0)}^{\tau} \frac{-0.0060\left[\frac{x}{300}\right]\left[\frac{\sin\phi}{1}\right]}{0.1667\left[\frac{x}{300}\right]\left[\frac{z}{1175}\right]-0.0236\left[\frac{\sin\phi}{1}\right]}\,d\tau + \left[\frac{z(0)}{1175}\right], \qquad (2.6.3b)$$

$$\left[\frac{\phi}{\pi}\right] = -\int_{\phi(0)}^{\phi} -\left[\frac{1}{\pi}\right]\,d\tau + \left[\frac{\phi(0)}{\pi}\right]. \qquad (2.6.3c)$$

Using the amplitude and time scaled equations (i.e., (2.6.3)) the corresponding scaled computer flow diagram can be drawn as in Figure 2.6.1. With the scaled diagram it is possible to patch the computer panel. Unfortunately, when this was attempted on the available computer it was discovered that the available computer elements were insufficient. This approach had to be abandoned for this reason but it is noticed that optimistically, it should be possible to obtain some plots from a machine with more elements. The numerical methods that have been investigated in this work are now compared in the next section.

Section (2.7) Comparison of Numerical Methods

So far, the numerical methods used in the investigation of the differential equations system of the drop shaped shell (namely (a) explicit Euler, (b) explicit improved or modified Euler, (c) implicit Euler, (d) Runge-Kutta and (e) Adams-Bashforth/Adams-Moulton) have been moderately described. In this section, they are discussed further, compared and evaluated with their merits and demerits assessed in a reasonably definitive way. For this work EMAS (Edinburgh multi-access system) of the Edinburgh Regional Computing Centre[49,50] is used.

In general terms it is very difficult to say that one numerical method is better than another unless one of them is very "poor", e.g., the solution set of a method not converging when a reduction in step-length is being made. Explaining this further, it is known that by using different compilers for the same problem and methods, the effectiveness of these methods may be affected. Hence, the outcome of a computer investigation is dependent to some extent on the available hardware and software.

As there is no universal standardisation of the above, it is possible to obtain different effects and results for the same problem and methods depending on the prevailing circumstances.

It is also interesting to note that even when one finds a particular method more useful than another in a particular investigation, this situation may be reversed if the problem being considered is slightly altered, e.g., by introducing a discontinuity to the region of investigation or by changing some parameters of the problem, etc.

In this comparison, some of the factors to be considered are:

1) the preparation of input data;

2) the accuracy of the solutions;

3) the number of function, i.e. derivative evaluations; and

4) the time used in executing a problem.

The problems used for the comparison are those with parameter-values:

(i) Design head = 1000.0 mm, Thickness = 4.0 mm, Design stress = 0.15 MN/m^2, Unit weight of Fluid = 11.61 KN/m^3, step-lengths, DX, DZ varying from 8.00 mm to 0.25 mm, and

(ii) Design head = 1525.0 mm, Thickness = 2.5 mm, Design stress = 0.70 MN/m^2, Unit weight of Fluid = 9.81 KN/m^3 and step-lengths, DX, DZ varying from 8.00 mm to 0.25 mm.

The computational results obtained by using methods (a) to (e) in generating the coordinates of shells for parameter values in problem (i) are as in Tables 2.4.1 to 2.4.5. It is noticed that for these methods, no special starting procedures are needed except in (e) where (d) was employed as an initiator. It is also observed that for these methods the same input data is used, i.e., the data preparation is the same for all the methods. From Table 2.4.1, it is observed that the results for explicit Euler, using such small step-lengths, are very poor in terms of convergence. The situation is the same for implicit

Euler (Table 2.4.3), Adams-Bashforth/Adams-Moulton (Table 2.4.5), but slightly improved in Runge-Kutta (Table 2.4.4). In the case of explicit improved or modified Euler, the convergence is encouraging (Table 2.4.2). Therefore it can be stated that in terms of convergence of solution, the methods can be ranked in a decreasing order of effectiveness as follows:

(1) explicit improved or modified Euler,

(2) Runge-Kutta, and

(3) Adams-Bashforth/Adams-Moulton, implicit Euler, explicit Euler.

Next, the problem of accuracy is studied. The accuracy of any numerical method depends on the errors acquired during its usage. In the numerical solution of ordinary differential equations there are three types of possible errors. These are (1) round-off, (2) truncation, and (3) propagation or inherited.

The round-off error is dependent on the computer used. Since the methods utilised the same precision in their arithmetic - double precision - one expects that the machine used will affect them in the same way. The truncation error is not machine caused but dependent on the method. It is this type of error that one can investigate here to some extent. As for the inherited or propagated error, it is caused by the use of previously calculated points which are already erroneous due to round-off and truncation errors. This is a very difficult error to overcome and is quite complicated and random in nature as it depends on the type of computer used, the sequence in which the computations are carried out, the method of rounding-off, etc.[51]. One only hopes that this does not affect appreciably the solutions of the problems using the various methods.

Because of these problems one realises that it is very difficult to obtain an overall estimate of errors in a numerical investigation. Nonetheless an exercise involving the estimation of the local truncation error of problem (1) using explicit Euler method is carried out and reported in appendix A 2.7.1. Other methods can similarly be considered, although some are more complicating as special formulae may have to be used. Since for small step-length, h, the order of these methods is (a) first, (b) second, (c) first, (d) fourth and (e) fourth, it is expected that the truncation error in using (a) and (c) is of order h^2, for (b) it is of order h^3 and for (d) and (e) of order h^5. Therefore the contribution of the truncation errors in the obtained results will be highest with the low-order methods, e.g., (a) and (c), and least with the highest ones, i.e., (d) and (e). For this reason, the results obtained by explicit Euler and implicit Euler should be more contaminated for the same step-length by their truncation errors than the results of explicit improved or modified Euler, Runge-Kutta and Adams-Bashforth/Adams-Moulton methods.

Generally, it must be emphasised that the analysis of errors in a numerical work is very important even when the amount of work that may be required is enormous. The importance of the analysis is in the lack of confidence which one has in results that are unsupported by a knowledge of possible magnitude of errors. Therefore one must endeavour to carry out an error analysis whenever possible. Appendix A 2.7.1 gives a small outline of what should be done for a particular method and problem.

It should also be mentioned that all the methods used in this chapter are numerically stable[52]. For step-size adjustments which may be necessary in some problems methods (a) to (d) are easy to manipulate as they are one-step methods. Method (e) is quite difficult to treat in this case as special formulae are required.

Next the number of function (derivative) evaluations and time of run for each method using the same input data are compiled and given in Tables 2.7.1 and 2.7.2. On the multi-access system used it is very difficult to state exactly a time of run for a particular problem and method as this is greatly affected by several factors, e.g. number of users. Luckily in the cases reported the number of users for the methods remain the same throughout the runs.

From Tables 2.7.1 and 2.7.2, it is noticed that the number of function evaluations for method (a) is least whilst for method (c) greatest. The times of runs of method (a) is also least whilst that of (c) is

Method	Total function Evalulations	Total CPU Time
a) explicit Euler	66	0.326
b) explicit improved/modified Euler	126	0.335
c) implicit Euler	612	0.520
d) Runge-Kutta	264	0.430
e) Adams-Bashforth/Adams-Moulton	276	0.442

TABLE 2.7.1

Total function evaluations and time for problem (i) where Design Head = 1000.0 mm, Thickness = 4.0 mm, Design Stress = 0.15 MN/m^2, Unit weight of Fluid = 11.61 KN/m^3, Step-lengths DX = DZ = 8.000 mm.

Method	Total function Evaluations	Total CPU Time
a) explicit Euler	136	0.451
b) explicit improved/modified Euler	266	0.528
c) implicit Euler	1056	0.778
d) Runge-Kutta	544	0.694
e) Adams-Bashforth/Adams-Moulton	580	0.725

TABLE 2.7.2

Total function evaluations and time for problem (ii) where Design Head = 1525.0 mm, Thickness = 2.5mm, Design Stress = 0.70 MN/m^2, Unit weight of Fluid = 9.81 KN/m^3, Step-lengths DX = DZ = 8.000 mm.

greatest. Recalling the accuracy of these methods, it will appear like methods (a), (c) and (e) are to be discarded as they are quite unreliable in the present circumstance (see also Section 2.4). One is now left with explicit improved or modified Euler method and Runge-Kutta method (refer to Tables 2.4.2, 2.7.3 for explicit improved or modified Euler and Tables 2.4.4, 2.7.4 for Runge-Kutta). In the case of Runge-Kutta it is very difficult to see a pattern indicating a convergence of the solution set, whereas in explicit modified or improved Euler this is noticeable easily. The total number of function evaluations and time also lend more weight to the reliability of explicit improved or modified Euler. With these in mind one is made to recommend this method as the most suitable or reliable for investigating the solution of the system of equations given the meridian of the drop shaped shell in the present circumstance.

x-values	z- values for step-length 8.0	4.0	2.0	1.0	0.5	0.25
8	0.1369	0.1369	0.1369	0.1369	0.1369	0.1369
16	0.5480	0.5479	0.5479	0.5479	0.5479	0.5479
24	1.2349	1.2347	1.2346	1.2346	1.2346	1.2346
32	2.2003	2.1999	2.1998	2.1998	2.1997	2.1997
40	3.4479	3.4472	3.4471	3.4470	3.4470	3.4470
48	4.9826	4.9817	4.9815	4.9814	4.9814	4.9814
56	6.8109	6.8096	6.8093	6.8092	6.8092	6.8092
64	8.9404	8.9387	8.9382	8.9381	8.9381	8.9381
72	11.3806	11.3783	11.3777	11.3775	11.3775	11.3775
80	14.1426	14.1396	14.1389	14.1387	14.1386	14.1386
88	17.2398	17.2361	17.2351	17.2349	17.2348	17.2348
96	20.6882	20.6836	20.6824	20.6821	20.6820	20.6820
104	24.5068	24.5010	24.4995	24.4992	24.4991	24.4991
112	28.7182	28.7110	28.7092	28.7088	28.7087	28.7086
120	33.3496	33.3408	33.3386	33.3380	33.3379	33.3379
128	38.4341	38.4232	38.4205	38.4198	38.4197	38.4196
136	44.0120	43.9986	43.9952	43.9944	43.9942	43.9941
144	50.1334	50.1169	50.1127	50.1117	50.1114	50.1113
152	56.8618	56.8411	56.8359	56.8346	56.8343	56.8342
160	64.2789	64.2527	64.2461	64.2444	64.2440	64.2439

(x and z values in mm)

TABLE 2.7.3

Coordinates of first part of shell obtained by explicit improved or modified Euler method for problem (ii) with parameters

Design Head = 1525.0 mm,

Thickness = 2.50 mm,

Design Stress = 0.70 MN/m^2,

Unit weight of Fluid = 9.81 KN/m^3,

Step-lengths (DX,DZ) varying from 8.0 mm to 0.25 mm.

x-values	z-values for step-length 8.0	4.0	2.0	1.0	0.5	0.25
8	0.0265	0.1241	0.1343	0.1368	0.1376	0.1328
16	0.4976	0.5378	0.5452	0.5479	0.5485	0.5413
24	1.1962	1.2247	1.2317	1.2346	1.2352	1.2265
32	2.1653	2.1894	2.1966	2.1997	2.2003	2.1905
40	3.4137	3.4360	3.4437	3.4470	3.4475	3.4369
48	4.9476	4.9695	4.9778	4.9814	4.9819	4.9705
56	6.7737	6.7963	6.8052	6.8092	6.8098	6.7977
64	8.9001	8.9239	8.9338	8.9381	8.9388	8.9262
72	11.3361	11.3618	11.3728	11.3775	11.3783	11.3653
80	14.0929	14.1210	14.1333	14.1386	14.1396	14.1261
88	17.1837	17.2150	17.2289	17.2348	17.2360	17.2222
96	20.6243	20.6595	20.6753	20.6820	20.6835	20.6694
104	24.4334	24.4734	24.4914	24.4991	24.5009	24.4866
112	28.6332	28.6791	28.6997	28.7086	28.7110	28.6964
120	33.2506	33.3036	33.3275	33.3379	33.3408	33.3261
128	38.3178	38.3795	38.4075	38.4196	38.4234	38.4086
136	43.8744	43.9469	43.9797	43.9941	43.9989	43.9841
144	49.9694	50.0552	50.0941	50.1113	50.1173	50.1026
152	56.6642	56.7667	56.8134	56.8341	56.8418	56.8273
160	64.0380	64.1620	64.2185	64.2439	64.2536	64.2396

(x and z values in mm)

TABLE 2.7.4

Coordinates of first part of shell obtained by Runge-Kutta method for problem (ii) with parameters

Design Head = 1525.0 mm,

Thickness = 2.5 mm,

Design Stress = 0.70 MN/m^2,

Unit weight of Fluid = 9.81 KN/m^3,

Step-lengths (DX, DZ) varying from 8.0 mm to 0.25 mm.

Section (2.8) Summary and Conclusions

In this chapter the ordinary differential equations system giving the meridional shape of the drop shaped tank are stated. Some of the properties of the equations are mentioned and some possible methods of solution discussed. For the system of equations, it appeared like no closed form (or analytic) solution exists and the realisation of its solution using an analogue computer may only be possible on a large machine. The numerical methods considered are compared and a "most" reliable method suggested, thus settling the problem of shape prediction regarding the drop shaped tank.

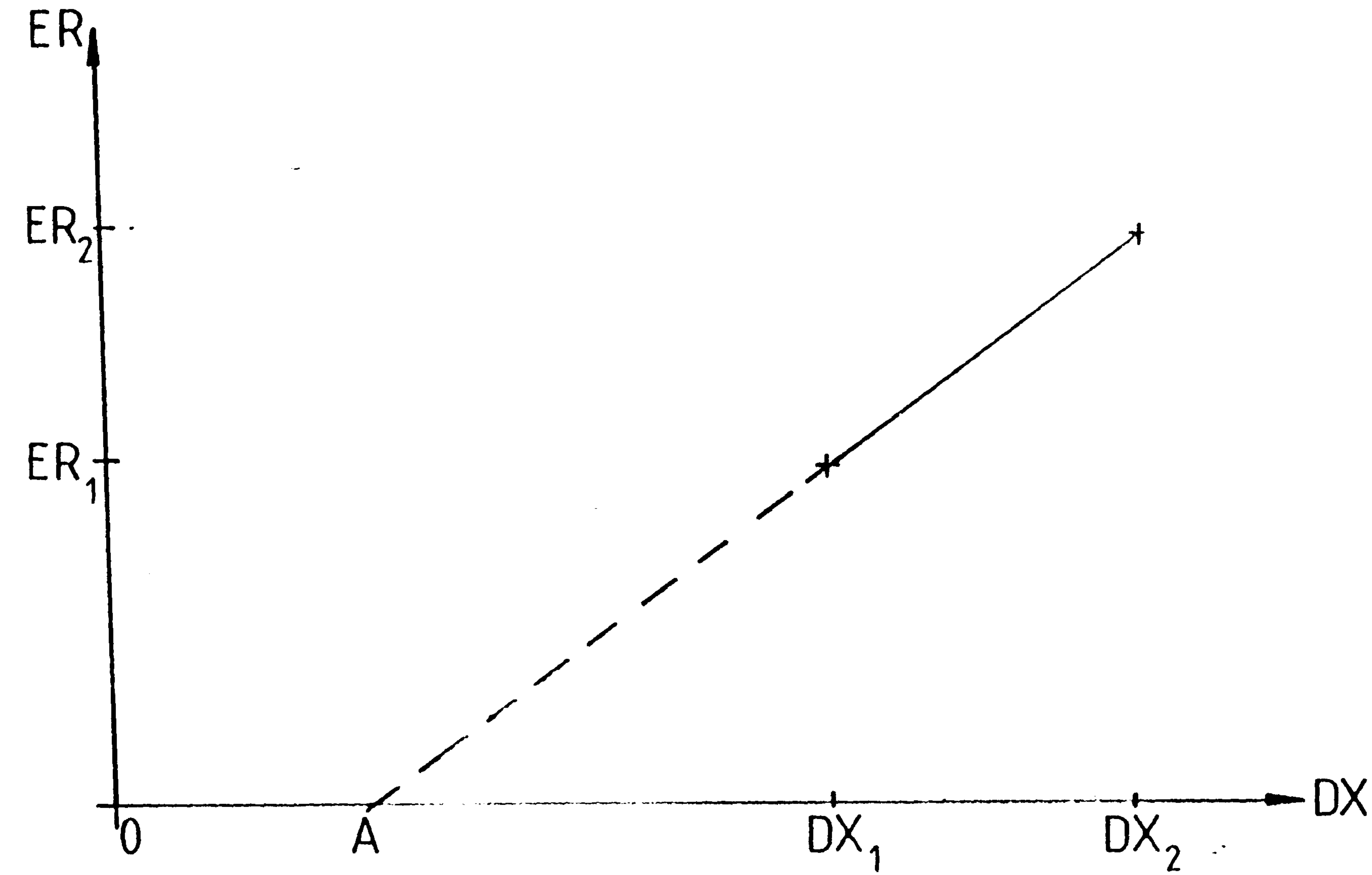

Fig. 2.3.1 An illustration showing the graph of error v. step length

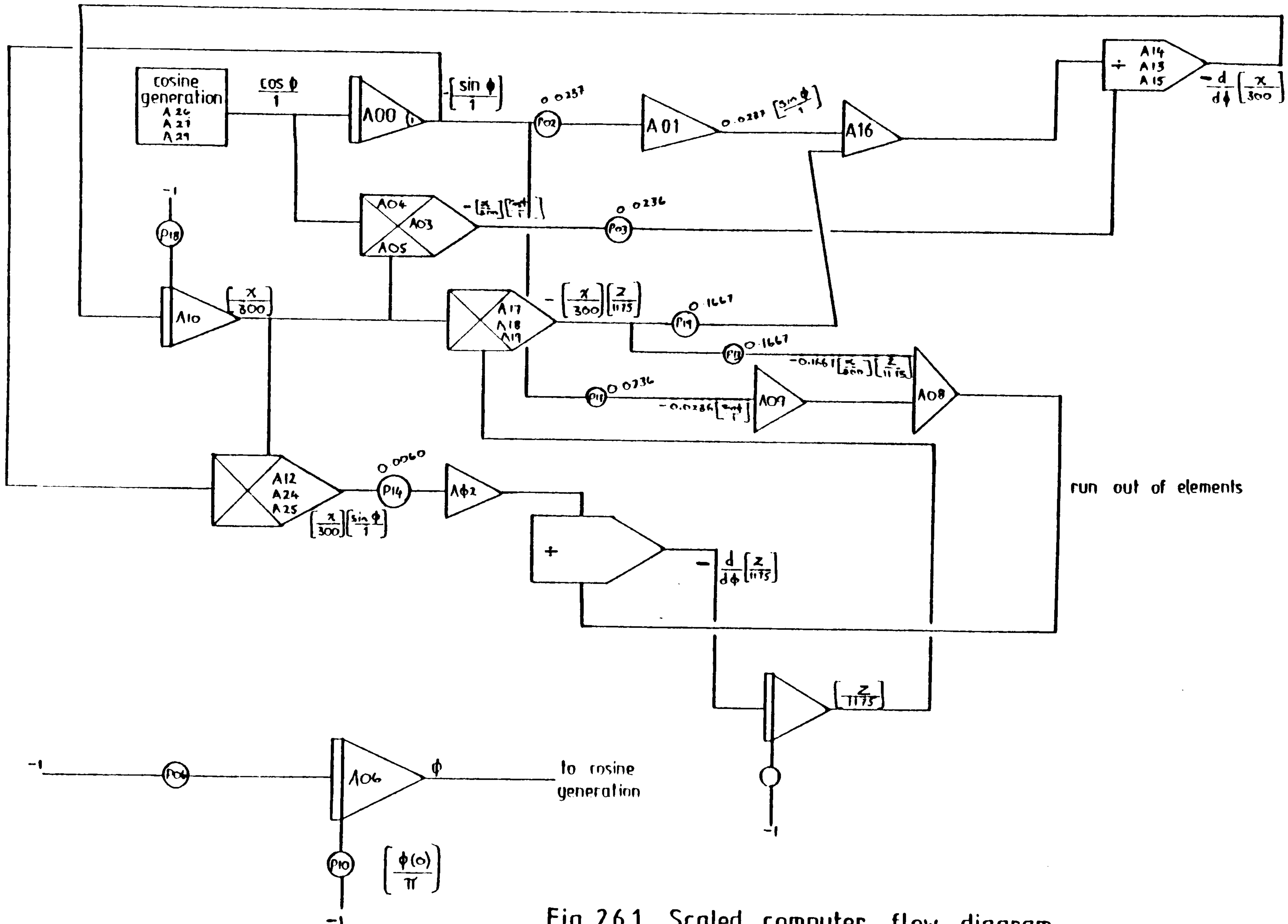

Fig. 2.6.1 Scaled computer flow diagram

CHAPTER THREE: ON DESIGN CURVES

CHAPTER THREE: On Design Curves

Section (3.0): Introduction

In the last chapter, the system of ordinary differential equations and the initial condition of the drop shaped shell were stated. Subsequently the proposed methods of solution for this system suggested by various workers were discussed. Thereafter, some other methods of solution were stated and considered by the author. In particular, the numerical methods investigated were compared. As a result, explicit improved or modified Euler method was considered to be the most reliable for solving this system under the present circumstance.

In this chapter, this method is used for generating some design curves in the manner of Currie[36]. The design curves are necessary from a practical viewpoint as it is useful to have available an easy source of reference from which a person interested in the drop shaped shell can make an appropriate choice. By following the procedure which is explained below a person is saved an enormous labour, time and cost which will be expended if many possible shapes have to be generated before a final selection is made.

It is appreciated now that the computer program used by Currie[36] contained a few errors. They have been corrected and it is therefore necessary to update the work.

Another relevant reason for the revision of work done by [36] is that the method of integration used earlier (explicit Euler) had been shown to be unreliable in the last chapter. This is made more obvious in section (3.2) where the step-lengths used in some of the previous exercises are continuously halved. This is done in order to ascertain the convergence of the solutions obtained by this method.

As will be shown, the results obtained do not support the use of explicit Euler method. When explicit improved or modified Euler is used instead for the same problems, the outcome of the exercise is favourable.

What then are design curves and how are they used in practice? The answer to this is given in the next section.

Section (3.1) Design Curves and their Purpose

The design curves provide a designer with a systematic procedure by which certain alternative choices are eliminated before arriving at an accepted scheme satisfying a specific set of parameter values. This procedure, if adopted, should save labour, time and cost which will be expended if the appropriate design has to be calculated from first principles or by some other lengthy process.

If the drop shaped shell is to be used for submarine storage or habitation some parameters, namely, the depth at which shell will operate, the volume it will enclose, and the unit weight of surrounding medium will be known. Others, namely, the material of construction, thickness and design stress of shell will be unknown. For a specific case, it may be possible to have more than one design. Through the design curves it should be possible to obtain these parameter values without calculating each and every one of them.

To generate these curves after deciding on a particular material, various values of parameters such as design head, shell thickness, shell design stress and surrounding medium's unit weight are used in computer-runs of the Fortran program used in shape generation. The results from these runs give values of diameter, height, volume, etc. for the inputs. These are then compiled and used in drawing the following graphs:

Graph 1: volume against operating depth,

Graph 2: volume against design head,

Graph 3: maximum diameter against design head,

Graph 4: shell height/maximum diameter against design head,

Graph 5: volume of material against design head.

The above graphs are grouped together according to the shell thickness which depends among other things on the environment in which the shell is to operate. For a particular design under consideration its operating depth and volume will be known. Then one goes to Graph 1 of the first thickness group to verify the compatibility of the known parameters with this thickness value. If there is no compatibility one moves to Graph 1 of the next thickness group. This is continued until a minimum thickness value which is compatible with the known parameters is reached. From the appropriate Graph 1, the possible design stress for the shell is obtained. In this same thickness group, Graph 2 is employed in obtaining the design head of the shell. The shell's diameter is obtained from Graph 3 and Graph 4 can be used as a check for the design head obtained earlier. Finally, Graph 5 gives the volume of material required in constructing the shell and so the relative costs of the design can be estimated. This process can then be repeated for other thicknesses after which the final choice based on various constraints, e.g. cost, availability of construction materials and facilities is made by the designer.

Having described the design curves and how to use them a convergence exercise is considered in the next section. This exercise illustrates the inappropriateness of explicit Euler method as used previously[36] and establishes the appropriateness of explicit improved or modified Euler method that will be used as an alternative.

Section (3.2): A Convergence Exercise

In this section a deeper look is taken at some of the computed results obtained by Currie[36] using the Fortran computer program ECHIDOM (refer to appendix A 2.4.1). It is recalled that the obtained values can be used in drawing the outline of some drop shaped shells whenever the need arises. The last chapter cast some doubts on the reliability of the numerical method of integration used in the past[36]. It is therefore necessary to settle the usage or not of this method in obtaining the design curve and consequently the shapes of the shells. In order to do this, the convergence exercise described below is carried out.

In this exercise, the step-lengths used previously are continuously reduced. The effect of this on the computed values are observed. Since it is known that for a reliable numerical method, the changes in the computed values should not vary much over a range of step-length values for which the solution set is convergent, it is hoped that this exercise will either support the usage of explicit Euler method or direct attention to another numerical method. The parameter values of Table (3.2.1) obtained from[36] are used in the exercise.

Employing the computer program of explicit Euler method (see appendix A 2.4.2) which is an updated form of ECHIDOM the results in Tables 3.2.2a to 3.2.2h are obtained for these parameter values. The same exercise is repeated using the computer program of explicit improved or modified Euler method (see appendix A 2.4.3), another numerical method of integration. The results obtained are listed in Tables 3.2.3a to 3.2.3h. From these tables the difference in absolute value between successive computed values of coordinates using the considered step-length values are evaluated. It is expected that for a more reliable method the evaluated differences will be small and smaller than those of a less reliable method. Typical graphs illustrating this are drawn as shown in Figures 3.2.1 and 3.2.2. From these Figures (and/or Tables 3.2.2a', 3.2.2h', 3.2.3a', 3.2.3h') it is observed that there is little or no agreement between the respective computed values for the considered step-lengths in the case of explicit Euler method used by Currie thereby casting doubt on the reliability of this method. In the case of explicit improved or modified Euler, there is an agreement between the respective computed values indicating the reliability of the method and its possible usage. Therefore it is applied to the establishment of design curves.

Design head (mm)	Thickness (mm)	Design stress (MN/m^2)	Unit weight of medium (KN/m^3)	Step-lengths (DX, DZ) (mm)
5000	100	35	9.81	200
5000	100	35	9.81	100
5000	100	35	9.81	50
5000	100	35	9.81	25
10000	100	35	9.81	100
10000	100	35	9.81	50
10000	100	35	9.81	25
60000	150	35	9.81	100
60000	150	35	9.81	50
60000	150	35	9.81	25
80000	150	35	9.81	100
80000	150	35	9.81	50
80000	150	35	9.81	25
100000	150	35	9.81	100
100000	150	35	9.81	50
100000	150	35	9.81	25
160000	150	35	9.81	100
160000	150	35	9.81	50
160000	150	35	9.81	25
180000	150	35	9.81	100
180000	150	35	9.81	50
180000	150	35	9.81	25
200000	150	35	9.81	100
200000	150	35	9.81	50
200000	150	35	9.81	25

TABLE 3.2.1

x values (mm)	z values (mm) for step-lengths (DX,DZ) = 200	100	50	25
0	5000.0000	5000.0000	5000.0000	5000.0000
5000	5084.4208	5086.2099	5087.1052	5087.5531
10000	5349.3588	5353.1791	5355.0928	5356.0505
15000	5809.7609	5816.1354	5819.3311	5820.9311
20000	6492.5698	6502.3757	6507.2958	6509.7602
25000	7439.9257	7454.5368	7461.8739	No values of x correspond to these exactly
30000	8715.3233	8736.8816	8747.7171	
35000	10416.0265	10448.0122	10464.1062	
40000	12701.2552	12749.8827	12774.3894	
45000	15870.6854	15949.3623	15989.1335	

TABLE 3.2.2a

x values (mm)	z values (mm) for step-lengths (DX,DZ) = 200	100	50	25
0	10000.0000	10000.0000	10000.0000	10000.0000
5000	10168.9931	10172.5779	10174.3719	10175.2694
10000	10701.4139	10709.1190	10712.9788	10714.9106
15000	11635.0097	11648.0481	11654.5861	11657.8597
20000	13043.2558	13063.8703	13074.2185	13079.4030
25000	15058.5059	15090.8780	15107.1513	No values of x correspond to these exactly
30000	17936.4104	17989.4822	18016.2239	

TABLE 3.2.2b

x values	z values (mm) for step-lengths (DX,DZ) =		
(mm)	100	50	25
0	60000.0000	60000.0000	60000.0000
5000	60702.4681	60709.8751	60713.5810
10000	63079.7877	63098.0641	63107.2184

TABLE 3.2.2c

x values	z values (mm) for step-lengths (DX,DZ) =		
(mm)	100	50	25
0	80000.0000	80000.0000	80000.0000
5000	80952.1702	80962.3941	80967.5110
7500	82285.4128	82303.0593	82311.8982

TABLE 3.2.2d

x values	z values (mm) for step-lengths (DX,DZ) =		
(mm)	100	50	25
0	100000.0000	100000.0000	100000.0000
2500	100284.2376	100290.2594	100293.2721
5000	101217.2667	101230.6798	101237.3957

TABLE 3.2.2e

x values (mm)	z values (mm) for step-lengths (DX,DZ) = 100	50	25
0	160000.0000	160000.0000	160000.0000
1200	160099.4108	160103.9680	160106.2479
2500	160464.9117	160474.9995	160480.0503

TABLE 3.2.2f

x values (mm)	z values (mm) for step-lengths (DX,DZ) = 100	50	25
0	180000.0000	180000.0000	180000.0000
1200	180112.0622	180117.2103	180119.7863
2500	180528.3299	180539.9247	180545.7321

TABLE 3.2.2g

x values (mm)	z values (mm) for step-lengths (DX,DZ) = 100	50	25
0	200000.0000	200000.0000	200000.0000
1000	200084.7633	200089.5144	200091.8917
2000	200367.8122	200377.8796	200382.9214
3000	200887.2910	200904.2390	200912.7378

TABLE 3.2.2h

x values (mm)	z values (mm) for step-lengths (DX,DZ) = 200	100	50	25
0	5000.0000	5000.0000	5000.0000	5000.0000
5000	5087.9998	5088.0008	5088.0010	5088.0011
10000	5357.0036	5357.0075	5357.0084	5357.0085
15000	5822.5204	5822.5296	5822.5318	5822.5324
20000	6512.2040	6512.2216	6512.2260	6512.2271
25000	7469.1915	7469.2222	7469.2298	No x-values correspond to these exactly
30000	8758.5234	8758.5738	8758.5864	
35000	10480.1608	10480.2416	10480.2617	
40000	12798.8546	12798.9834	12799.0156	
45000	16028.9199	16029.1292	16029.1815	

TABLE 3.2.3a

x values (mm)	z values (mm) for step-lengths (DX,DZ) = 200	100	50	25
0	10000.0000	10000.0000	10000.0000	10000.0000
5000	10176.1649	10176.1666	10176.1670	10176.1671
10000	10716.8343	10716.8413	10716.8430	10716.8434
15000	11661.1146	11661.1311	11661.1352	11661.1363
20000	13084.5511	13084.5836	13084.5917	13084.5937
25000	15123.4059	15123.4640	15123.4785	No x-values correspond to these exactly
30000	18042.9726	18043.0719	18043.0967	

TABLE 3.2.3b

x values (mm)	z values (mm) for step-lengths (DX,DZ) = 100	50	25
0	60000.0000	60000.0000	60000.0000
5000	60717.2929	60717.2897	60717.2888
10000	63116.4132	63116.3910	63116.3854

TABLE 3.2.3c

x values (mm)	z values (mm) for step-lengths (DX,DZ) = 100	50	25
0	80000.0000	80000.0000	80000.0000
5000	80972.6445	80972.6344	80972.6319
7500	82320.7907	82320.7584	82320.7503

TABLE 3.2.3d

x values (mm)	z values (mm) for step-lengths (DX,DZ) = 100	50	25
0	100000.0000	100000.0000	100000.0000
2500	100296.2920	100296.2875	100296.2863
5000	101244.1498	101244.1258	101244.1198

TABLE 3.2.3e

x values (mm)	z values (mm) for step-lengths (DX,DZ) =		
	100	50	25
0	160000.0000	160000.0000	160000.0000
1200	160108.5347	160108.5302	160108.5291
2500	160485.1360	160485.1132	160485.1075

TABLE 3.2.3f

x values (mm)	z values (mm) for step-lengths (DX,DZ) =		
	100	50	25
0	180000.0000	180000.0000	1 0000.0000
1200	180122.3722	180122.3657	180122.3641
2500	180551.5924	180551.5578	180551.5491

TABLE 3.2.3g

x values (mm)	z values (mm) for step-lengths (DX,DZ) =		
	100	50	25
0	200000.0000	200000.0000	200000.0000
1000	200094.2785	200094.2723	200094.2707
2000	200388.0071	200387.9782	200387.9710
3000	200921.3697	200921.2823	200921.2604

TABLE 3.2.3h

x value (mm)	$\lvert z_{200}-z_{100} \rvert$ (mm)	$\lvert z_{100}-z_{50} \rvert$ (mm)	$\lvert z_{50}-z_{25} \rvert$ (mm)
5000	1.8	0.9	0.4
10000	3.8	1.9	1.0
15000	6.4	3.2	1.6
20000	9.8	4.9	2.5
25000	14.6	7.3	No exact z values correspond to these x values
40000	48.6	24.5	
45000	78.7	39.8	

TABLE 3.2.2a'

Difference in absolute value of successive values of coordinates computed with the step-lengths considered in Table 3.2.2a (values correct to 1 decimal place) (also see Figure 3.2.1)

x value (mm)	$\lvert z_{200}-z_{100} \rvert$ (mm)	$\lvert z_{100}-z_{50} \rvert$ (mm)	$\lvert z_{50}-z_{25} \rvert$ (mm)
5000	0	0	0
10000	0	0	0
15000	0	0	0
20000	0	0	0
25000	0	0	No exact z values correspond to these x values
40000	0.1	0	
45000	0.2	0.1	

TABLE 3.2.3a'

Difference in absolute value of successive values of coordinates computed with the step-lengths considered in Table 3.2.3a (see also Figure 3.2.1)

x value (mm)	$\lvert z_{100}-z_{50} \rvert$ (mm)	$\lvert z_{50}-z_{25} \rvert$ (mm)
1000	4.8	2.4
2000	10.1	5.0
3000	16.9	8.5

TABLE 3.2.2h'

Difference in absolute value of successive values of coordinates computed with the step-lengths considered in Table 3.2.2h (values correct to 1 decimal place) (see Figure 3.2.2 also)

x value (mm)	$\lvert z_{100}-z_{50} \rvert$ (mm)	$\lvert z_{50}-z_{25} \rvert$ (mm)
1000	0	0
2000	0	0
3000	0.1	0

TABLE 3.2.3h'

Difference in absolute value of successive values of coordinates computed with the step-lengths considered in Table 3.2.3h (see Figure 3.2.2 also)

Section (3.3) Revision of Currie's Work

From the last section, it is observed that the integration of the system of equations of the drop shaped shell is more accurately carried out by using explicit improved or modified Euler instead of explicit Euler method. Since it is very useful and important to have a quick and accurate reference in any practical work involving the design of a drop shaped shell, some of the parameter-values used by Currie[36] are employed in revising his work. Instead of using explicit Euler method, explicit improved or modified Euler is used.

It is important to draw attention to the fact that this replication is not as elaborate as in the original work of Currie[36]. This is because it is believed that the important thing is to show the way some typical design curves are obtained. Once the general method of generation is understood, any more design curves can be obtained in the same manner.

For this establishment the following steps are taken:

(i) A suitable set of parameter values are chosen (see Tables 3.3.1a to 3.3.1c).

(ii) Using the computer program listed in appendix A 2.4.3 which employs explicit improved or modified Euler method for integration, tables for the set of parameter values of (i) are obtained (see Tables 3.3.2a to 3.3.4e).

(iii) Finally, from the tables in (ii) Graphs 1 to 5 of the design curves are drawn (refer to Section (3.1) and see Figures 3.3.1a to 3.3.3e).

Using Tables 3.3.1a to 3.3.1c and above computer program the design curves for thicknesses 50, 75 and 100 mm are drawn as in Figures 3.3.1a to 3.3.3e. These curves can be used in practice as described in Section (3.1). The rigours of plotting the coordinates generated by the computer program could be eased by an automatic form of graphical presentation initiated by the computer.

Design head (mm)	Thickness (mm)	Design stress (MN/m^2)	Unit weight of medium (KN/m^3)	Step-lengths (DX,DZ) (mm)
20000	50	20	9.81	100
40000	50	20	9.81	100
60000	50	20	9.81	100
80000	50	20	9.81	100
100000	50	20	9.81	100
120000	50	20	9.81	100
140000	50	20	9.81	100
160000	50	20	9.81	100
180000	50	20	9.81	100
200000	50	20	9.81	100
20000	50	30	9.81	100
40000	50	30	9.81	100
60000	50	30	9.81	100
80000	50	30	9.81	100
100000	50	30	9.81	100
120000	50	30	9.81	100
140000	50	30	9.81	100
160000	50	30	9.81	100
180000	50	30	9.81	100
200000	50	30	9.81	100
20000	50	40	9.81	100
40000	50	40	9.81	100
60000	50	40	9.81	100
80000	50	40	9.81	100
100000	50	40	9.81	100
120000	50	40	9.81	100
140000	50	40	9.81	100
160000	50	40	9.81	100
180000	50	40	9.81	100
200000	50	40	9.81	100

TABLE 3.3.1a

Shell thickness = 50.0 mm

Design head (mm)	Thickness (mm)	Design stress (MN/m^2)	Unit weight of medium (KN/m^3)	Step-lengths (DX,DZ) (mm)
20000	75	20	9.81	100
40000	75	20	9.81	100
60000	75	20	9.81	100
80000	75	20	9.81	100
100000	75	20	9.81	100
120000	75	20	9.81	100
140000	75	20	9.81	100
160000	75	20	9.81	100
180000	75	20	9.81	100
200000	75	20	9.81	100
20000	75	30	9.81	100
40000	75	30	9.81	100
60000	75	30	9.81	100
80000	75	30	9.81	100
100000	75	30	9.81	100
120000	75	30	9.81	100
140000	75	30	9.81	100
160000	75	30	9.81	100
180000	75	30	9.81	100
200000	75	30	9.81	100
20000	75	40	9.81	100
40000	75	40	9.81	100
60000	75	40	9.81	100
80000	75	40	9.81	100
100000	75	40	9.81	100
120000	75	40	9.81	100
140000	75	40	9.81	100
160000	75	40	9.81	100
180000	75	40	9.81	100
200000	75	40	9.81	100

TABLE 3.3.1b

Shell thickness = 75.0 mm

Design head (mm)	Thickness (mm)	Design stress (MN/m^2)	Unit weight of medium (KN/m^3)	Step-lengths (DX,DZ) (mm)
20000	100	20	9.81	100
40000	100	20	9.81	100
60000	100	20	9.81	100
80000	100	20	9.81	100
100000	100	20	9.81	100
120000	100	20	9.81	100
140000	100	20	9.81	100
160000	100	20	9.81	100
180000	100	20	9.81	100
200000	100	20	9.81	100
20000	100	30	9.81	100
40000	100	30	9.81	100
60000	100	30	9.81	100
80000	100	30	9.81	100
100000	100	30	9.81	100
120000	100	30	9.81	100
140000	100	30	9.81	100
160000	100	30	9.81	100
180000	100	30	9.81	100
200000	100	30	9.81	100
20000	100	40	9.81	100
40000	100	40	9.81	100
60000	100	40	9.81	100
80000	100	40	9.81	100
100000	100	40	9.81	100
120000	100	40	9.81	100
140000	100	40	9.81	100
160000	100	40	9.81	100
180000	100	40	9.81	100
200000	100	40	9.81	100

TABLE 3.3.1c

Shell thickness = 100.0 mm

Thickness = 50.0 mm
Design stress = 20.0 MN/m^2

Operating Depth (m)	Volume enclosed (m^3)
32.8	2345.3
48.4	440.5
66.1	144.7
84.8	63.2
103.9	32.8
123.3	19.0
142.8	12.0
162.5	8.0
182.2	5.6
202.0	4.0

Thickness = 50.0 mm
Design stress = 30.0 MN/m^2

Operating Depth (m)	Volume enclosed (m^3)
37.2	6551.8
51.9	1370.4
68.8	469.8
87.0	209.7
105.7	110.1
124.9	64.5
144.2	40.8
163.7	27.4
183.3	19.2
203.0	14.0

Thickness = 50.0 mm
Design stress = 40.0 MN/m^2

Operating Depth (m)	Volume enclosed (m^3)
41.0	13293.6
55.0	3013.7
71.4	1070.4
89.1	486.3
107.5	258.0
126.4	152.1
145.6	96.7
164.9	65.1
184.4	45.8
204.0	33.4

TABLE 3.3.2a

Thickness = 50.0 mm

Design head (m)	Volume enclosed (m^3) for Design Stress (MN/m^2) 20	30	40
20	2345.3	6551.8	13293.6
40	440.5	1370.4	3013.7
60	144.7	469.8	1070.4
80	63.2	209.7	486.3
100	32.8	110.1	258.0
120	19.0	64.5	152.1
140	12.0	40.8	96.7
160	8.0	27.4	65.1
180	5.6	19.2	45.8
200	4.0	14.0	33.4

TABLE 3.3.2b

Thickness = 50.0 mm

Design head (m)	Maximum Diameter (m) for Design Stress (MN/m^2) 20	30	40
20	18.0	25.9	33.3
40	9.8	14.5	19.0
60	6.7	9.9	13.1
80	5.0	7.5	10.0
100	4.0	6.1	8.0
120	3.4	5.1	6.7
140	2.9	4.3	5.8
160	2.5	3.8	5.1
180	2.3	3.4	4.5
200	2.0	3.0	4.1

TABLE 3.3.2c

Thickness = 50.0 mm

Design head (m)	Height/maximum Diameter for Design Stress (MN/m^2) 20	30	40
20	0.71	0.67	0.63
40	0.86	0.82	0.79
60	0.92	0.89	0.87
80	0.94	0.93	0.91
100	0.96	0.95	0.93
120	0.97	0.96	0.95
140	0.98	0.97	0.96
160	0.98	0.97	0.97
180	0.98	0.98	0.97
200	0.99	0.98	0.98

TABLE 3.3.2d

Thickness = 50.0 mm

Design head (m)	Volume of shell material (m^3) for design stress (MN/m^2) 20	30	40
20	40.4	79.2	125.9
40	13.7	29.0	48.5
60	6.7	14.5	24.8
80	3.9	8.5	14.9
100	2.5	5.6	9.8
120	1.8	3.9	6.9
140	1.3	2.9	5.1
160	1.0	2.2	4.0
180	0.8	1.8	3.2
200	0.6	1.4	2.6

TABLE 3.3.2e

Thickness = 75.0 mm
Design Stress = 20.0 MN/m^2

Operating depth (m)	Volume enclosed (m^3)
37.2	6532.0
51.9	1363.1
68.8	466.2
87.0	207.6
105.7	108.7
124.9	63.5
144.2	40.1
163.7	26.8
183.3	18.8
203.0	13.6

Thickness = 75.0 mm
Design Stress = 30.0 MN/m^2

Operating depth (m)	Volume enclosed (m^3)
42.8	17633.2
56.5	4127.8
72.6	1486.9
90.1	680.0
108.4	361.8
127.2	213.6
146.2	135.9
165.5	91.5
184.9	64.4
204.5	46.9

Thickness = 75.0 mm
Design Stress = 40.0 MN/m^2

Operating depth (m)	Volume enclosed (m^3)
47.6	34933.4
60.6	8885.5
76.1	3335.6
93.1	1559.6
110.9	840.9
129.4	500.6
148.2	320.4
167.3	216.7
186.5	153.0
205.9	111.9

TABLE 3.3.3a

Thickness = 75.0 mm

Design head (m)	Volume enclosed (m^3) for Design Stress (MN/m^2) 20	30	40
20	6532.0	17633.2	34933.4
40	1363.1	4127.8	8885.5
60	466.2	1486.9	3335.6
80	207.6	680.0	1559.6
100	108.7	361.8	840.9
120	63.5	213.6	500.6
140	40.1	135.9	320.4
160	26.8	91.5	216.7
180	18.8	64.4	153.0
200	13.6	46.9	111.9

TABLE 3.3.3b

Thickness = 75.0 mm

Design head (m)	Maximum Diameter (m) for Design Stress (MN/m^2) 20	30	40
20	25.9	36.8	47.0
40	14.5	21.2	27.7
60	9.9	14.7	19.4
80	7.5	11.2	14.8
100	6.1	9.0	12.0
120	5.1	7.6	10.1
140	4.3	6.5	8.6
160	3.8	5.7	7.6
180	3.4	5.1	6.8
200	3.0	4.6	6.1

TABLE 3.3.3c

Thickness = 75.0 mm

Design head (m)	Height/maximum Diameter for Design Stress (MN/m^2) 20	30	40
20	0.67	0.62	0.59
40	0.82	0.78	0.74
60	0.89	0.86	0.83
80	0.93	0.90	0.88
100	0.95	0.93	0.91
120	0.96	0.95	0.93
140	0.97	0.96	0.95
160	0.97	0.96	0.96
180	0.98	0.97	0.96
200	0.98	0.98	0.97

TABLE 3.3.3d

Thickness = 75.0 mm

Design head (m)	Volume of shell material (m^3) for Design Stress (MN/m^2) 20	30	40
20	118.7	227.3	356.1
40	43.4	89.8	148.3
60	21.7	46.4	78.9
80	12.8	27.9	48.2
100	8.4	18.5	32.2
120	5.9	13.1	23.0
140	4.4	9.8	17.1
160	3.4	7.5	13.3
180	2.7	6.0	10.6
200	2.2	4.9	8.6

TABLE 3.3.3e

Thickness = 100.0 mm
Design Stress = 20.0 MN/m^2

Operating depth (m)	Volume enclosed (m^3)
41.0	13230.6
55.0	2989.4
71.4	1058.0
89.1	478.9
107.5	253.1
126.4	148.6
145.6	94.2
164.9	63.1
184.4	44.3
204.0	32.2

Thickness = 100.0 mm
Design Stress = 30.0 MN/m^2

Operating depth (m)	Volume enclosed (m^3)
47.6	34874.1
60.6	8860.8
76.1	3322.5
93.1	1551.6
110.9	835.5
129.4	496.8
148.2	317.6
167.3	214.5
186.5	151.3
205.9	110.4

Thickness = 100.0 mm
Design Stress = 40.0 MN/m^2

Operating depth (m)	Volume enclosed (m^3)
53.2	67978.4
65.7	18762.6
80.4	7357.7
96.8	3524.8
114.2	1927.7
132.2	1157.9
150.7	745.6
169.5	506.3
188.6	358.6
207.8	262.7

TABLE 3.3.4a

Thickness = 100.0 mm

Design head (mm)	Volume enclosed (m^3) for Design Stress (MN/m^2) 20	30	40
20	13230.6	34874.1	67978.4
40	2989.4	8860.8	18762.6
60	1058.0	3322.5	7357.7
80	478.9	1551.6	3524.8
100	253.1	835.5	1927.7
120	148.6	496.8	1157.9
140	94.2	317.6	745.6
160	63.1	214.5	506.3
180	44.3	151.3	358.6
200	32.2	110.4	262.7

TABLE 3.3.4b

Thickness = 100.0 mm

Design head (mm)	Maximum Diameter (m) for Design Stress (MN/m^2) 20	30	40
20	33.3	47.0	59.7
40	19.0	27.7	36.0
60	13.1	19.4	25.5
80	10.0	14.9	19.6
100	8.0	12.0	15.9
120	6.7	10.1	13.3
140	5.8	8.6	11.5
160	5.1	7.6	10.1
180	4.5	6.8	9.0
200	4.1	6.1	8.1

TABLE 3.3.4c

Thickness = 100.0 mm

Design head (mm)	Height/maximum Diameter for Design Stress (MN/m^2) 20	30	40
20	0.63	0.59	0.56
40	0.79	0.74	0.71
60	0.87	0.83	0.80
80	0.91	0.88	0.86
100	0.93	0.91	0.89
120	0.95	0.93	0.92
140	0.96	0.95	0.93
160	0.97	0.96	0.95
180	0.97	0.96	0.95
200	0.98	0.97	0.96

TABLE 3.3.4d

Thickness = 100.0 mm

Design head (mm)	Volume of shell material (m^3) for Design Stress (MN/m^2) 20	30	40
20	251.8	474.8	736.8
40	97.1	197.7	322.9
60	49.7	105.2	177.2
80	29.7	64.2	109.9
100	19.6	42.9	74.3
120	13.9	30.6	53.3
140	10.3	22.8	40.0
160	8.0	17.7	31.1
180	6.3	14.1	24.8
200	5.1	11.5	20.2

TABLE 3.3.4e

Section (3.4) On Graph Plottings

In this section, the work on design curves is extended to include automatic graph plottings. By so doing, the rigour of having to plot the coordinates of points generated by the computer program manually is minimized. This new step entails employing the graph plotting facility of the computing installation in drawing some typical curves that show the relative sizes, and the variation in the sizes of the drop shaped shell when some of the parameters used in generating its shape are varied. For illustration, only one of the parameters namely the design head is varied. The effect of varying more than one parameter is the same and is not considered here.

It is known that if other parameters remain unchanged the size of the drop shaped shell decreases when its design head increases(53). By employing a slightly modified version of the computer program of explicit modified or improved Euler method (refer to appendix A 3.4.1), the coordinates of the shells are generated for the various heads considered. Another computer program (appendix A 3.4.2) is then used in producing the graphs of the computed shapes. This program utilises subroutines of the graph plotting package CALCOMP(54) which is available in Edinburgh Regional Computing Centre.

To illustrate the above procedure consider the following example with the stated parameter values:

Design head (mm)	Thickness (mm)	Design Stress (MN/m^2)	Unit Weight of fluid (KN/m^3)	Step-lengths (DX,DZ) (mm)
500	4.0	0.15	11.61	8
1000	4.0	0.15	11.61	8
2000	4.0	0.15	11.61	8
3000	4.0	0.15	11.61	8

Using the explicit improved Euler program, the coordinates of the shells for these design heads are generated. The obtained output file is edited to produce the input data file for the graph plotting program (appendix A 3.4.2). Running this program with the obtained data file gives the graphical output shown in Figure 3.4.1.

The coordinates of the graphs are joined with straight lines. This can be improved upon by joining them with curves or generating more coordinates for the shapes as in Figure 3.4.2. In this Figure, more coordinates are generated for design heads 2m and 3m by halving the step-lengths formerly used. Finally, the automatic graph plotting approach can be extended to the drawing of design curves considered in previous sections. The curves for the exercise (refer to Section (3.3)) where z0 varies from 20 to 200m, T = 50 mm, DS varies from 20 to 40 MN/m^2, G = 9.81 KN/m^3, DX = DZ = 100 mm previously considered are obtained by this approach and are as illustrated in Figures 3.4.3. to 3.4.7.

Section (3.5) Summary and Conclusions

This chapter is concerned with "design curves". As it closely followed the work of Currie[36] two main reasons for replication are given, viz.,(a) the detection of some errors in the Fortran computer program used by Currie, necessitating their correction. After this, there is a need to re-apply the new corrected program. (b) The inefficiency and unreliability of the method of integration used by Currie is established. This is followed by a demonstration of the reliability of a new method of integration, which is used in a new computer program.

After completing the revision, automatic graph-plotting using available computer-graphics is introduced. It is believed that by employing the procedure, i.e., design-curves, outlined in this chapter, a designer is saved enormous labour, time and cost in practice.

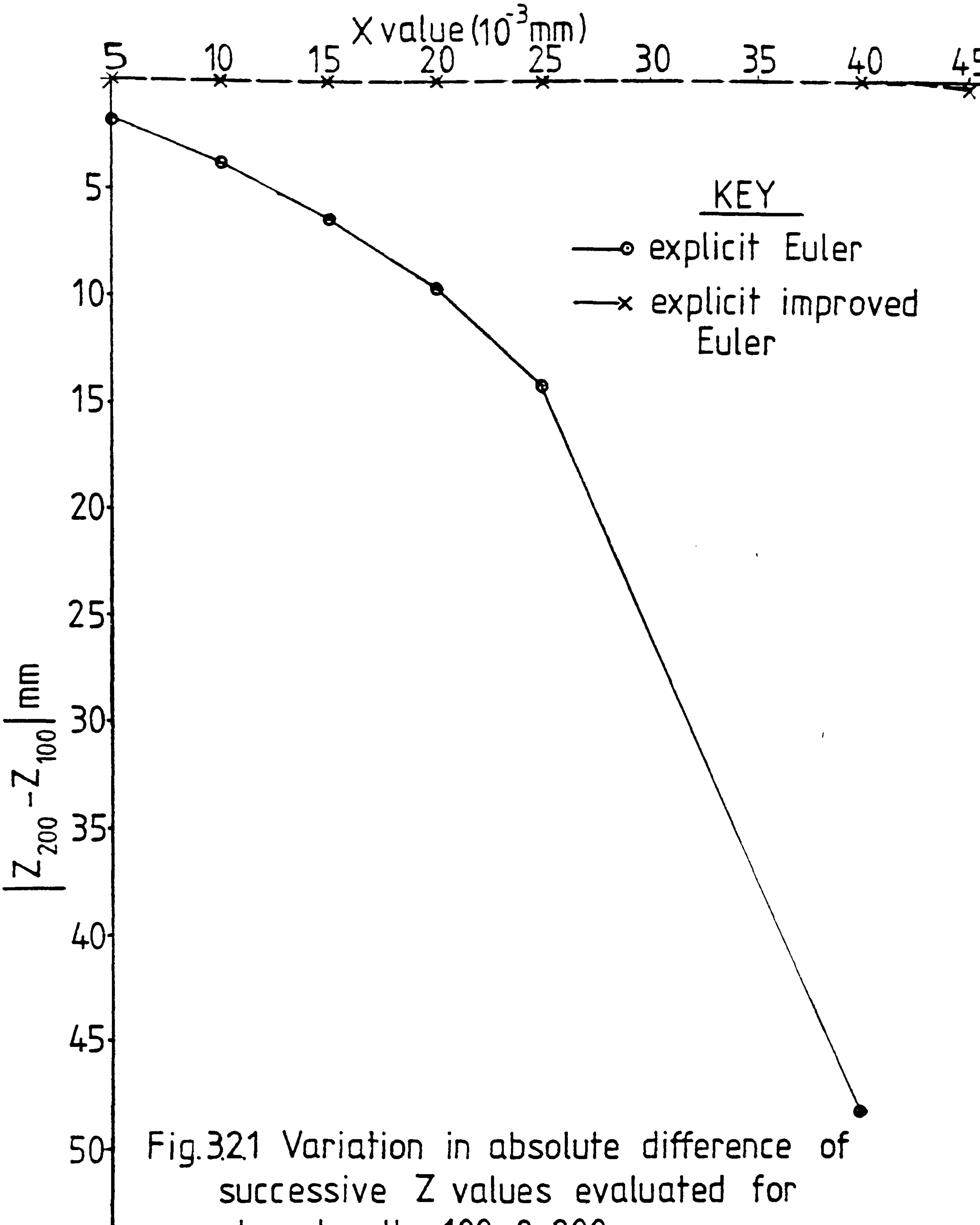

Fig. 3.21 Variation in absolute difference of successive Z values evaluated for step lengths 100 & 200 mm.

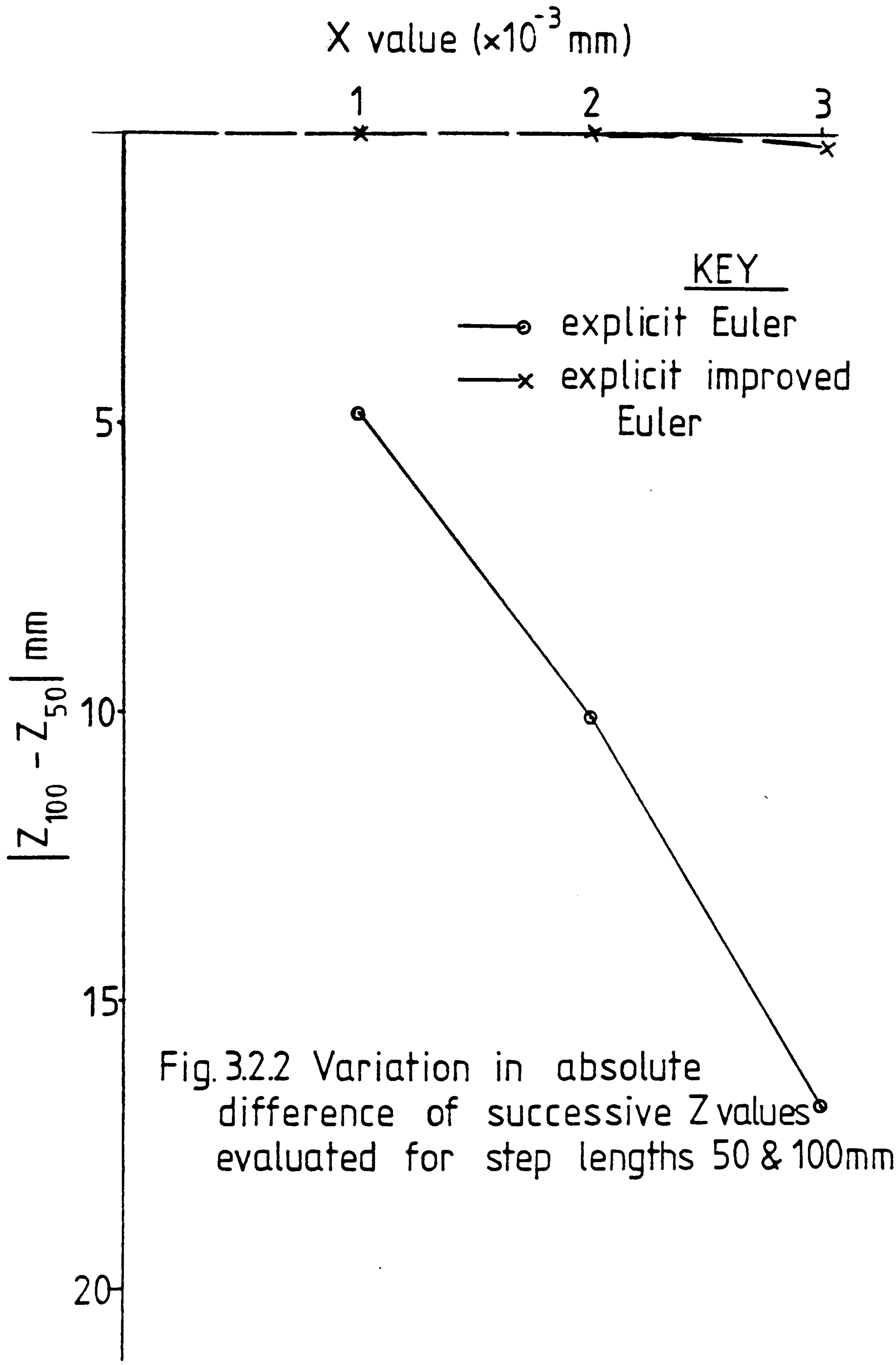

Fig. 3.2.2 Variation in absolute difference of successive Z values evaluated for step lengths 50 & 100mm

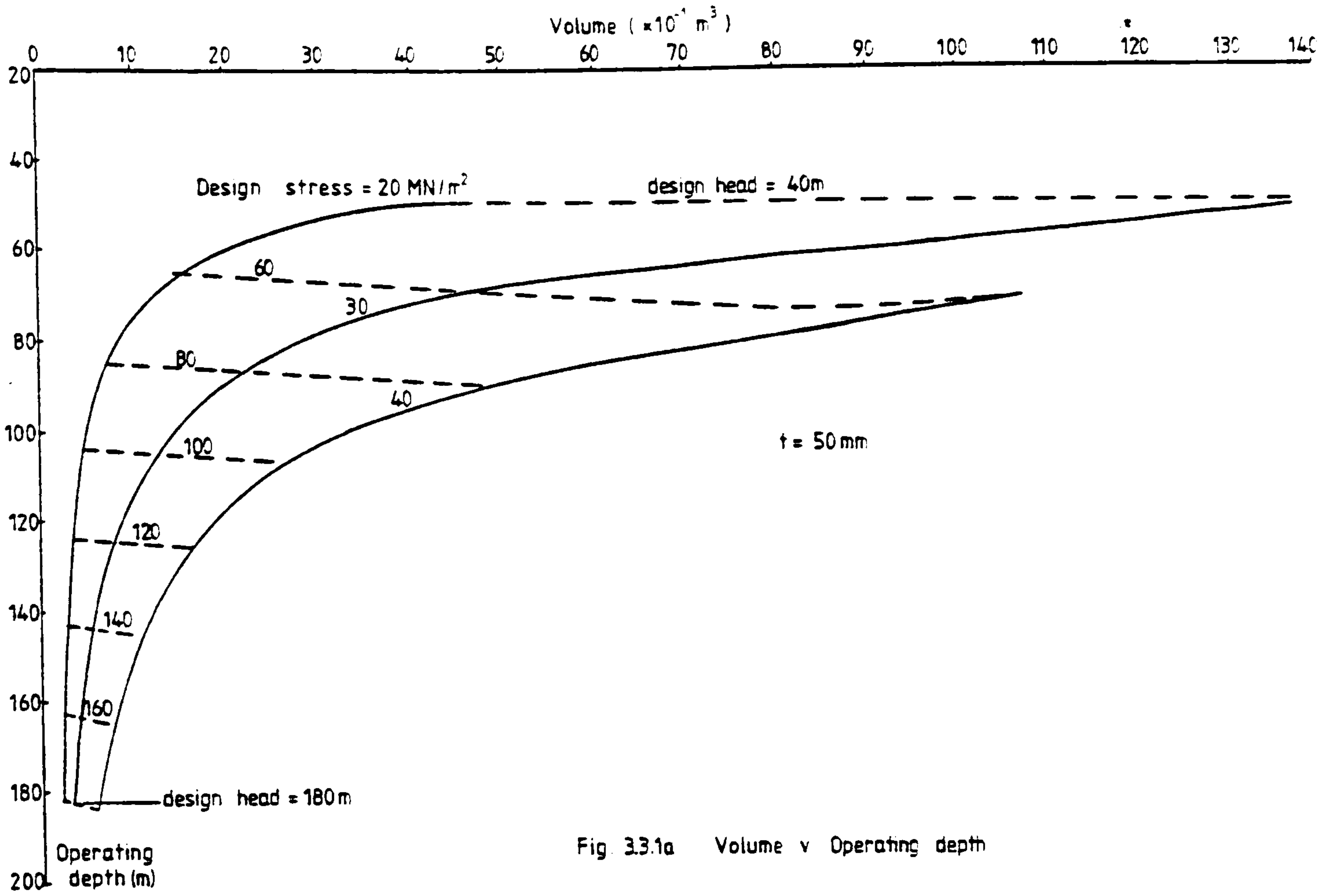

Fig 3.3.1a Volume v Operating depth

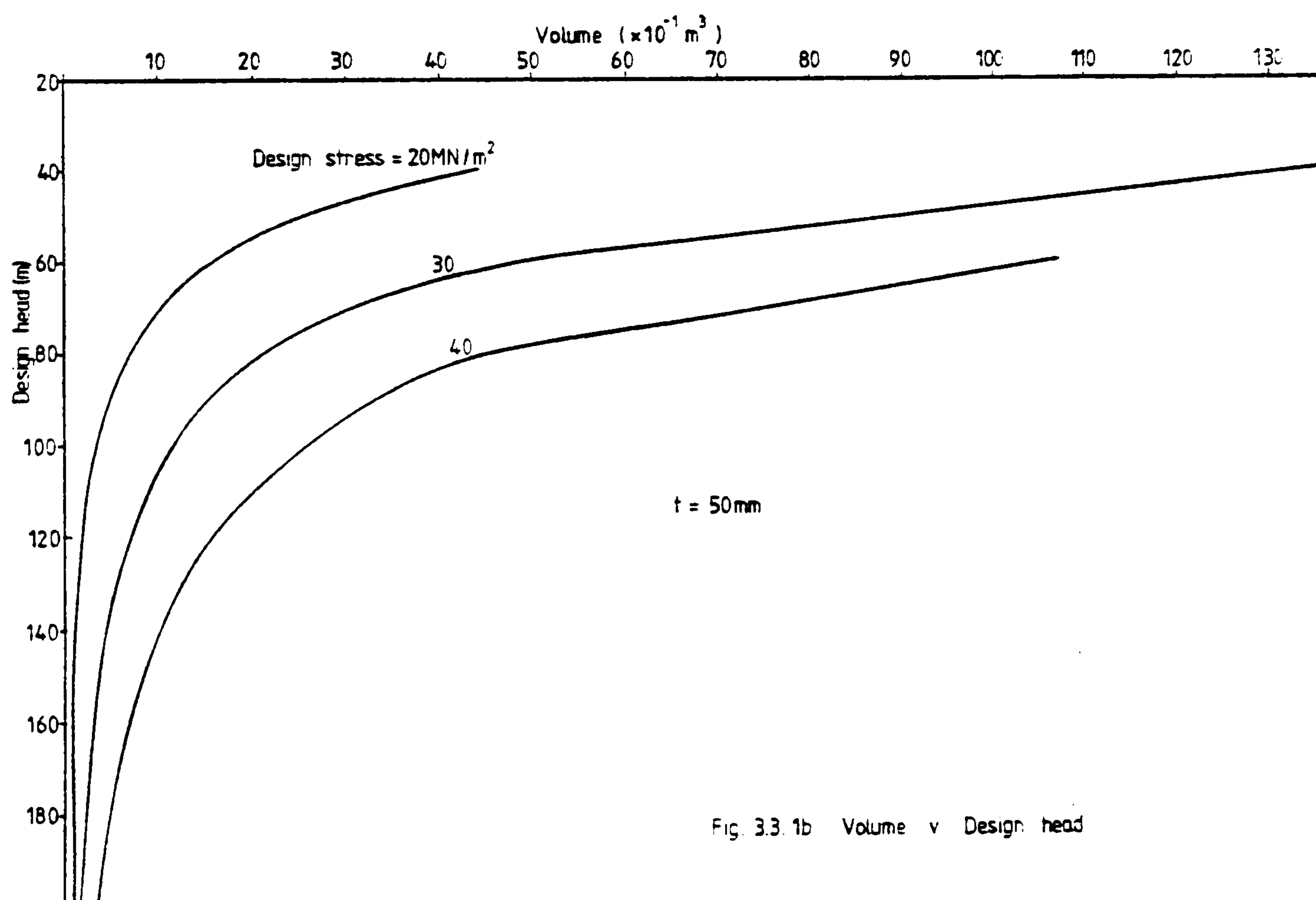

Fig. 3.3.1b Volume v Design head

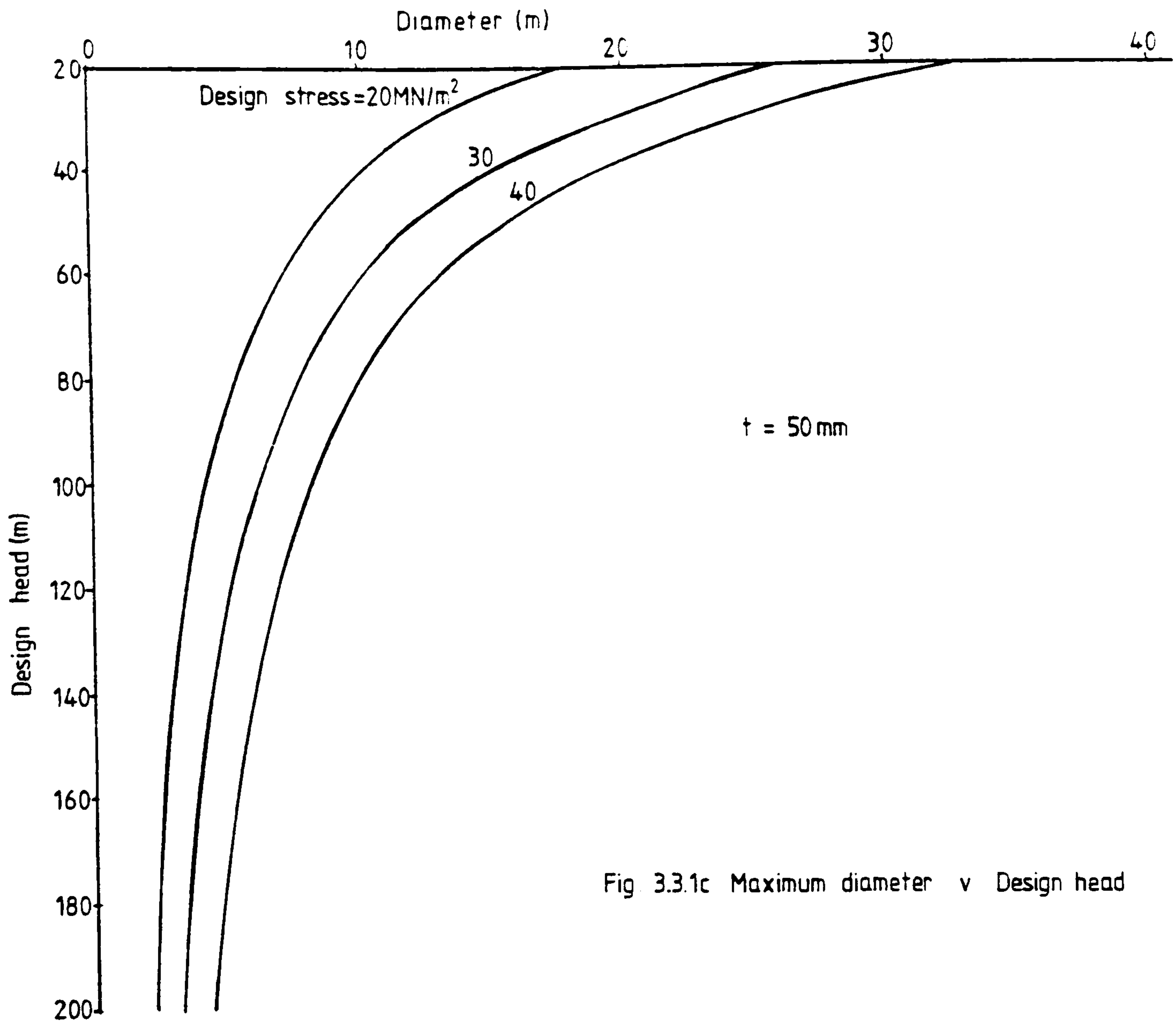

Fig. 3.3.1c Maximum diameter v Design head

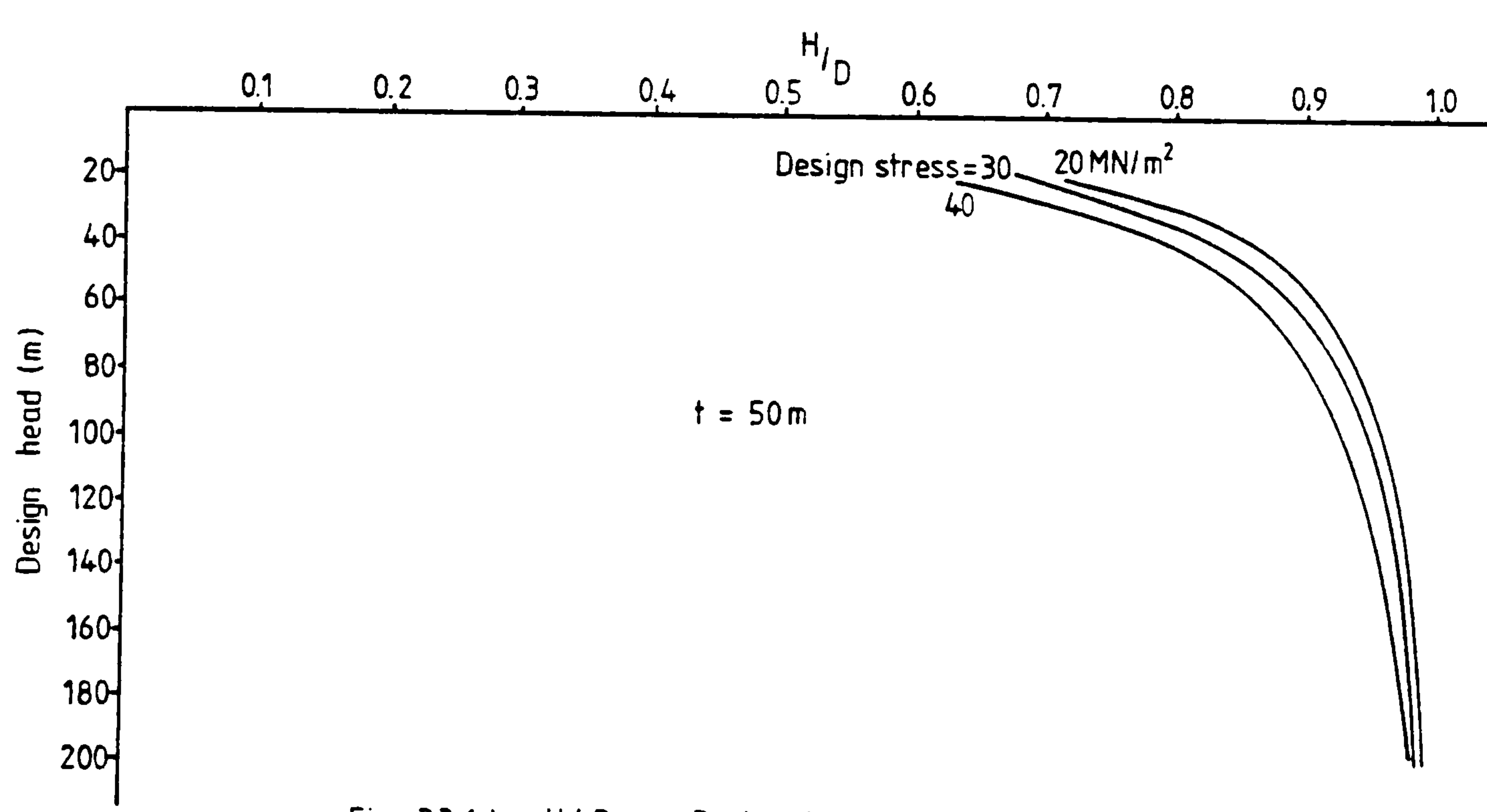

Fig. 3.3.1d H / D v Design head

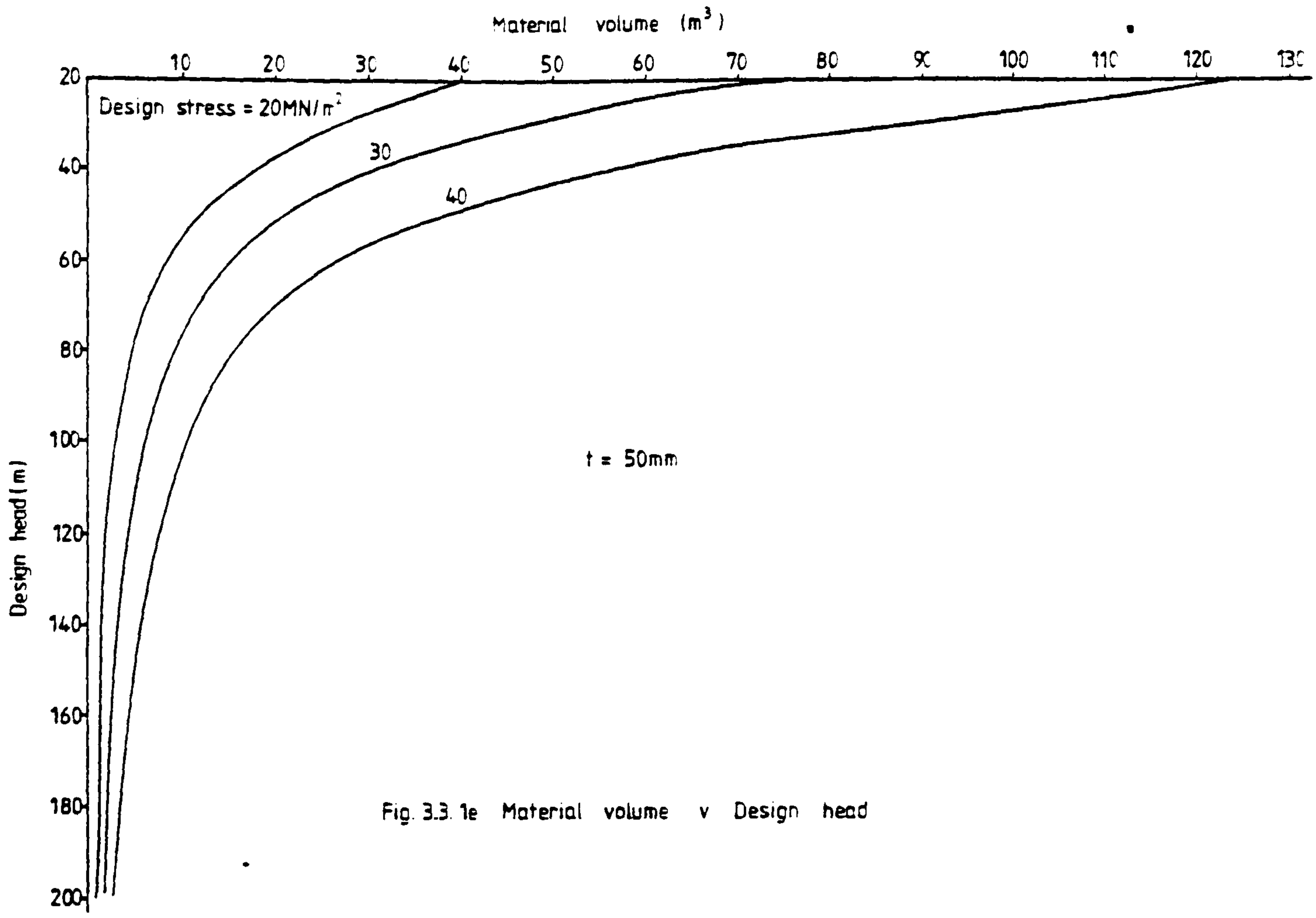

Fig. 3.3. 1e Material volume v Design head

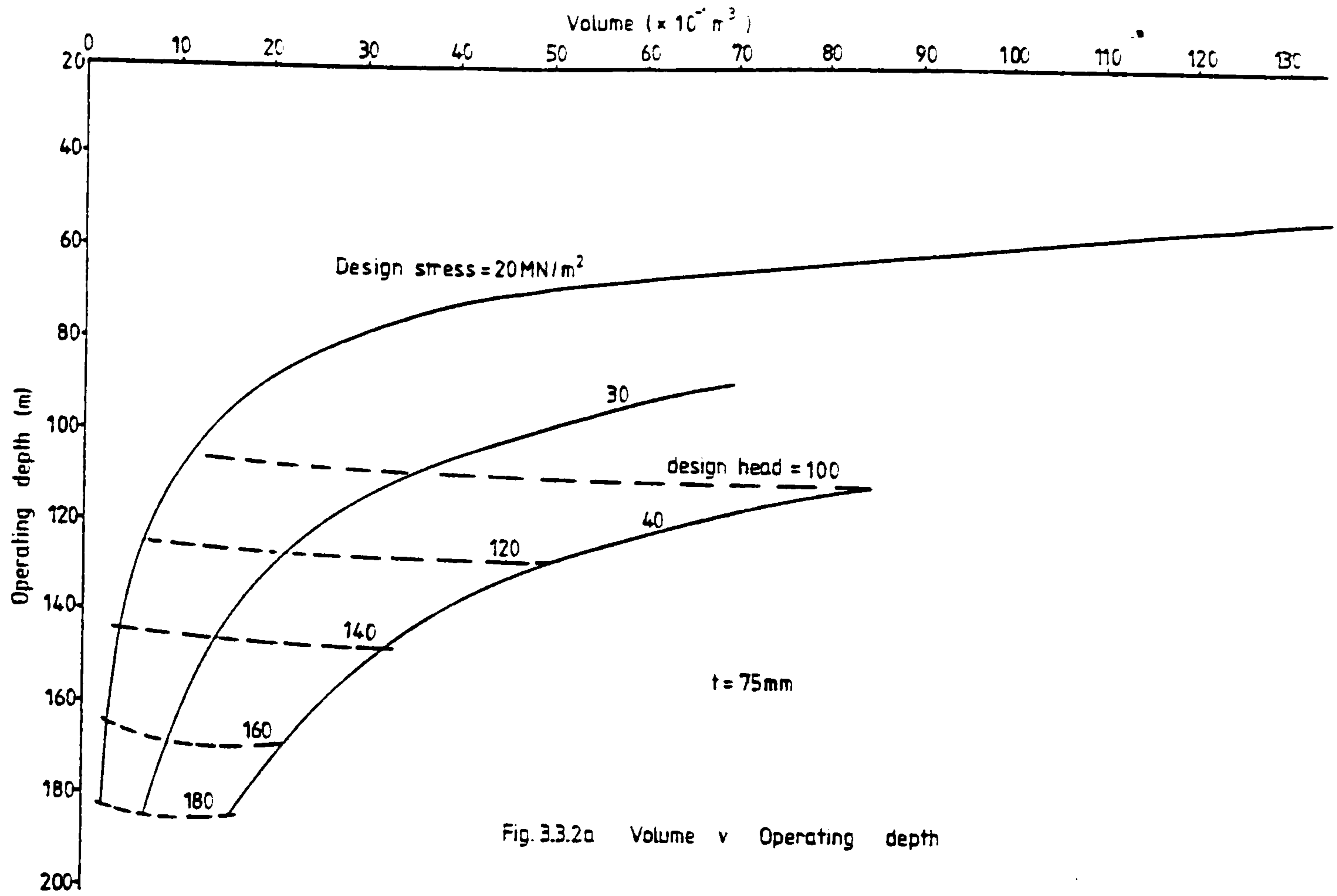

Fig. 3.3.2a Volume v Operating depth

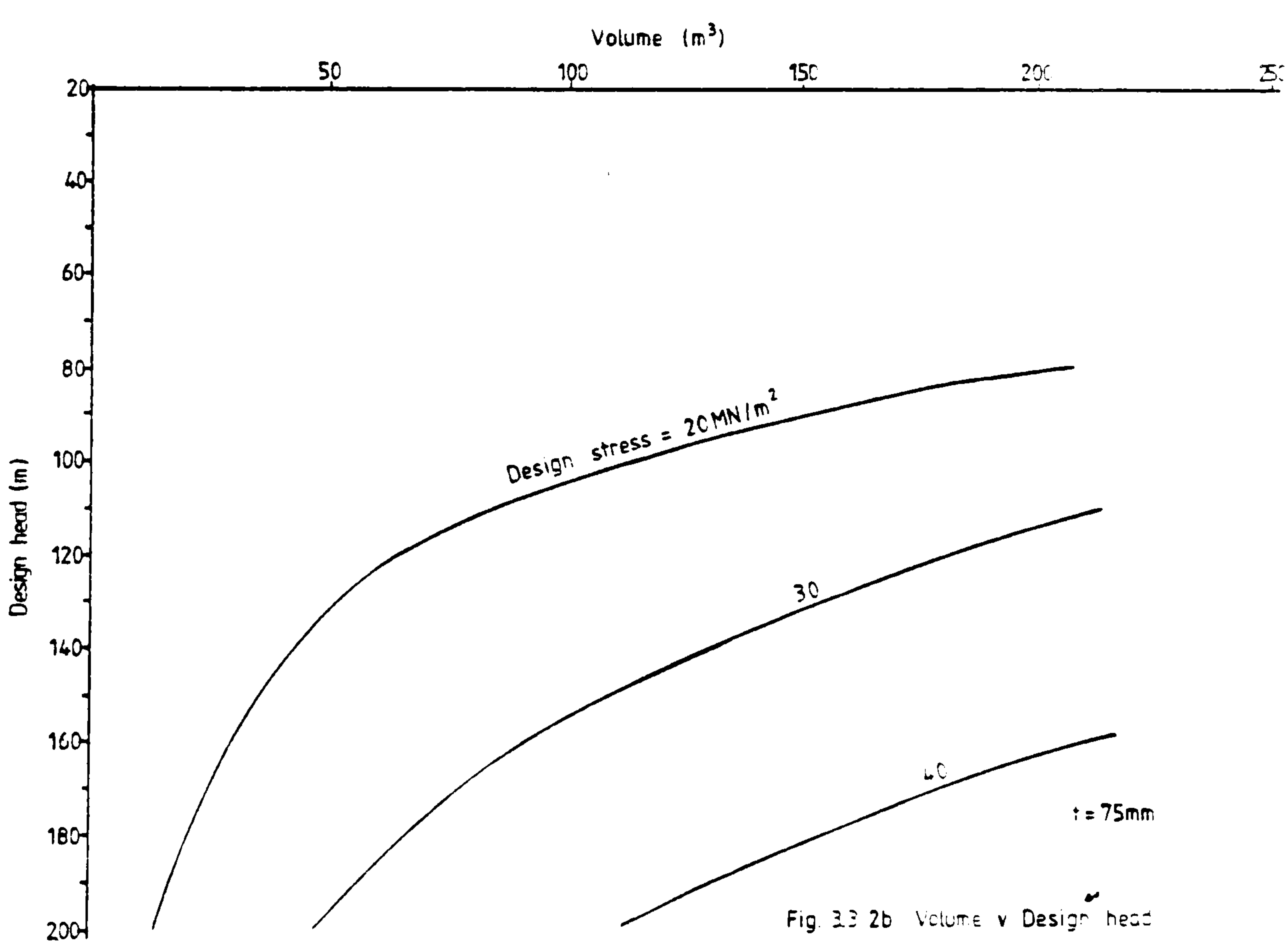

Fig. 3.3.2b Volume v Design head

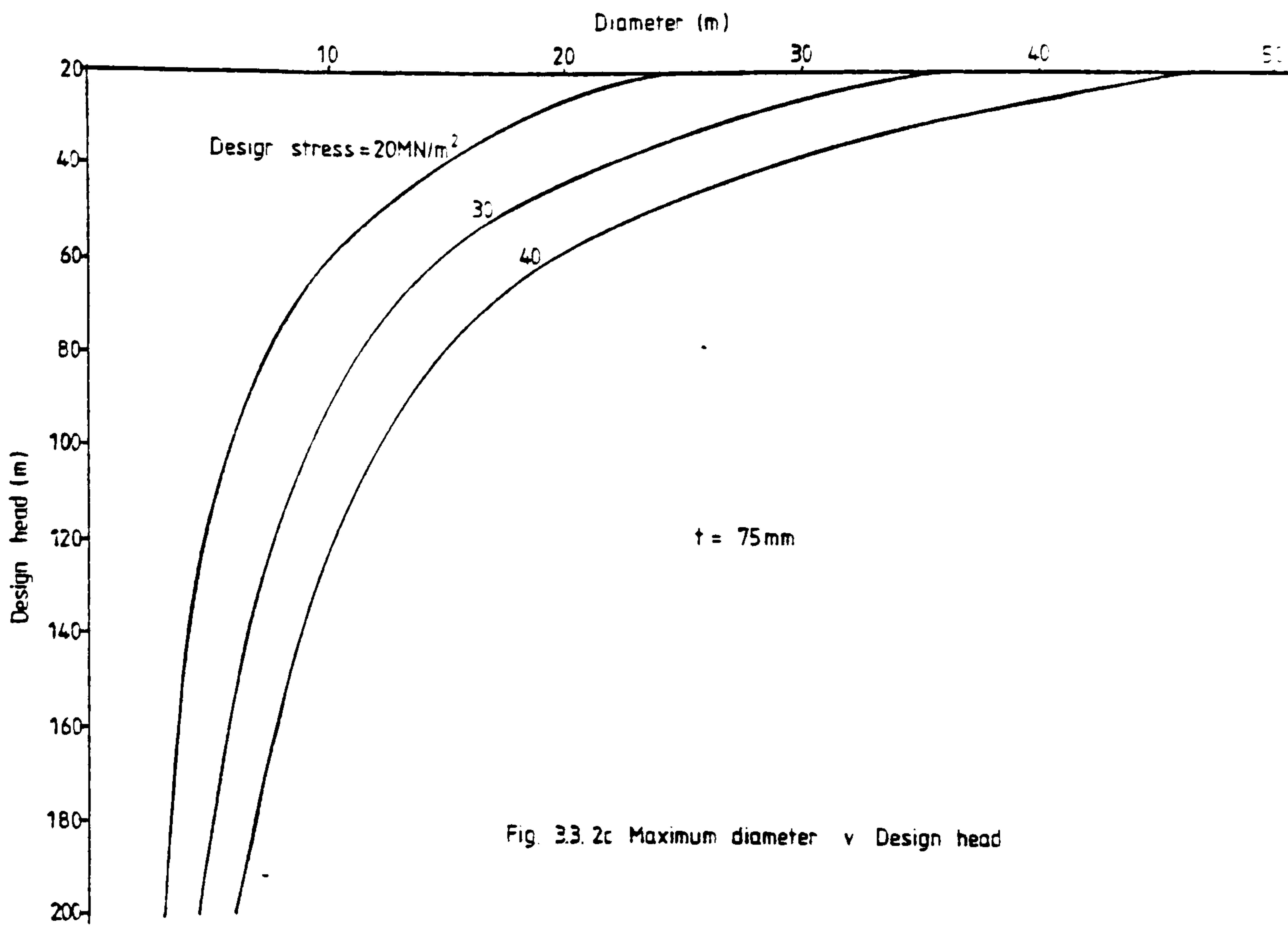

Fig. 3.3. 2c Maximum diameter v Design head

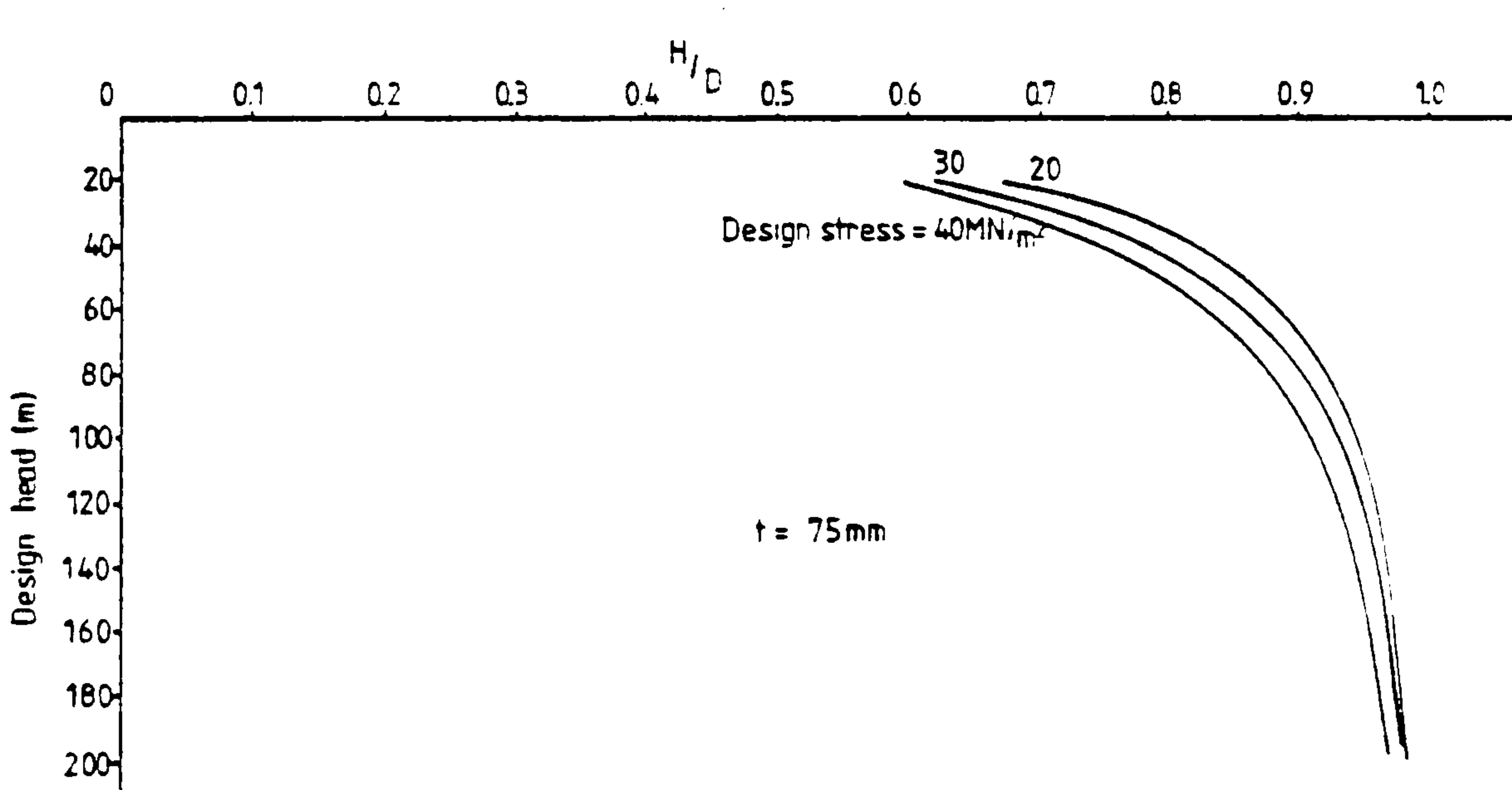

Fig. 3.3. 2d H/D v Design head

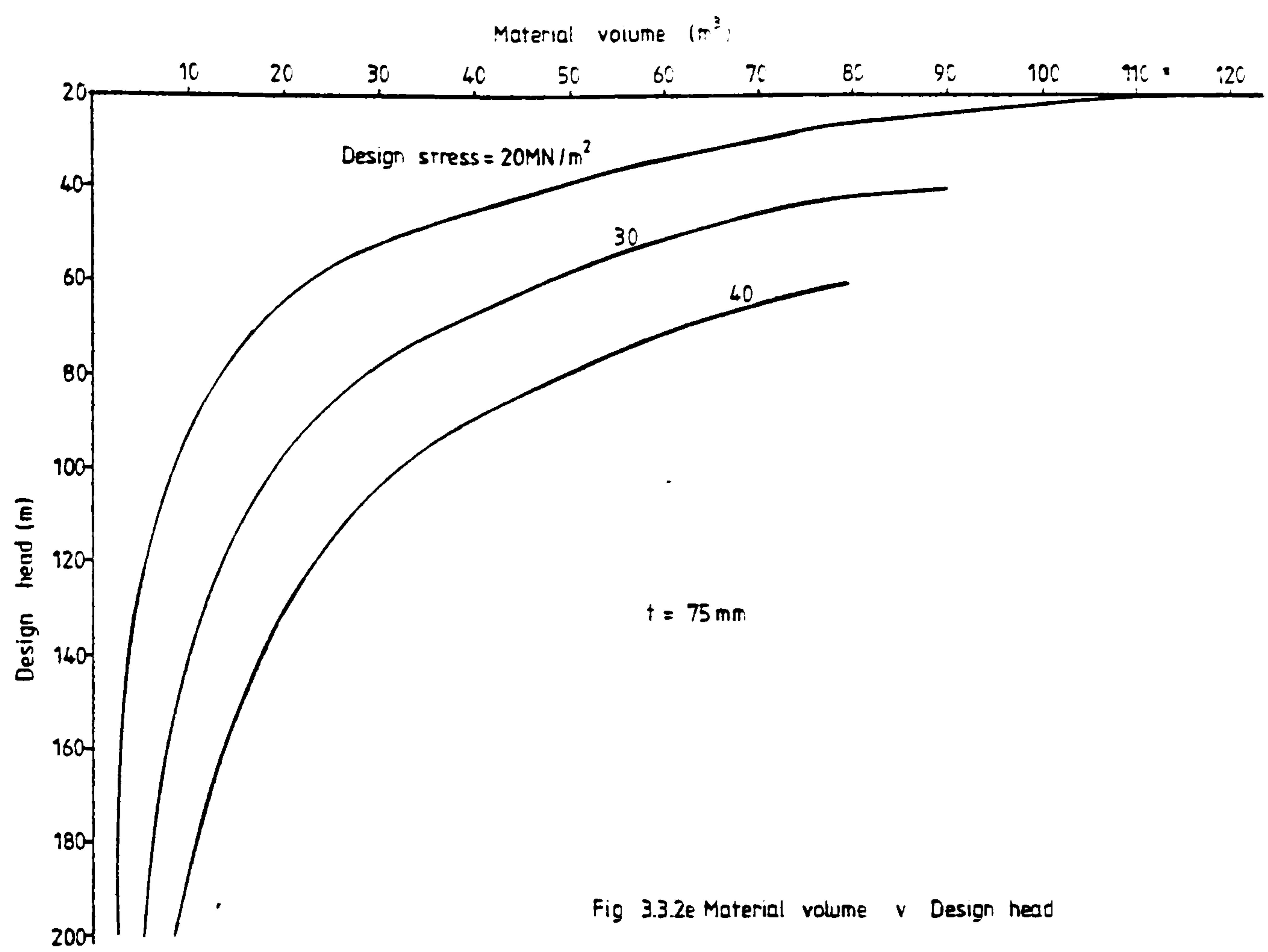

Fig 3.3.2e Material volume v Design head

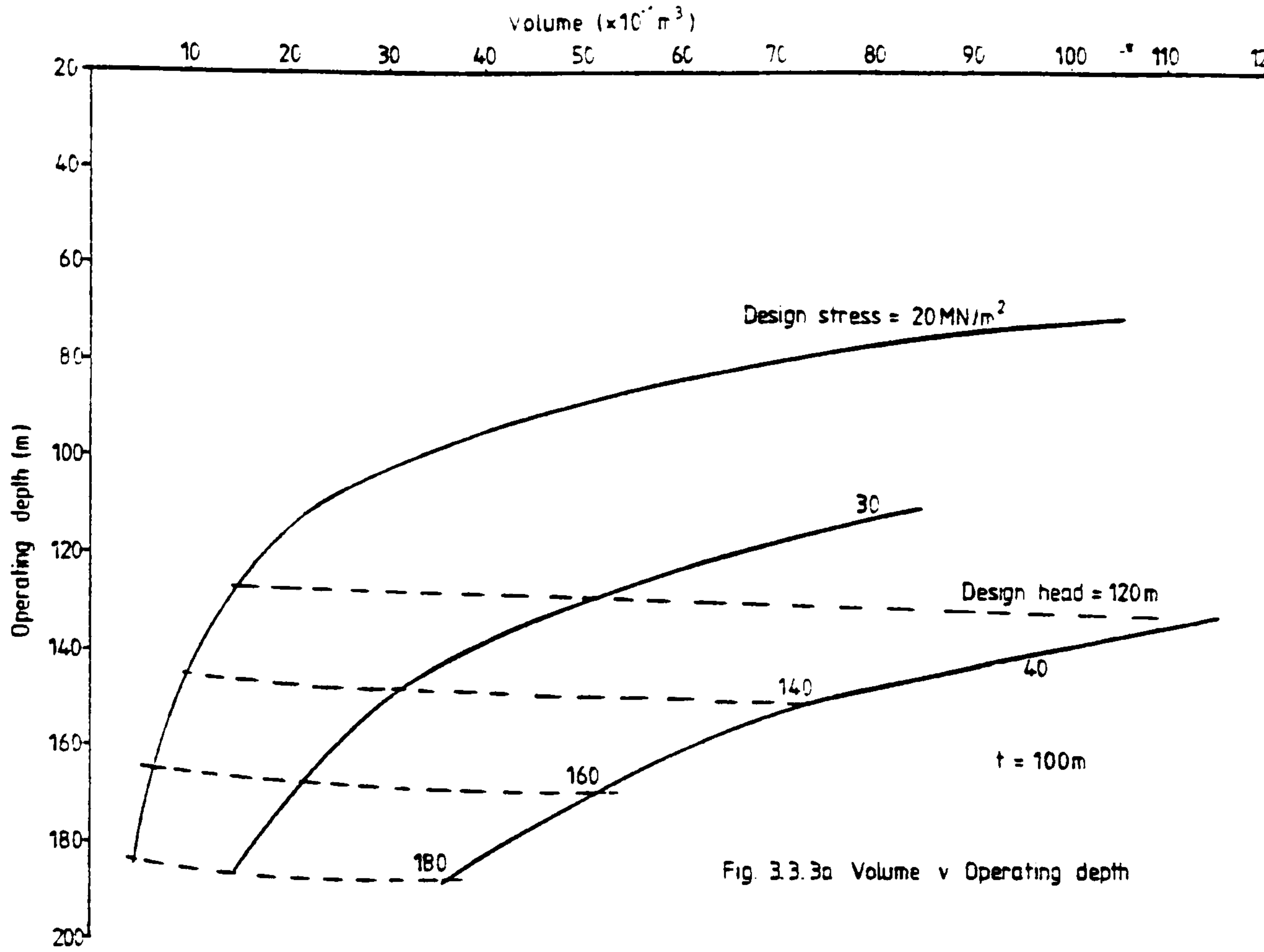

Fig. 3.3.3a Volume v Operating depth

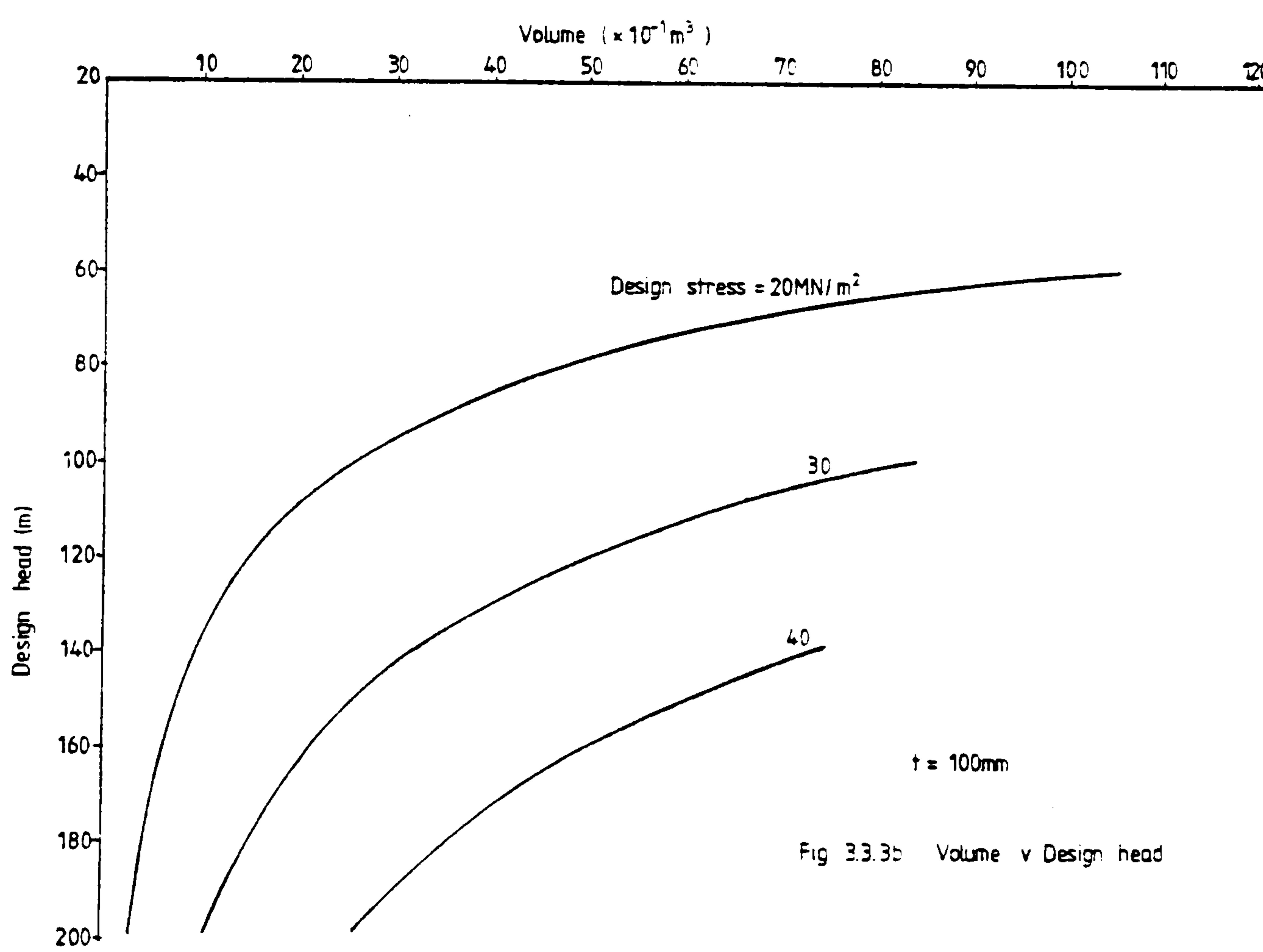

Fig. 3.3.3b Volume v Design head

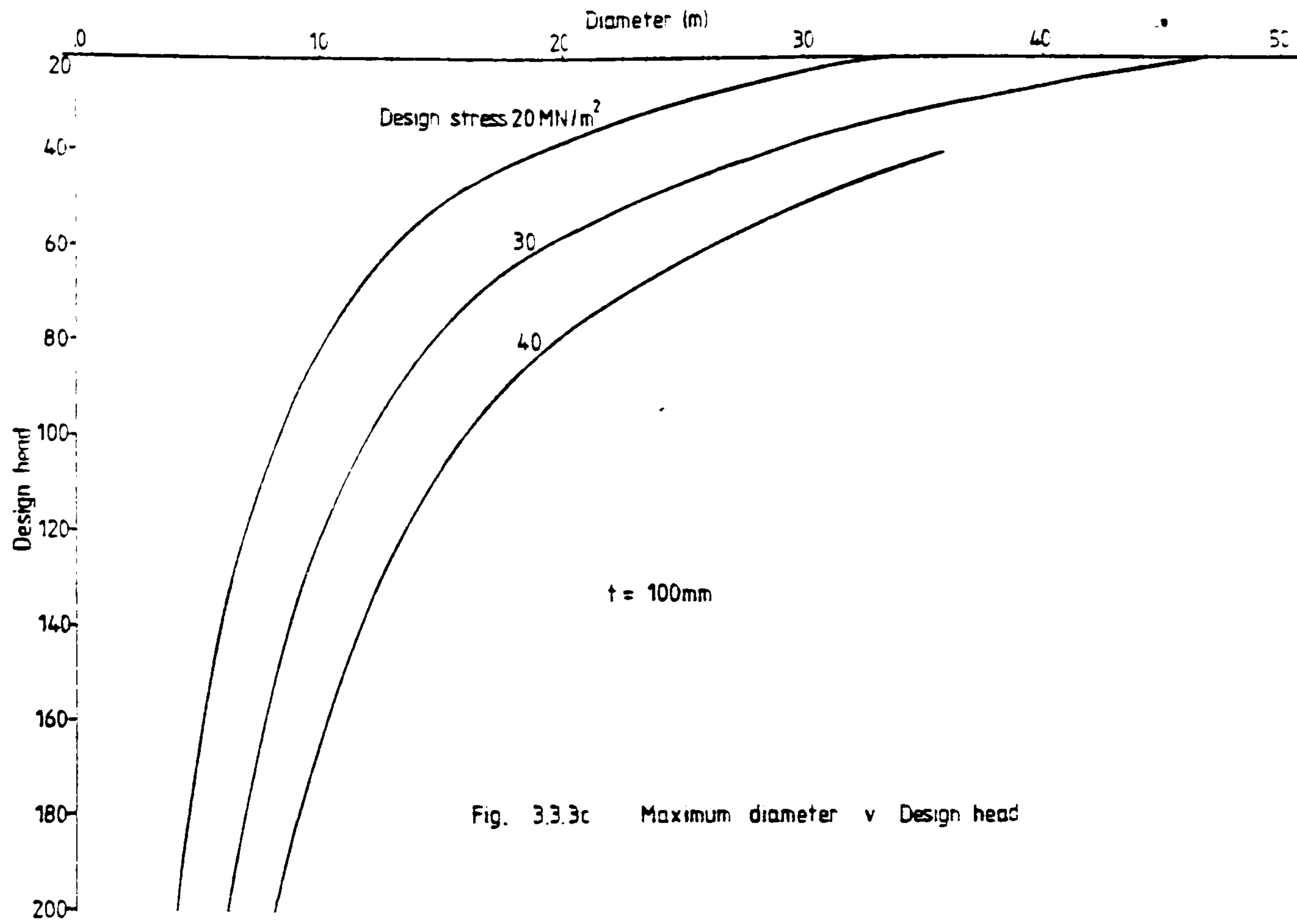

Fig. 3.3.3c Maximum diameter v Design head

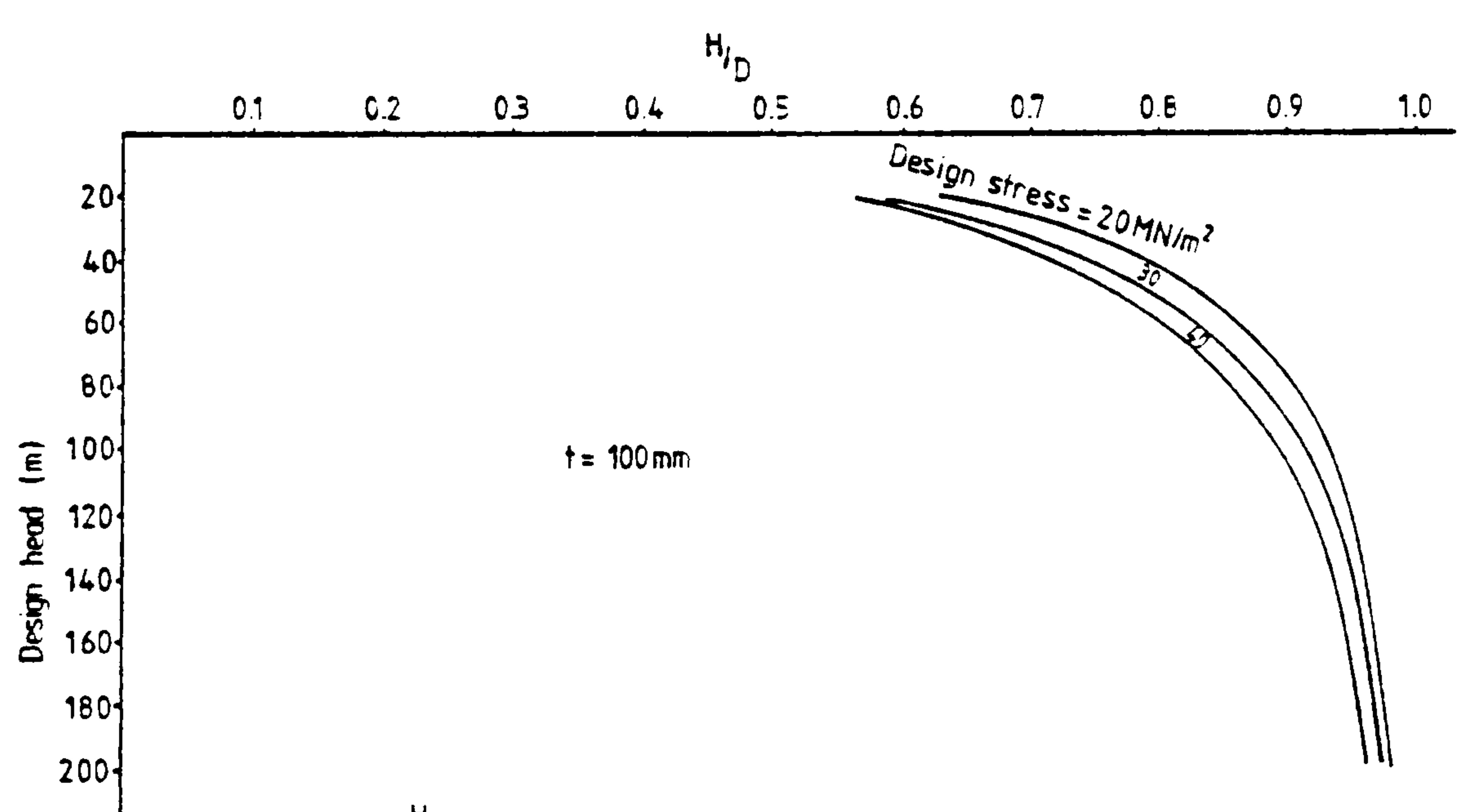

Fig 3.3.3d H/D v Design head

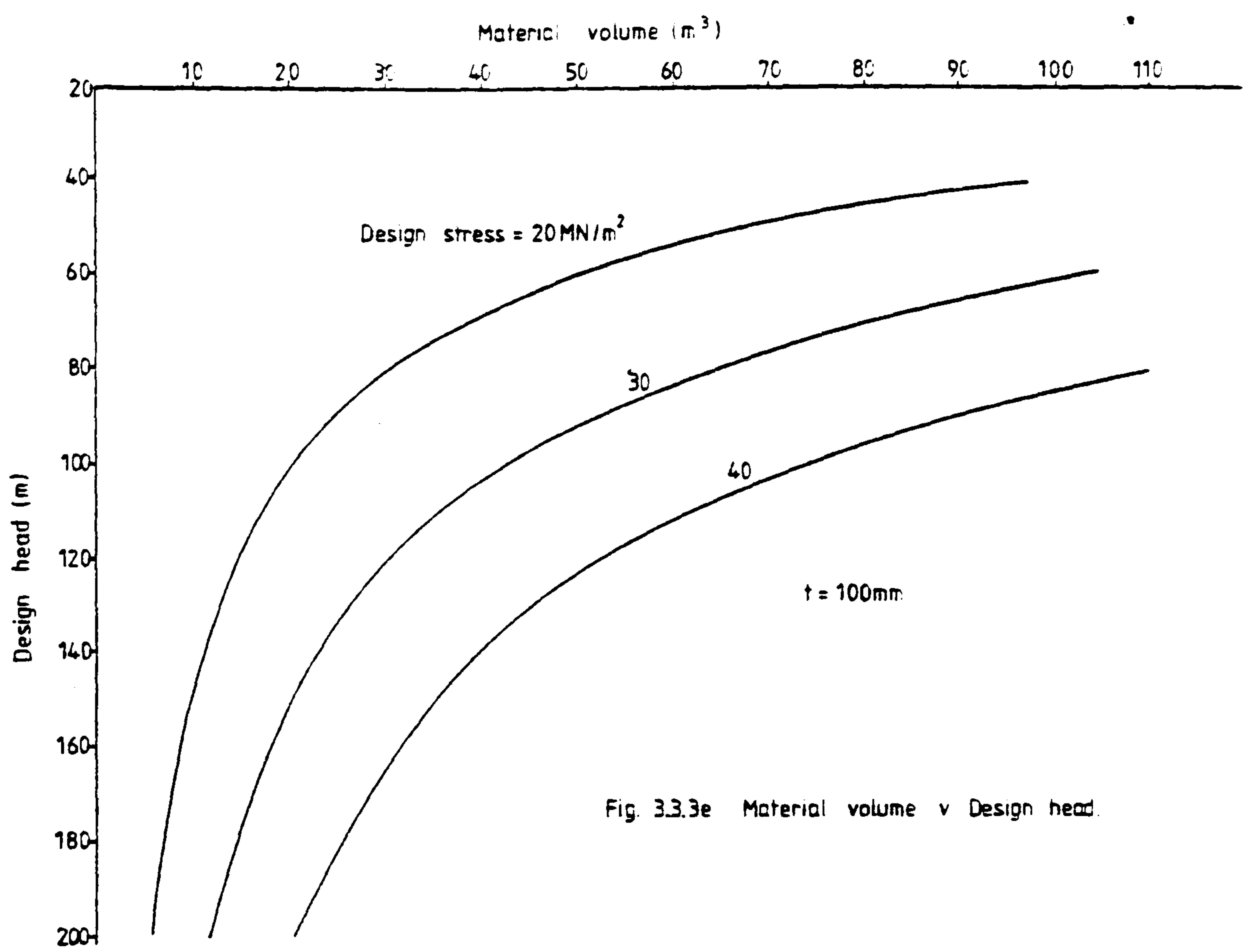

Fig. 3.3.3e Material volume v Design head.

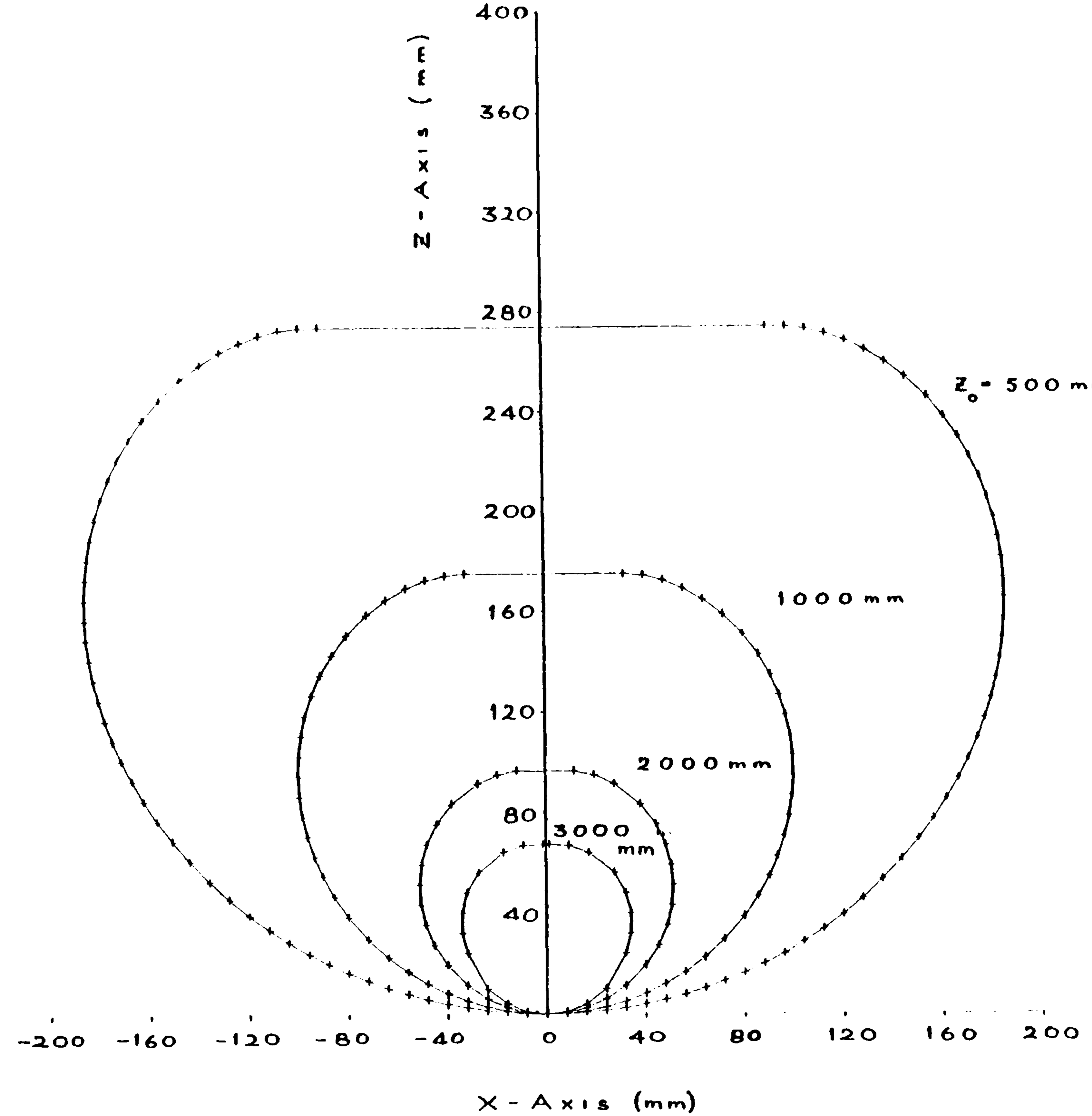

Figure 3.4.1
Shell sizes for various design heads

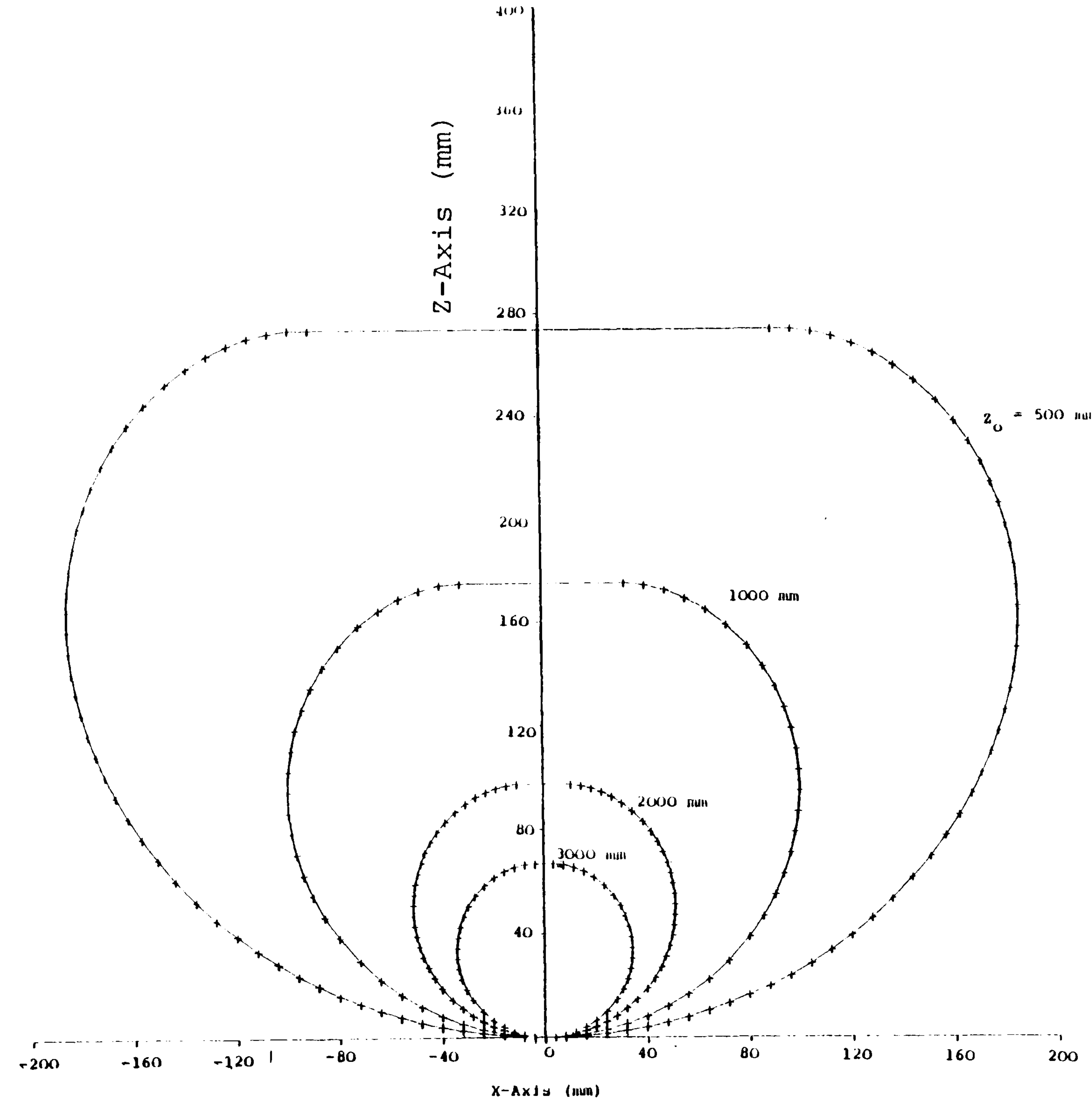

Figure 3.4.2

Shell sizes for various design heads (half step-lengths for heads 2 and 3 m)

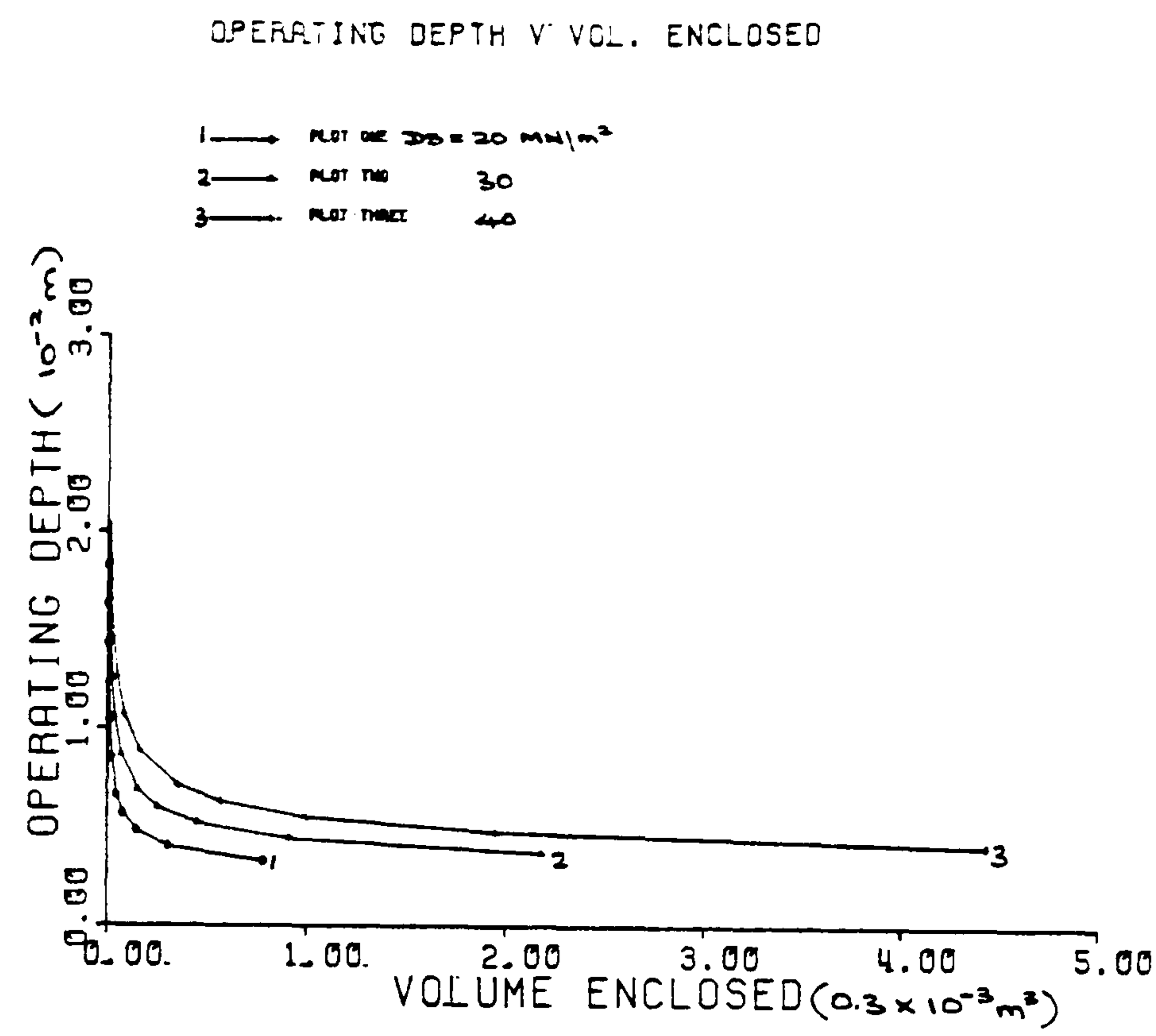

Figure 3.4.3

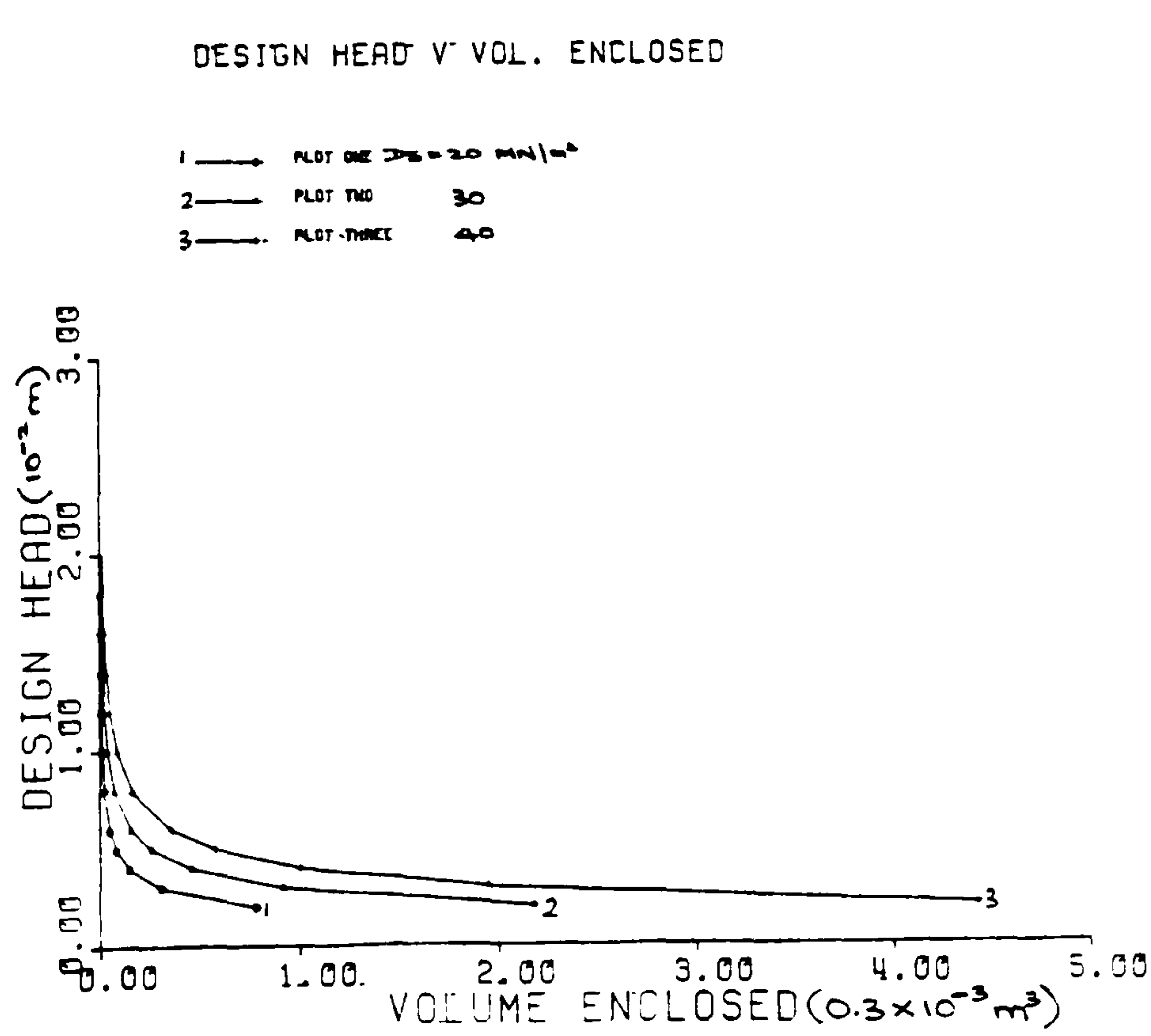

Figure 3.4.4

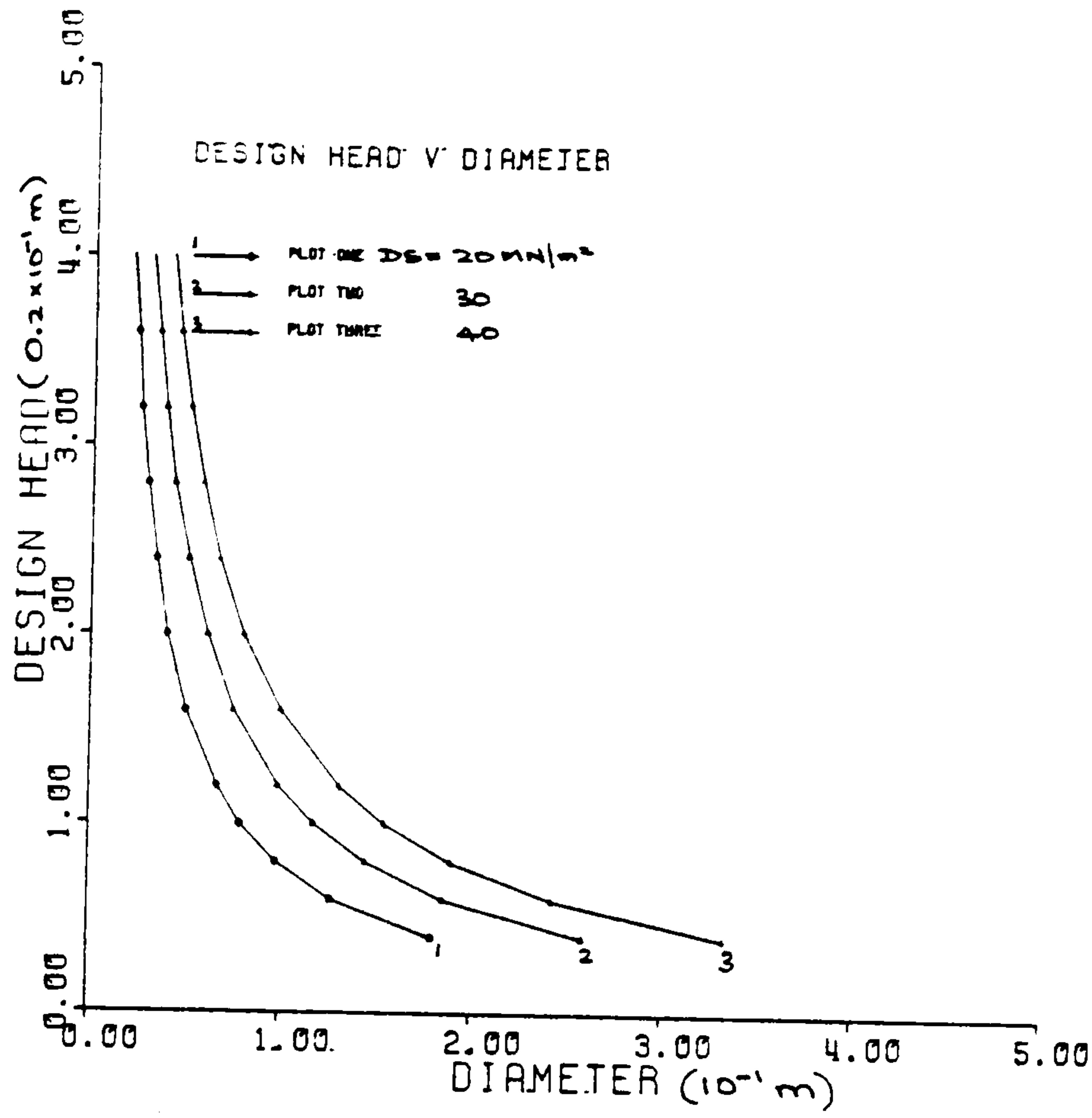

Figure 3.4.5

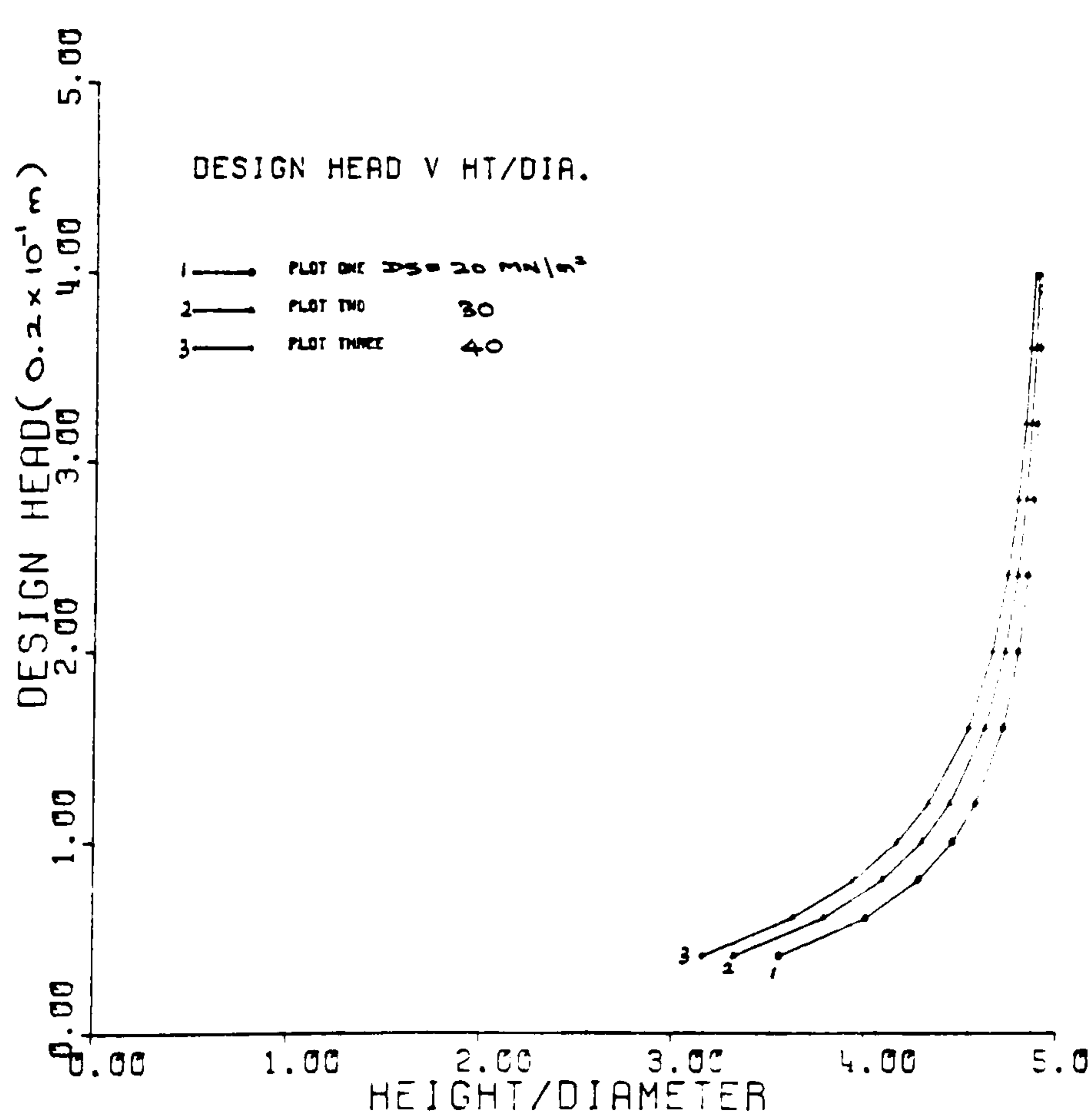

Figure 3.4.6

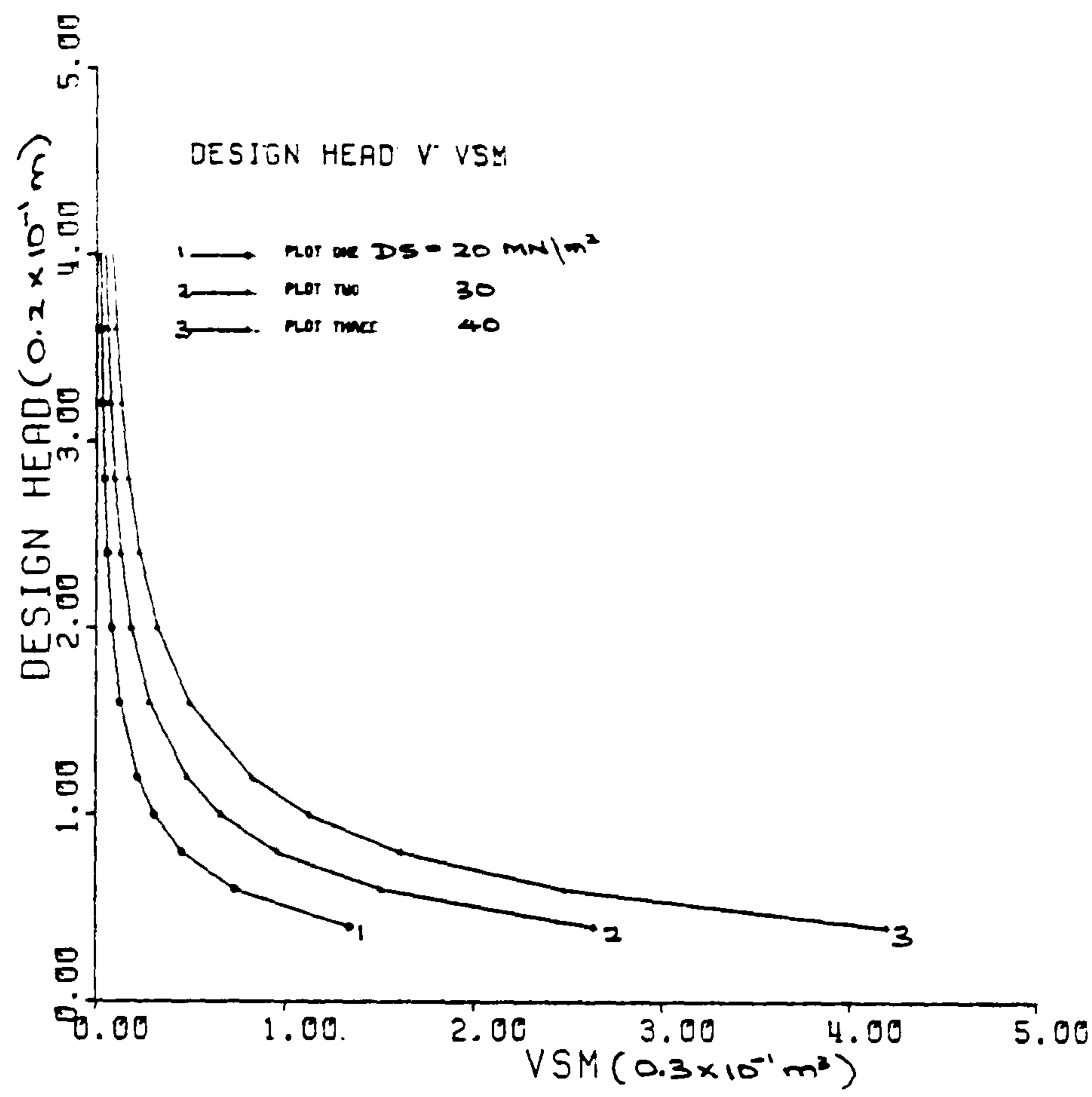
DESIGN HEAD V' VSM
1 PLOT ONE DS = 20 MN\m²
2 PLOT TWO 30
3 PLOT THREE 40
DESIGN HEAD (0.2 x 10⁻¹ m)
VSM (0.3 x 10⁵ m³)
0.00
1.00
2.00
3.00
4.00
5.00
1
2
3

Figure 3.4.7

CHAPTER FOUR: AN APPROXIMATE ANALYSIS OF THE DROP SHAPED SHELL

CHAPTER FOUR: An approximate Analysis of the Drop Shaped Shell

Section (4.0): Introduction

Having discussed some methods for generating the form of the drop shaped shell and an easy process of choosing an appropriate shape for a particular situation using design curves, an investigation of the possible behaviour of this shell when used in reality is required. In this respect the focal point of interest is in predicting the response of the shell when subjected to various practical loadings. Among the loadings that may be of interest and therefore should be considered are those due to varying pressure heads, launching, impact, explosions, earthquakes, tidal waves, etc. The response investigation may be carried out experimentally and/or theoretically.

In this chapter, a theoretical study of the effect of varying hydrostatic pressure loading on the behaviour of the drop shaped shell is considered. Due to a limitation of time, the effects of other types of loadings are not pursued.

Section (4.1) Membrane Analysis and Drop Shaped Shell

In deriving the system of equations giving the meridian of the shell of uniform strength, membrane theory is employed. In so doing, the effects of bending moments and radial shears are assumed small compared with the membrane forces N_ϕ and N_θ, and are therefore negligible. This of course is not entirely true of the behaviour of the shell in reality, since, for example, if the pressure head for a particular tank is different from the design pressure, the forces per unit length N_ϕ and N_θ are no longer uniform. Then, it is incorrect to apply membrane theory in the analysis of the tank. However, it is interesting to try and deduce the variation of the stress resultants in the shell as the pressure head varies using membrane theory as an approximate method. In chapter five, a more accurate analysis of this problem will be attempted using the finite element method.

Before considering this approximation it is interesting to report an observation regarding the work of Novozhilov[18] on a reservoir of uniform strength having a zero pressure at its apex. The objective was to study the effect of pressure other than the design head on the tank. Without explaining the procedure taken too clearly it is shown that the stress distribution in the shell is not uniform in such circumstances. It is suspected that the graphical representation of the variation of the stress resultants over the depth of the tank were obtained on the basis of membrane theory. Following the definition of

$$\lambda = \frac{P_0}{\sqrt{T_0 \rho}} \tag{4.1.1}$$

given by Novozhilov [18], where P_0 is the pressure at the top of the reservoir, T_0 is the design stress and ρ is the specific gravity of fluid contained, and choosing appropriate design parameters yielding the same value of λ, it can be shown that the stress resultant variations illustrated by Novozhilov are of doubtful form. With slight adjustment they are more acceptable.

The equations giving the normal and tangential forces following membrane approximations are derived in the next section.

Section (4.2) Application of Membrane Theory to Derivation of Stress Resultants in Thin Shells

In appendix (A 4.2.1) the following membrane equations are stated:

$$2\pi r_0 N_\phi \sin\phi + R = 0 \qquad (4.2.1a),$$

$$N_\phi = \frac{-R}{2\pi r_0 \sin\phi} \qquad (4.2.1b),$$

$$\frac{N_\phi}{r_1} + \frac{N_\theta}{r_2} + Z = 0 \qquad (4.2.2).$$

Using these equations the two forces N_ϕ and N_θ theoretically can be evaluated. The drawback to this approach is the evaluation of R as one moves from the apex of a shell to its base.

Consider a strip of the shell given by a small arc δs. The arc is obtained by a radius r_1 moving through a small angle $\delta\phi$ as shown in Figure 4.2.1 in appendix (A 4.2.1). Then

$$\delta s = r_1 \delta\phi \qquad (4.2.3)$$

and the surface area of the strip is

$$2\pi r_0 \delta s = 2\pi r_0 r_1 \delta\phi \quad .$$

If the unit weight of fluid acting on shell is γ, then the vertical force component on a strip at depth z is given by

$$\gamma z\, 2\pi r_0 r_1 \delta\phi \cos\phi \qquad (4.2.4) .$$

Hence for this strip,

$$\delta R = \gamma z \; 2\pi r_0 r_1 \delta\phi \cos \phi \qquad (4.2.5) \; ,$$

where δR is the resultant of the external load on the strip acting vertically downwards. From equation (4.2.5) it is observed that δR can be evaluated for any strip of the shell and the accuracy of the subsequent procedure for obtaining the forces N_ϕ and N_θ will depend on the size of $\delta\phi$.

To set up the iterative process which will be used for evaluating these stress resultants for a particular angle $\phi = \phi'$, the shell is considered to consist of many strips with various δR's acting on them.

Above the level defined by ϕ' ,

$$R_{\phi'} = \sum_{\phi=0}^{90^\circ} \gamma z r_1 \delta\phi 2\pi r_2 \sin\phi \cos\phi - \sum_{\phi=90^\circ}^{\phi'} \gamma z r_1 \delta\phi 2\pi r_2 \sin(180-\phi)\cos(180-\phi) \qquad (4.2.6)$$

(using the fact that $r_0 = r_2 \sin\phi$ and R becoming a buoyant force for $\phi > 90^\circ$).

Simplifying this equation by noting that $\delta x = r_1 \cos\phi\delta\phi$,

$$R_{\phi'} = \sum_{\phi=0}^{90^\circ} \gamma z 2\pi x \delta x + \sum_{\phi=90^\circ}^{\phi'} \gamma z 2\pi x \delta x \qquad (4.2.7) \; .$$

(Note that in equation (4.2.7) δx is negative for $\phi > 90^\circ$).

From equations (4.2.1b) and (4.2.7), it follows that

$$N_{\phi'} = \frac{-\left\{\sum\limits_{\phi=0}^{90} \gamma z 2\pi x \;\delta x + \sum\limits_{\phi=90}^{\phi'} \gamma z 2\pi x \;\delta x\right\}}{2\pi r_2' \sin\phi' \sin\phi'}$$

$$= \frac{-\gamma\left\{\sum\limits_{\phi=0}^{90} zx \;\delta x + \sum\limits_{\phi=90}^{\phi'} zx \;\delta x\right\}}{x' \sin\phi'} \qquad (4.2.8)$$

since $x' = r_2' \sin\phi'$ and noting that $\delta x = x_{i+1} - x_i$ $(i = 0,1,2, \ldots, M)$ and this is negative for $\phi > 90^o$. [x_{M+1} is the last x coordinate at the base of shell.]

$$N_{\theta'} = -\gamma z' r_2' + \frac{r_2'}{r_1'} \gamma \frac{\left[\sum\limits_{\phi=0}^{90} zx \;\delta x + \sum\limits_{\phi=90}^{\phi'} zx \;\delta x\right]}{x' \sin\phi'} \qquad (4.2.9)$$

It should be noted that equations (4.2.8) and (4.2.9) hold good for $\phi = \phi' > 0^o$. Under symmetrical loading if the shell resists the applied forces partly in flexure there would be a relationship between R and the moments at a level $\phi = \phi'$. However as R is the resultant of force above level $\phi = \phi'$, when $\phi' = 0$ it seems highly likely that $R = 0$. Under unsymmetrical loading bending and shears could be set up round a parallel circle. The forces shown in Figure 4.2.1 would be augmented by internal moments of resistance and radial shears existing at the level $\phi = \phi'$ in addition to $N_{\phi'}$ and $N_{\theta'}$. The resultant R would not necessarily act through the apex or be vertical for that matter. Consequently under general loading at $\phi' = 0$

$R \neq 0$ is highly probable.

Now the problem of pressure head varying from design value for a drop shaped shell can be examined using membrane theory. In Figure 4.2.2 let z be measured from the design head datum, i.e., z_0 above apex of shell. At its design head, the (x,z) coordinates of the shell are established and $N_\phi = N_\theta =$ a specified constant for all value of ϕ. Using the established equations (4.2.8) and (4.2.9) should result in $N_{\phi'} = N_{\theta'} =$ the specified constant for $\phi = \phi'$.

If the pressure head at the apex is changed from z_0 to $z_0 + h$ say, then the coordinates of all points on the shell can be written relative to the new zero pressure datum as $(x, z+h)$. The meridional and hoop stresses at any level defined by $\phi = \phi'$ can now be found from equations (4.2.8) and (4.2.9) as

$$N_{\phi'} = \frac{-\gamma\left[\sum\limits_{\phi=0}^{90}(z+h)x\delta x + \sum\limits_{\phi=90}^{\phi'}(z+h)x\delta x\right]}{x' \sin\phi'} \qquad (4.2.10),$$

$$N_{\theta'} = -\gamma(z+h)r_2' - \frac{r_2'}{r_1'}N_{\phi'} \qquad (4.2.11).$$

Equation (4.2.10) can be rewritten as

$$N_{\phi'} = \frac{-\gamma\left[\left(\sum\limits_{\phi=0}^{90} zx\delta x + \sum\limits_{\phi=90}^{\phi'} zx\delta x\right) + h\left(\sum\limits_{\phi=0}^{90} x\delta x + \sum\limits_{\phi=90}^{\phi'} x\delta x\right)\right]}{x' \sin\phi'}$$

$$= N_\phi(\text{DESIGN}) - \frac{\gamma h\left[\sum\limits_{\phi=0}^{90} x\delta x + \sum\limits_{\phi=90}^{\phi'} x\delta x\right]}{x' \sin\phi'} \qquad (4.2.12) .$$

Section (4.3) Implementation of Derived Equations on the Computer

In the previous section, the equations that give the meridional and hoop stresses at any level of the drop shaped shell when subjected to varying pressure head were considered. In this section, the implementation of these equations on the computer is investigated.

For this exercise, the Fortran computer program used in generating the coordinates of the meridian of the shell listed in appendix (A 2.4.3) is modified. In so doing the values of the stress resultants at the levels of interest are evaluated using equations (4.2.10) or (4.2.12) and (4.2.11), (see appendices (A 4.3.1) and (A 4.3.2)). In appendix (A 4.3.1) the stress resultants evaluated are given correct to one decimal place, whilst they are given correct to eight places of decimal in appendix (A 4.3.2). This is the only difference between these two programs.

To illustrate the usage of this program the exercise described below is carried out. Notice that by carrying out the procedure, the correct working of the program is also checked.

A Computer Exercise

In this exercise, one of the above computer programs, namely STRESS 10 (see appendix (A 4.3.1)) is used in conjunction with the set of parameter values: Design head = 1000.0 mm, Thickness = 4.0 mm, Design stress = 0.15 MN/m^2, Unit weight of fluid = 11.61 KN/m^3. The step-lengths (DX, DZ, DXX) are allowed to vary over a suitable range of values. Notice that up until now DXX has been made to have the same values as DX, but there is no reason for it not to be different.

For this set of parameter values, the expected value of the stress resultants when the pressure head is the same as the design head is 0.60 N/mm. One therefore expects STRESS 10 to give this value when the parameter CONST is set equal to zero.

By using various values of DX, DZ, DXX, one tries to obtain a set of values where the region of agreement between the computed and expected values of stress resultants are about ninety percent or more. The region of disagreement may be improved upon in some cases by considering other values of DX, DZ, DXX, but one has to bear in mind the effect of computer rounding errors if the step-length values used are very small.

Continuing with this exercise, the schematic shown in appendix (A 4.3.3a to d) is obtained. There the regions of agreement (segment BC) and disagreement (segments AB and CD) for the step-lengths considered are shown. It is observed that the percentage of agreement increases as the values of the step-lengths decrease. With DX = DZ = DXX = 0.125 mm, the percentage is 99.4 and is considered adequate in the present circumstance.

Having obtained a suitable set of step-length values as above and having been satisfied that the computer program is functioning as expected, the effect of pressure heads different from the design head of the shell is now investigated. What this entails is allowing the parameter CONST in the computer program to vary. In this example it is allowed to equal -1000, 1000, 2000 and 3000 (mm). These correspond to pressure heads (mm) zero, 2000, 3000 and 4000. The values obtained for N_ϕ and N_θ are used in drawing the graphs in Figures 4.3.1a and b. From the graphs it can be observed that except in the case of zero pressure head, N_ϕ is compressive in nature and increases generally after initial decrement as one moves to the base of the shell. N_θ is also compressive but decreases as one moves to the base. In the case of zero pressure head, N_ϕ becomes tensile towards the base of the shell. N_θ in this case is still compressive but increases towards the base of shell.

Section (4.4) On Novozhilov's Graphs

The evaluation of the forces N_ϕ and N_θ acting on a drop shaped shell with varying pressure head was discussed in previous sections. This problem was also considered by Novozhilov [18]. In his discussion an illustration of a case with $\lambda \doteqdot 1.1$, where

$$\lambda = \frac{P_0}{\sqrt{T_0 \rho}}$$

(see equation 4.1.1 in Section (4.1)) having a zero pressure head was given. Assuming that his graphs were obtained by using membrane analysis as an approximate method, it is interesting to investigate a possibility of obtaining for a similar problem similar graphs. In this case it is hoped that by choosing an appropriate set of parameter values such that $\lambda \doteqdot 1.1$ one should end up with graphs which are either in support of the above hypothesis or against the same.

Consider the set of parameter values: Design head = 250.1 mm, Thickness = 4.0 mm, Design stress = 0.15 MN/m^2 and Unit weight of fluid = 11.61 KN/m^3. For this set, $\lambda \doteqdot 1.1$. By considering different values of step-lengths and setting CONST = 0.0 in the computer program STRESS 10 (see appendix A 4.3.1), a suitable set of step-lengths is obtained where the expected N_ϕ and N_θ values agree with the computed ones up to an acceptable percentage. With these step-lengths (values) the case of zero pressure head above this shell is considered by setting CONST = -250.1 mm.

The obtained values of N_ϕ and N_θ are plotted in Figures 4.4.1 and 4.4.2. Notice that here the acting pressure is external whilst in Novozhilov it is internal. The qualitative effect of this is a reversal in sign of force and thus of graphical representations.

Discussion of Graphs

Figure 4.4.3 is obtained from Novozhilov[18]. In his text T_1 and T_2 correspond to N_ϕ and N_θ respectively. However in this figure where he shows T_2 it really should be T_1 and vice versa. As he considers the problem of a tension skin $N_\theta(T_2)$ should at all times be tensile and one might expect $N_\phi(T_1)$ to be tensile over most of the depth of the shell becoming compressive near the base.

In Figure 4.4.3 the nature of the stresses (tensile or compressive) is not indicated. However, by deduction assuming the internal pressure P to be positive, then T_1 (which should be shown as T_2) could be expected to be positive, i.e. tensile, and to the right of datum line as shown. Whereas T_2 (which should be shown as T_1) should be tensile over most of the depth, i.e. plotted to right of vertical datum, going slightly compressive at bottom.

In this thesis the problem considered is that of an underwater structure. Here N_θ should at all times be compressive and one expects N_ϕ to be compressive over most of the depth of the shell becoming tensile near the base. This is supported by the plotted results in Figures 4.4.1 and 4.4.2.

Section (4.5) Summary and Conclusions

The effect of varying hydrostatic pressure loading on the behaviour of the drop shaped shell is theoretically investigated in this chapter. The basis of the investigation is membrane shell theory which is employed in deriving the equations giving the membrane forces in the specified condition. Then the derived equations are implemented on the computer. An example is given and used to illustrate the way the computer program can be put into effect. The chapter also considered a problem which was investigated by Novozhilov [18].

From the work carried out in the chapter, the following can be stated:

(i) membrane theory could be a good theoretical method for investigating the type of problem considered in this chapter provided the obtained results are in agreement with experimental results.

(ii) Membrane theory is limited in scope and application as it does not take bending into consideration. A more accurate analysis could be obtained using finite element method. But it might be possible to have some reasonable agreement between the obtained results from the two approaches. Since a finite element simulation on the computer is likely to be more expensive than the membrane approach in terms of computer time and storage, it may be advantageous to use the membrane approximation in the region of agreement.

(iii) Finally it is possible that the results of this investigation are misleading and if no agreement is found to exist with experimental findings, then it would be necessary to discard this approach.

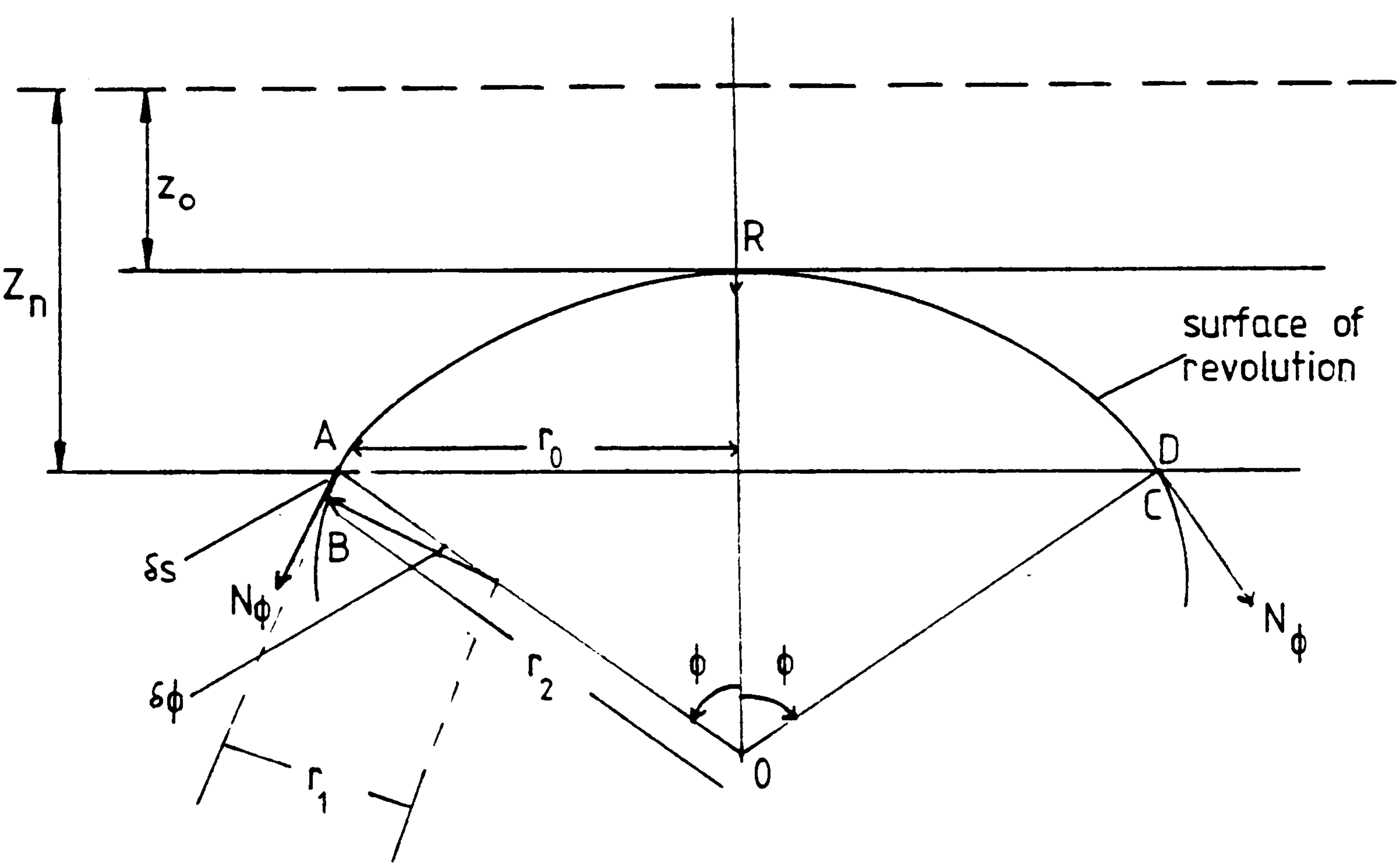

Fig. 4.2.1 Section of a shell

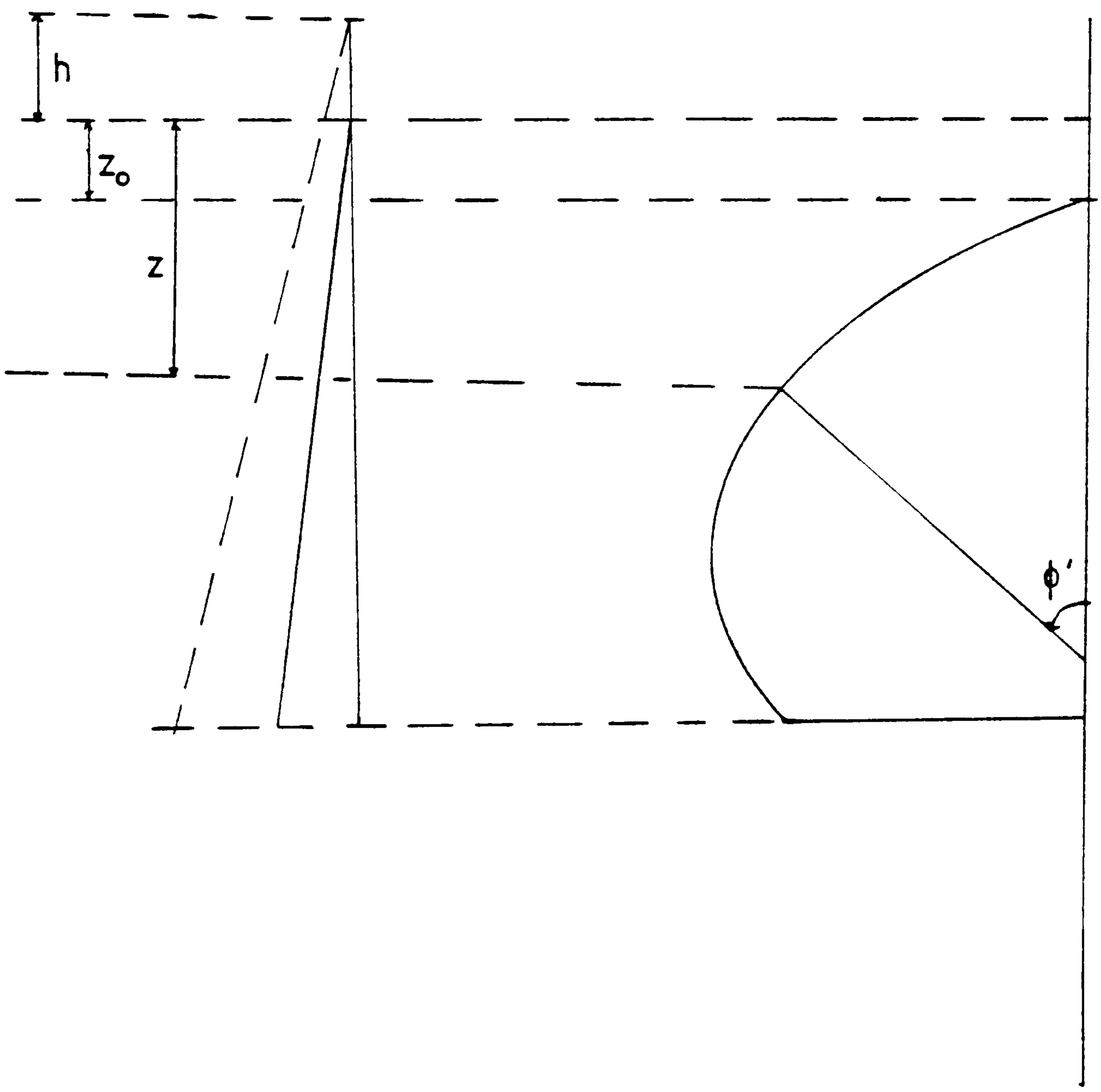

Fig. 4.2.2 Drop shaped shell with varying pressure head

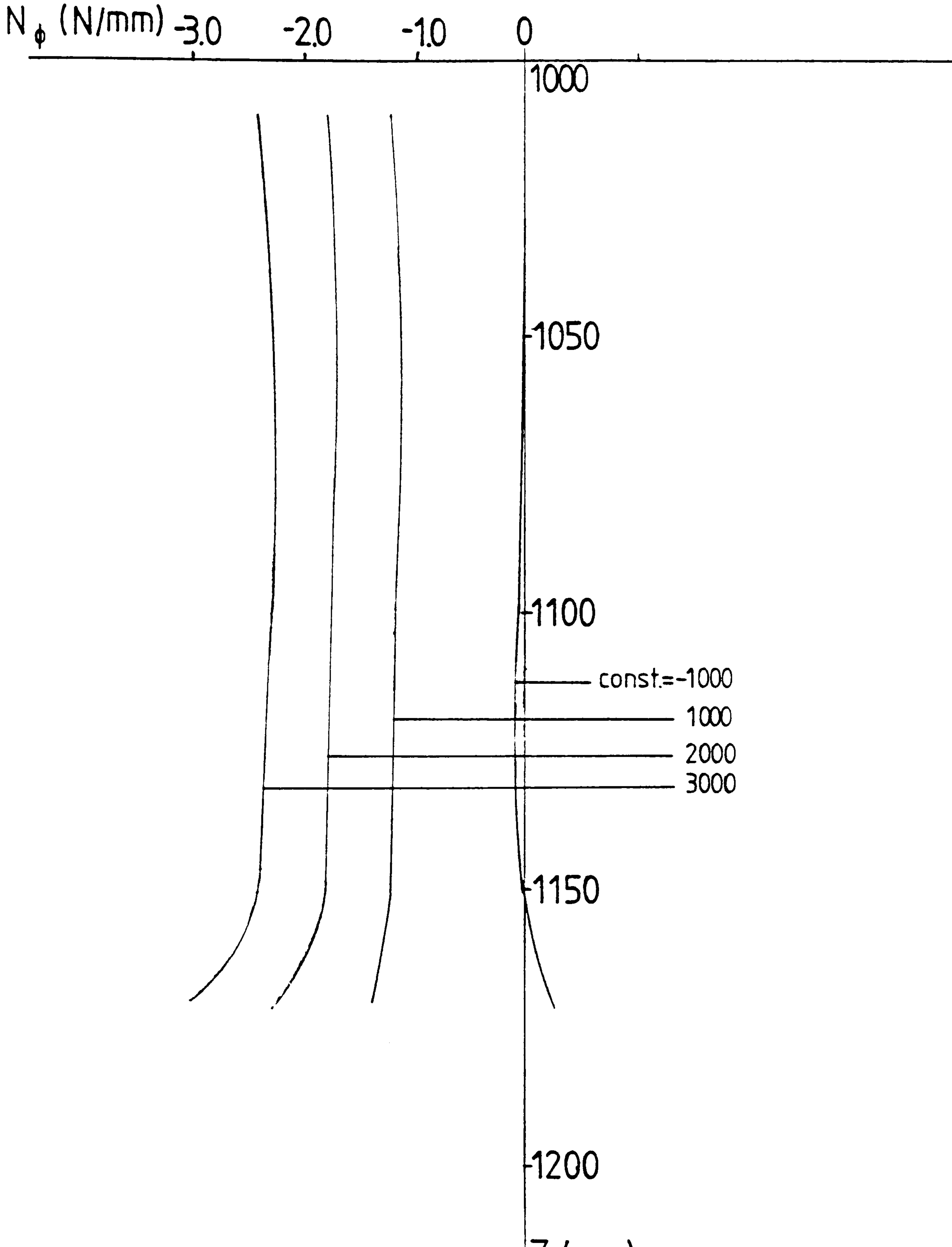

Fig. 4.3.1a Graph showing the effect of varying head on Nφ values (Shell parameters: ZO =1000 mm, T= 4.0 mm, DS = 0.15 MN/m^2, G = 11.61 KN/m^3, DX=DZ=DXX = 0.125)

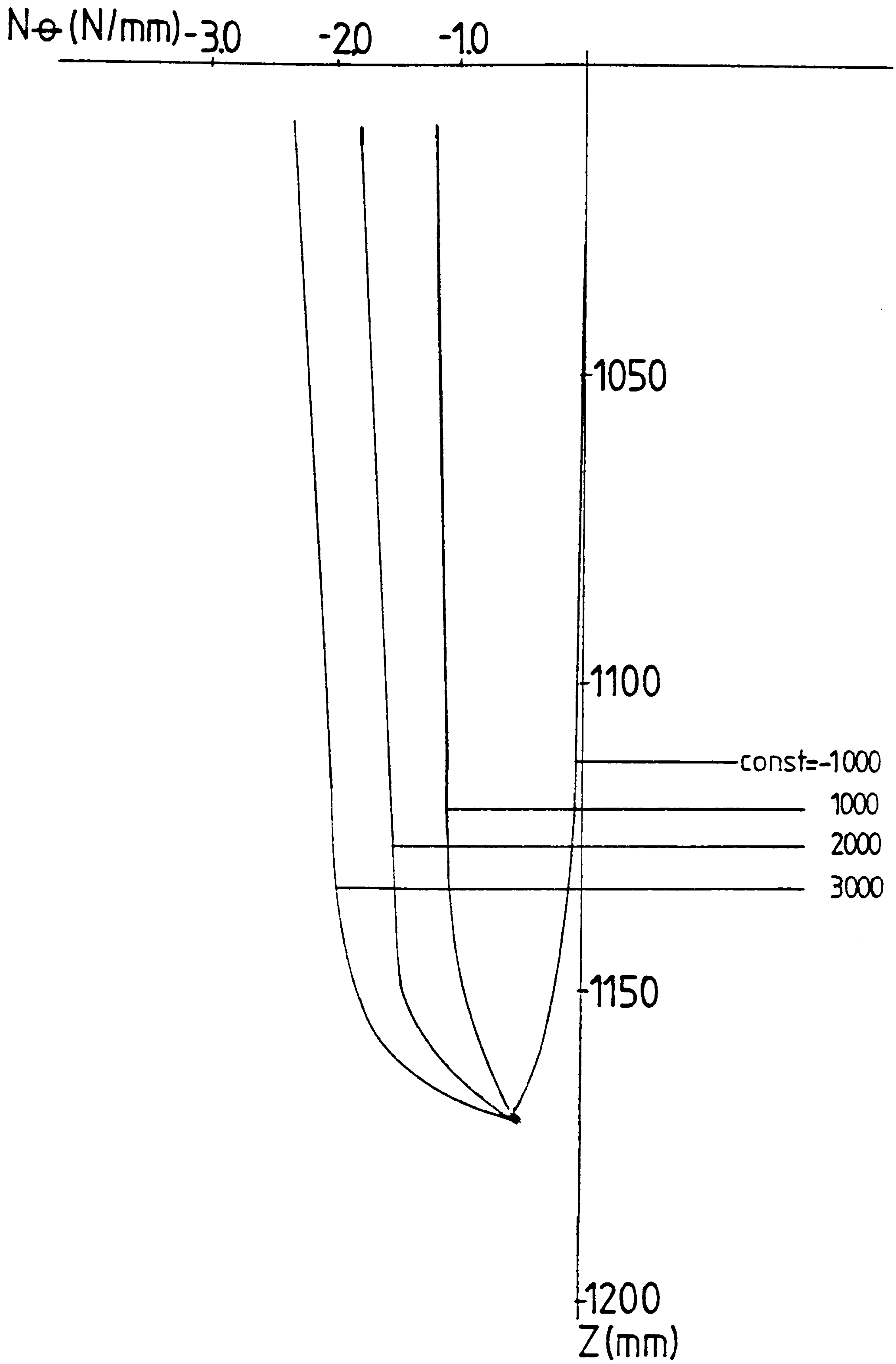

Fig. 4.3.1b Graph showing the effect of varying head on N_θ values (Shell parameters: Z0 = 1000mm, T = 4.0 mm, DS = 0.15 MN/m^2, G = 11.61 KN/m^3, DX = DZ = DXX = 0.125mm)

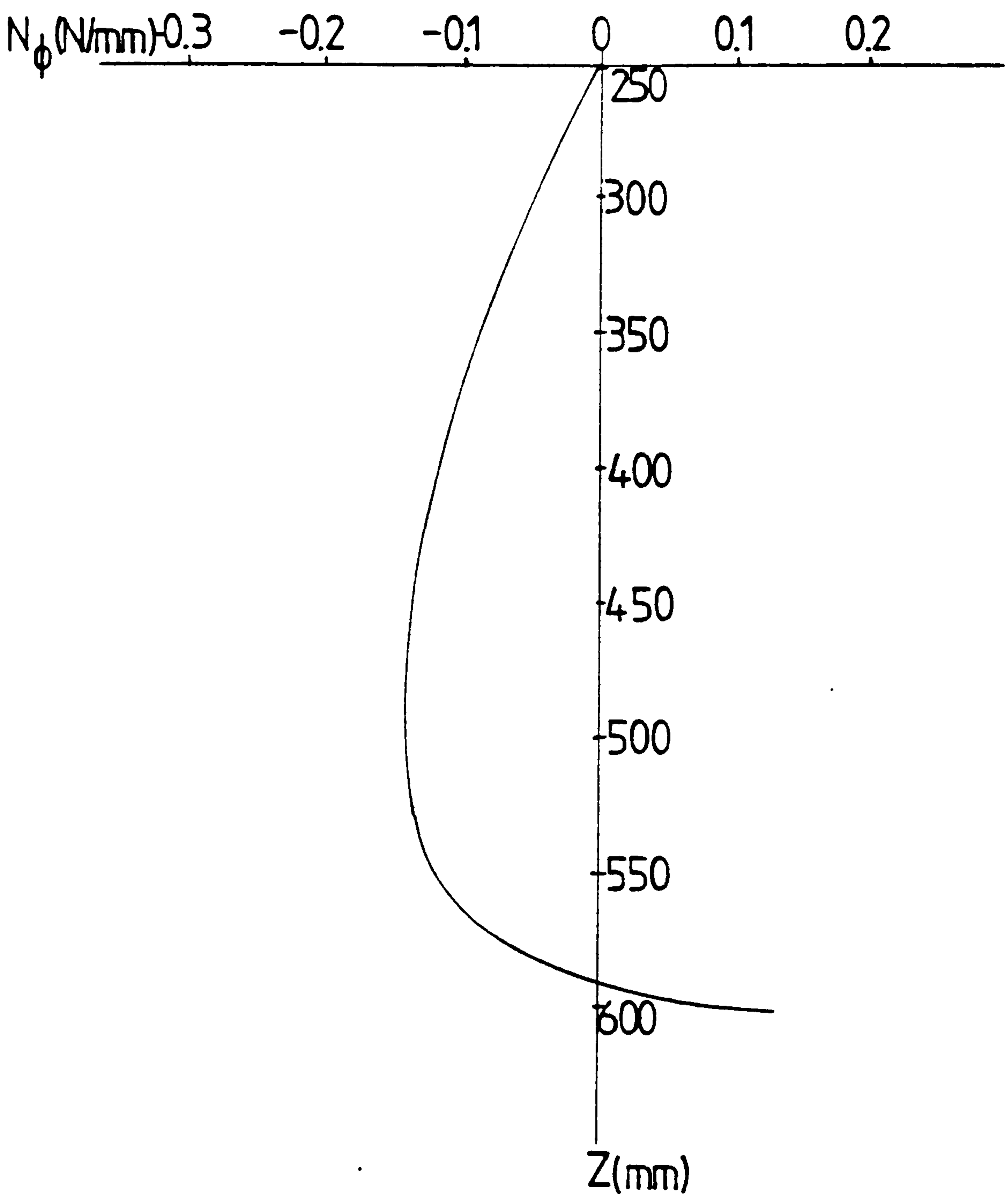

Fig. 4.4.1 Stress resultant in meridional direction
(Case of zero pressure head)

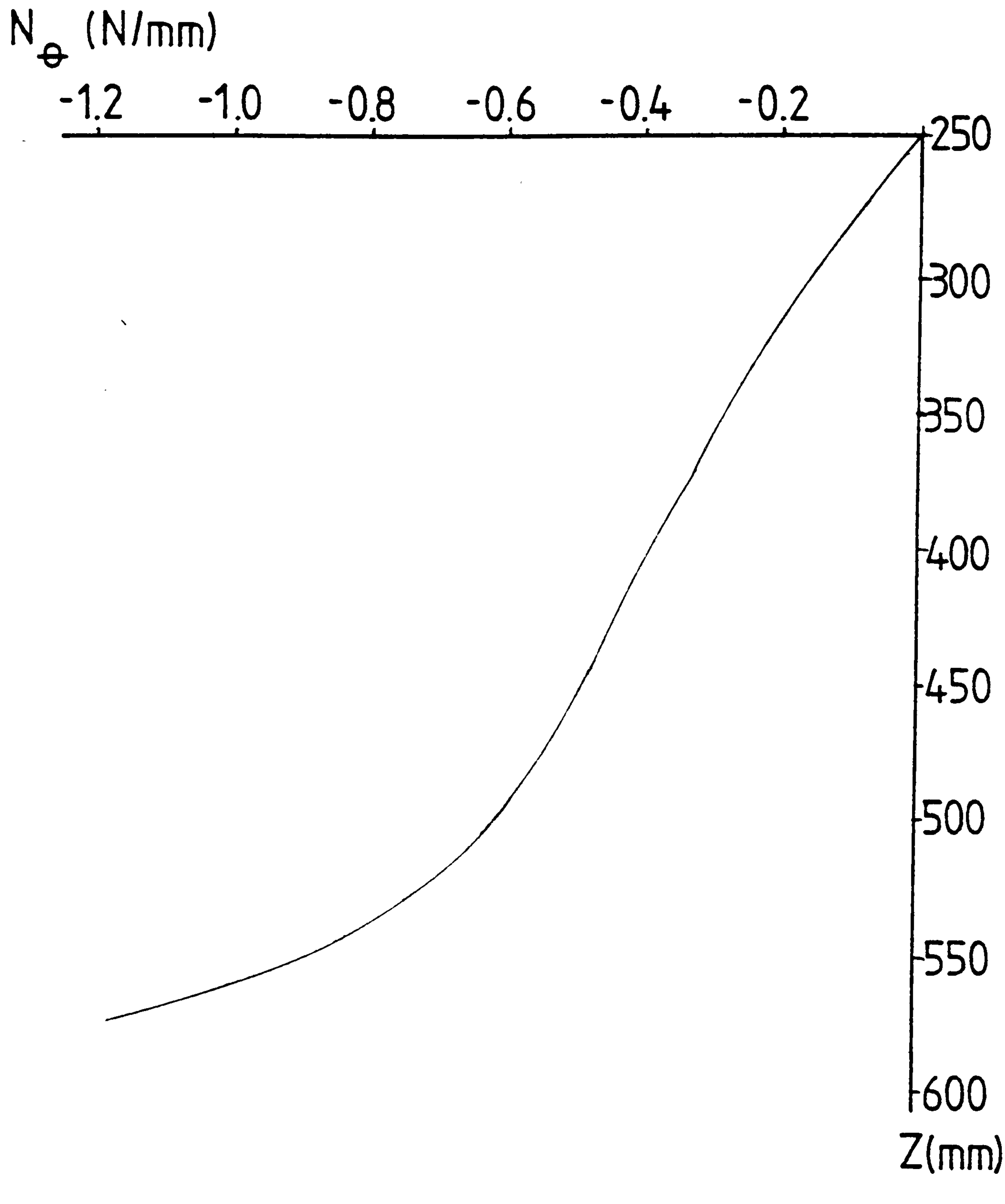

Fig. 4.4.2 Stress resultant in circumferential direction (case of zero pressure head)

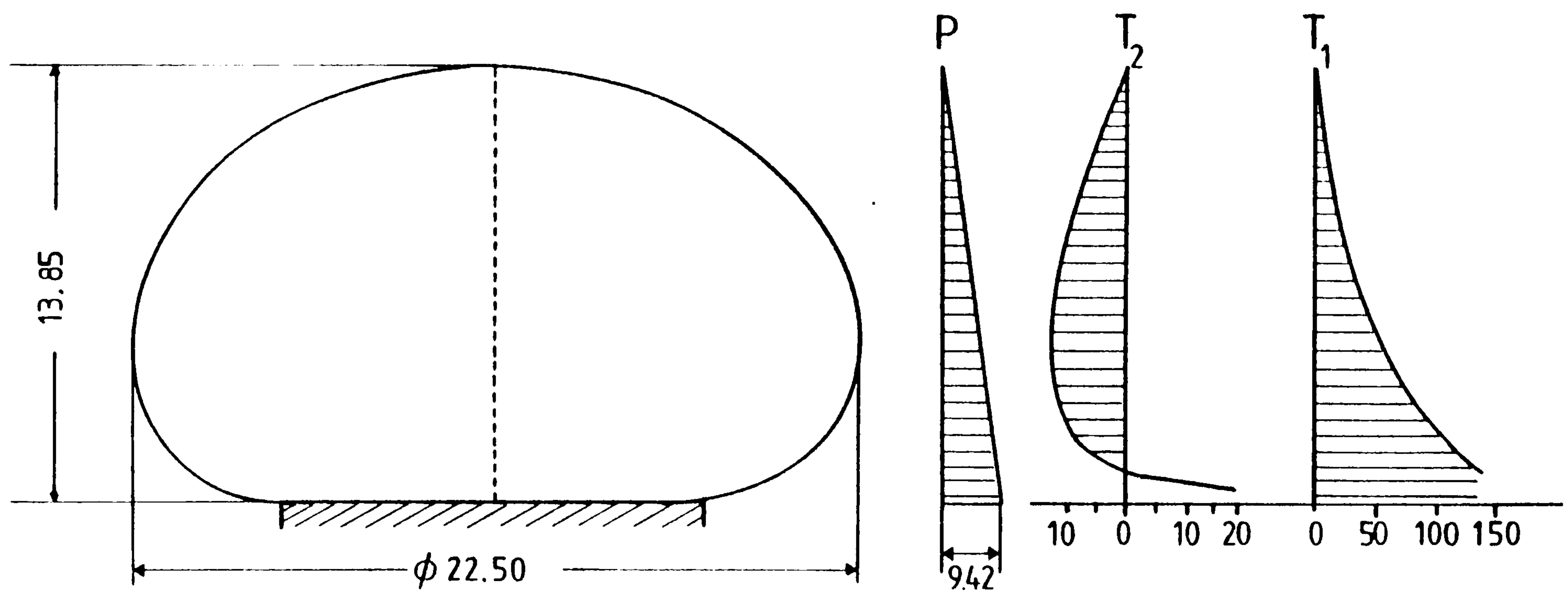

Fig. 4.4.3

Courtesy of P. Noordhoff.

CHAPTER FIVE: ANALYSIS OF THE DROP SHAPED SHELL USING FINITE ELEMENT METHOD

Chapter Five: Analysis of the Drop Shaped Shell using Finite Element Method

Section (5.0) Introduction

An attempt was made in the preceding chapter to investigate the behaviour of the drop shaped shell when the pressure head acting at its apex was different from that for which the shell was designed. The investigation was carried out using membrane shell analysis which does not take some aspects of the natural behaviour of the shell, e.g. bending, into consideration, and so the outcome of such investigation must be limited in scope and application. In this chapter, it is intended to improve on the situation by considering a more realistic model and method of analysis. The results are compared with those obtained by membrane analysis. Firstly types and methods of shell analysis are reviewed.

Section (5.1) Types and Methods of Shell Analysis

In order to understand the response of a shell to static and dynamic loading the following aspects of its behaviour must be examined: (a) stress/deflection, (b) buckling, (c) vibration, and (d) transient response. In stress/deflection analysis, the stresses and deformations in the shell under the applied loads are predicted and compared with permissible ones. In buckling analysis, the structural instability caused by bending and compression of the shell is of interest. In order to assess the possibilities of damage to a shell when subjected to oscillating forces a vibration analysis must be performed and if the shell is acted upon by suddenly applied short term forces its likely transient response must be established. Due to limitations of time only static stress/deflection behaviour is considered here.

The behaviour of shells can be investigated theoretically and experimentally. A theoretical analysis is classified either as analytical or numerical. In an analytical investigation, a classical method is employed in solving the governing differential equations. This invariably resulted in a closed form or series solution. In numerical methods, the equations after a transformation into an appropriate equivalent numerical form are solved by using the electronic digital computer. Numerical methods can further be classified into

a) methods that employ numerical forward technique of integration,

b) methods based on finite differences, and

c) methods based on finite elements.

A discussion of methods (a) and (b) can be found in other sources[55-57]. In this work it was felt that a finite element simulation of the problem would lend itself more readily to a solution.

Section (5.2) Finite Element Method

The finite element method can be regarded as a generalisation of matrix methods of structural analysis. Its basic concept is considering a structure as an assemblage of individual structural components with all of the material properties of the original structure retained in the individual components. In using finite elements method for the analysis of a structure the following steps are normally taken:

(i) the structure is idealised by replacing it with an assemblage of discrete elements,

(ii) the finite element properties of (i) are evaluated,

(iii) the structural analysis of the elements' assemblage is carried out making sure that the conditions of equilibrium, compatibility and force-deflection relationships are satisfied.

To implement the above steps on an electronic digital computer requires

(i) establishing the geometry of a structure,

(ii) declaring the material properties,

(iii) calculating the elements' stiffnesses,

(iv) assembling the overall structure stiffnesses from those obtained in (iii),

(v) establishing the boundary conditions of the problem and thus modifying the matrix in (iv),

(vi) introducing the loads,

(vii) solving the force-displacement equation for displacements,

(viii) calculating the element forces, and

(ix) listing the results obtained.

For a more detailed explanation of this method there are a number of articles and texts giving adequate coverage.[58,59,60]

With an understanding of the method, an appropriate finite element computer program for the problem at hand could be written. Alternatively one could consider the possibility of using an already established program either in its original or modified form. It is noticed that there are numerous general and special purpose finite element computer packages. Some of these are listed and discussed in.[61,62,63] The approach that is followed in this work is based on the approach of Mistry.[64]

Section (5.3) The Mistry Computer Program

Mistry's computer program[64] is a finite element program based on Bushnell's finite difference programs.[65] The description given below follows that given by Mistry.[64]

The computer program investigates the stresses and strains in thin axisymmetric shell structure to obtain their static and dynamic properties. Finite element method is used in deriving the shell stiffness and mass matrices to perform linear small displacement analyses. Geometric stiffness matrices for linear buckling calculations are also performed by this method. Using the program, one can evaluate (a) the displacements due to axisymmetric and certain non-axisymmetric loads and (b) stresses and strains under linear elastic (both geometric and material generated) conditions. In idealising a shell, it is assumed to be made up of a chain of segments, each having a uniform geometrical property, e.g., a cone, cylinder, spherical cap, a part of the torus or any general shape (defined by a set of coordinates). Each segment is further divided into ring finite elements which may take the form of conical or constant curvature frustra (refer to Figure 5.3.1). These nodal rings at the extremeties of each element represents the nodes of the element and each has four degrees of freedom, viz., meridian displacement (u), circumferential displacement (v), normal displacement (w), and rotation about the nodal ring (β). The degrees of freedom are referred to the middle surface of the element. The positional coordinates of the element are the meridian distance (s) and circumferential coordinates (θ). The global x and z axis of any shell segment lie along the axis of symmetry and perpendicular to the axis of symmetry radially outwards. The global y-axis is redundant. The displacements u and v are given by linear functions and for the w displacements a cubic relationship is assumed. A flow diagram and a copy of the updated form of Mistry's program used by the author on

ICL 4-75 and ICL 2900 machines are given in appendices A 5.3.1 and A 5.3.2.

For the problems considered a shell is idealised by a single segment having as many elements as required. Using the coordinate system shown in Figure 5.3.2, if the ends A and B of a shell are at depths DZ_A and DZ_B from water level and x_A, x_B from the origin of this coordinate system, the pressure acting at an arbitrary level, x from the coordinate system datum is given by

$$P = P_A + \rho_L \, g(DZ_A + x - x_A) \ , \qquad (5.3.1)$$

where P_A is atmospheric pressure, ρ_L is the liquid mass density and g is acceleration due to gravity. For external pressure, P_A and $\rho_L g$ are negative.

To use Mistry's computer program (appendix A 5.3.2) in investigating the effect of varying pressure head on the drop shaped shell, the input data that must be supplied are given in appendix A 5.3.3. From this, for every head considered the displacements and stress resultants are evaluated for each element of the segment. The deformed shapes of the shell at the heads may also be drawn by the graph plotter if needed. Finally, using the results of these calculations it should be possible to establish the appropriateness of the drop shaped shell as an underwater structure and to establish the order of the factor of safety it affords against collapse. The drop shaped shells analysed using Mistry's program are described next.

Section (5.4) Analysis of Drop Shaped Shells

(5.4.0) Description of Shells

The first shell shape analyzed arose from an envisaged photoelastic experiment[66]. Its coordinates were generated as described in chapter two with its height, maximum diameter and wall thickness being of the order of 175 mm, 200 mm and 4 mm respectively. The shell is to be constructed from araldite CT200 having material properties: Young's modulus = 0.30×10^{10} N/m^2 (or 3000 N/mm^2) and Poisson's Ratio = 0.38.

The second shell shape analyzed has coordinates of the fibreglass test model used by BAIG[67] and also in this work (chapter six). The coordinates were generated as described in chapter two and its height, maximum diameter and thickness are of the order of 380 mm, 450 mm and 2.5 mm respectively. From the material control tests carried out its material properties were Young's modulus = 0.80×10^{10}N/m^2 (or 8000 N/mm^2) and Poisson's Ratio = 0.36.

For both shells linear stress analysis was carried out. External forces other than those due to hydrostatic pressure were all set to zero and the boundary conditions at the apex of each shell were all free while at the base fixed. Each analysis is for pressure head varying from zero to ten times design head. A typical input data for the first shell after preparation is as in appendix A 5.4.1 and the corresponding output is in Table 5.4.1. The input data for the second shell is similar to that in appendix A 5.4.1 with changes occurring in the material properties, shell coordinates and the number of shell elements required for the problem.

(5.4.1) Numerical Results

The results obtained for the two shells considered are given in Tables 5.4.1 and 5.4.2. For each shell and the considered analysis the meridional (u), normal (w) displacements and rotation (β) are evaluated for each element at the considered head. The stress resultants (N_ϕ, N_θ, $N_{\phi\theta}$, M_ϕ, M_θ, $M_{\phi\theta}$), axisymmetric stresses (σ_ϕ, σ_θ) acting on the wall of the shell internally and externally and the equivalent (Von Misses) stresses are also evaluated. From the results it is possible to deduce the largest in terms of the computed values for each of the above variables and to compare them with allowable values.

(5.4.2) Discussion of Results

The prediction of the computer program for each problem considered is encouraging. For example, the program predicts a failure under static loading of the experimental fibreglass shell by axisymmetric collapse at more than nine times the design head[68]. The prediction for the araldite shell is similar. A typical computer prediction of the deflected form of the araldite shell at 2 × design head is given in Figure 5.4.1. This indicates little distortion and the zone of greatest distortion occurs near the base of shell. It will be interesting to investigate the characteristics of these shells experimentally. One will then be in a position to compare the results obtained by the two approaches.

Using Tables 5.4.1 and 5.4.2 the direct and bending stresses of elements of shells considered can be computed. From this it can be deduced that the bending stresses away from the bottom of the shells are small and therefore negligible when compared with the direct stresses. The table below shows the results of such calculations for the fibreglass shell at design head = 1.525 m (refer to Table 5.4.2.).

ELEMENT NO.		1	30	65
Bending stress, $\frac{6M_\phi}{t^2}$	MN/m^2	0.015	0.002	0.335
Direct stress, $\frac{N_\phi}{t}$	MN/m^2	0.696	0.700	0.800

TABLE 5.4.1

```
C ******** TABLE 5.4.1 ******************************************

        LINEAR STRESS ANALYSIS OF AN ECHIDOME MARCH 1979

    TYPE OF ANALYSIS    6

    NUMBER OF SEGMENTS IN THE STRUCTURE    1

    E= 0.30000D 10  NU= 0.38000D 00  RO= 0.10000D 05

      ATMOSPHERIC PRESSURE 0.00000D 00
      LIQUID DENSITY * G =-0.11610D 05
      DEPTH OF END A 0.00000D 01
      INCREMENT IN DEPTH 0.10000D 01
      MAXIMUM DEPTH OF END A 0.10000D 02

         SEGMENT THICKNESSES

         0.40000D-02

         ANALYSIS FOR N VARYING FROM     0  TO     0

    BOUNDARY CONDITIONS
             1    1    1    1
             0    0    0    0


    EXTERNAL LOADS AT ENDS 0.00000D 00 0.00000D 00 0.00000D 00 0.00000D 00
                           0.00000D 00 0.00000D 00 0.00000D 00 0.00000D 00

                NODE DATA FOR THE SEGMENTS IN THE STRUCTURE

    SEGMENT    1  IS A GENERAL CURVE


    NO. OF ELEMENTS IN THE SEGMENT   60
```

NODE	X	R
1	0.00000000D 00	0.00000000D 00
2	0.30000000D-03	0.80000000D-02
3	0.70000000D-03	0.12000000D-01
4	0.12000000D-02	0.16000000D-01
5	0.19000000D-02	0.20000000D-01
6	0.28000000D-02	0.24000000D-01
7	0.38000000D-02	0.28000000D-01
8	0.51000000D-02	0.32000000D-01
9	0.65000000D-02	0.36000000D-01
10	0.80000000D-02	0.40000000D-01
11	0.98000000D-02	0.44000000D-01
12	0.11800000D-01	0.48000000D-01
13	0.14100000D-01	0.52000000D-01
14	0.16500000D-01	0.56000000D-01
15	0.19300000D-01	0.60000000D-01
16	0.22300000D-01	0.64000000D-01
17	0.25700000D-01	0.68000000D-01
18	0.29400000D-01	0.72000000D-01
19	0.33400000D-01	0.75800000D-01
20	0.37400000D-01	0.79200000D-01

TABLE 5.4.1 (contd.)

21	0.41400000D-01	0.82300000D-01
22	0.45400000D-01	0.85000000D-01
23	0.49400000D-01	0.87500000D-01
24	0.53400000D-01	0.89700000D-01
25	0.57400000D-01	0.91700000D-01
26	0.61400000D-01	0.93500000D-01
27	0.65400000D-01	0.95000000D-01
28	0.69400000D-01	0.96400000D-01
29	0.73400000D-01	0.97500000D-01
30	0.77400000D-01	0.98500000D-01
31	0.81400000D-01	0.99200000D-01
32	0.85400000D-01	0.99800000D-01
33	0.89400000D-01	0.10020000D 00
34	0.93400000D-01	0.10040000D 00
35	0.97400000D-01	0.10050000D 00
36	0.10140000D 00	0.10030000D 00
37	0.10540000D 00	0.10000000D 00
38	0.10940000D 00	0.99500000D-01
39	0.11340000D 00	0.98800000D-01
40	0.11740000D 00	0.97900000D-01
41	0.12140000D 00	0.96900000D-01
42	0.12540000D 00	0.95500000D-01
43	0.12940000D 00	0.94000000D-01
44	0.13340000D 00	0.92200000D-01
45	0.13740000D 00	0.90200000D-01
46	0.14140000D 00	0.87800000D-01
47	0.14540000D 00	0.85200000D-01
48	0.14940000D 00	0.82100000D-01
49	0.15340000D 00	0.78600000D-01
50	0.15740000D 00	0.74600000D-01
51	0.16100000D 00	0.70600000D-01
52	0.16400000D 00	0.66600000D-01
53	0.16670000D 00	0.62600000D-01
54	0.16890000D 00	0.58600000D-01
55	0.17080000D 00	0.54600000D-01
56	0.17240000D 00	0.50600000D-01
57	0.17370000D 00	0.46600000D-01
58	0.17470000D 00	0.42600000D-01
59	0.17540000D 00	0.38600000D-01
60	0.17580000D 00	0.34600000D-01
61	0.17590000D 00	0.30600000D-01

SEGMENT NO. 1

ELEMENT	R(1)	R(2)	PHI(1)	PHI(2)	SL
1	0.0000000D 00	0.8000000D-02	0.8785241D 02	0.8785241D 02	0.8005623D-02
2	0.8000000D-02	0.1200000D-01	0.8428941D 02	0.8428941D 02	0.4019950D-02
3	0.1200000D-01	0.1600000D-01	0.8287498D 02	0.8287498D 02	0.4031129D-02
4	0.1600000D-01	0.2000000D-01	0.8007375D 02	0.8007375D 02	0.4060788D-02
5	0.2000000D-01	0.2400000D-01	0.7731962D 02	0.7731962D 02	0.4100000D-02
6	0.2400000D-01	0.2800000D-01	0.7596376D 02	0.7596376D 02	0.4123106D-02
7	0.2800000D-01	0.3200000D-01	0.7199584D 02	0.7199584D 02	0.4205948D-02
8	0.3200000D-01	0.3600000D-01	0.7070995D 02	0.7070995D 02	0.4237924D-02
9	0.3600000D-01	0.4000000D-01	0.6944395D 02	0.6944395D 02	0.4272002D-02
10	0.4000000D-01	0.4400000D-01	0.6577225D 02	0.6577225D 02	0.4386342D-02
11	0.4400000D-01	0.4800000D-01	0.6343495D 02	0.6343495D 02	0.4472136D-02
12	0.4800000D-01	0.5200000D-01	0.6010110D 02	0.6010110D 02	0.4614109D-02
13	0.5200000D-01	0.5600000D-01	0.5903624D 02	0.5903624D 02	0.4664762D-02
14	0.5600000D-01	0.6000000D-01	0.5500798D 02	0.5500798D 02	0.4882622D-02
15	0.6000000D-01	0.6400000D-01	0.5313010D 02	0.5313010D 02	0.5000000D-02

TABLE 5.4.1 (contd.)

16	0.6400000D-01	0.6800000D-01	0.4963546D 02	0.4963546D 02	0.5249762D-02
17	0.6800000D-01	0.7200000D-01	0.4723117D 02	0.4723117D 02	0.5448853D-02
18	0.7200000D-01	0.7580000D-01	0.4353120D 02	0.4353120D 02	0.5517246D-02
19	0.7580000D-01	0.7920000D-01	0.4036454D 02	0.4036454D 02	0.5249762D-02
20	0.7920000D-01	0.8230000D-01	0.3777568D 02	0.3777568D 02	0.5060632D-02
21	0.8230000D-01	0.8500000D-01	0.3401935D 02	0.3401935D 02	0.4825971D-02
22	0.8500000D-01	0.8750000D-01	0.3200538D 02	0.3200538D 02	0.4716991D-02
23	0.8750000D-01	0.8970000D-01	0.2881079D 02	0.2881079D 02	0.4565085D-02
24	0.8970000D-01	0.9170000D-01	0.2656505D 02	0.2656505D 02	0.4472136D-02
25	0.9170000D-01	0.9350000D-01	0.2422775D 02	0.2422775D 02	0.4386342D-02
26	0.9350000D-01	0.9500000D-01	0.2055605D 02	0.2055605D 02	0.4272002D-02
27	0.9500000D-01	0.9640000D-01	0.1929005D 02	0.1929005D 02	0.4237924D-02
28	0.9640000D-01	0.9750000D-01	0.1537625D 02	0.1537625D 02	0.4148494D-02
29	0.9750000D-01	0.9850000D-01	0.1403624D 02	0.1403624D 02	0.4123106D-02
30	0.9850000D-01	0.9920000D-01	0.9926246D 01	0.9926246D 01	0.4060788D-02
31	0.9920000D-01	0.9980000D-01	0.8530766D 01	0.8530766D 01	0.4044750D-02
32	0.9980000D-01	0.1002000D 00	0.5710593D 01	0.5710593D 01	0.4019950D-02
33	0.1002000D 00	0.1004000D 00	0.2862405D 01	0.2862405D 01	0.4004997D-02
34	0.1004000D 00	0.1005000D 00	0.1432096D 01	0.1432096D 01	0.4001250D-02
35	0.1005000D 00	0.1003000D 00	-0.2862405D 01	-0.2862405D 01	0.4004997D-02
36	0.1003000D 00	0.1000000D 00	-0.4289153D 01	-0.4289153D 01	0.4011234D-02
37	0.1000000D 00	0.9950000D-01	-0.7125016D 01	-0.7125016D 01	0.4031129D-02
38	0.9950000D-01	0.9880000D-01	-0.9926246D 01	-0.9926246D 01	0.4060788D-02
39	0.9880000D-01	0.9790000D-01	-0.1268038D 02	-0.1268038D 02	0.4100000D-02
40	0.9790000D-01	0.9690000D-01	-0.1403624D 02	-0.1403624D 02	0.4123106D-02
41	0.9690000D-01	0.9550000D-01	-0.1929005D 02	-0.1929005D 02	0.4237924D-02
42	0.9550000D-01	0.9400000D-01	-0.2055605D 02	-0.2055605D 02	0.4272002D-02
43	0.9400000D-01	0.9220000D-01	-0.2422775D 02	-0.2422775D 02	0.4386342D-02
44	0.9220000D-01	0.9020000D-01	-0.2656505D 02	-0.2656505D 02	0.4472136D-02
45	0.9020000D-01	0.8780000D-01	-0.3096376D 02	-0.3096376D 02	0.4664762D-02
46	0.8780000D-01	0.8520000D-01	-0.3302387D 02	-0.3302387D 02	0.4770744D-02
47	0.8520000D-01	0.8210000D-01	-0.3777568D 02	-0.3777568D 02	0.5060632D-02
48	0.8210000D-01	0.7860000D-01	-0.4118593D 02	-0.4118593D 02	0.5315073D-02
49	0.7860000D-01	0.7460000D-01	-0.4500000D 02	-0.4500000D 02	0.5656854D-02
50	0.7460000D-01	0.7060000D-01	-0.4801279D 02	-0.4801279D 02	0.5381450D-02
51	0.7060000D-01	0.6660000D-01	-0.5313010D 02	-0.5313010D 02	0.5000000D-02
52	0.6660000D-01	0.6260000D-01	-0.5598065D 02	-0.5598065D 02	0.4825971D-02
53	0.6260000D-01	0.5860000D-01	-0.6118921D 02	-0.6118921D 02	0.4565085D-02
54	0.5860000D-01	0.5460000D-01	-0.6459228D 02	-0.6459228D 02	0.4428318D-02
55	0.5460000D-01	0.5060000D-01	-0.6819859D 02	-0.6819859D 02	0.4308132D-02
56	0.5060000D-01	0.4660000D-01	-0.7199584D 02	-0.7199584D 02	0.4205948D-02
57	0.4660000D-01	0.4260000D-01	-0.7596376D 02	-0.7596376D 02	0.4123106D-02
58	0.4260000D-01	0.3860000D-01	-0.8007375D 02	-0.8007375D 02	0.4060788D-02
59	0.3860000D-01	0.3460000D-01	-0.8428941D 02	-0.8428941D 02	0.4019950D-02
60	0.3460000D-01	0.3060000D-01	-0.8856790D 02	-0.8856790D 02	0.4001250D-02

DEGREES OF FREEDOM, IDF= 179 MAXIMUM BAND SIZE, IBAND= 6

DISPLACEMENT NUMBERS ASSIGNED TO THE NODES

NODE	U	V	W	BETA
1	1	0	2	0
2	3	0	4	5
3	6	0	7	8
4	9	0	10	11
5	12	0	13	14
6	15	0	16	17
7	18	0	19	20

TABLE 5.4.1 (contd.)

8	21	0	22	23
9	24	0	25	26
10	27	0	28	29
11	30	0	31	32
12	33	0	34	35
13	36	0	37	38
14	39	0	40	41
15	42	0	43	44
16	45	0	46	47
17	48	0	49	50
18	51	0	52	53
19	54	0	55	56
20	57	0	58	59
21	60	0	61	62
22	63	0	64	65
23	66	0	67	68
24	69	0	70	71
25	72	0	73	74
26	75	0	76	77
27	78	0	79	80
28	81	0	82	83
29	84	0	85	86
30	87	0	88	89
31	90	0	91	92
32	93	0	94	95
33	96	0	97	98
34	99	0	100	101
35	102	0	103	104
36	105	0	106	107
37	108	0	109	110
38	111	0	112	113
39	114	0	115	116
40	117	0	118	119
41	120	0	121	122
42	123	0	124	125
43	126	0	127	128
44	129	0	130	131
45	132	0	133	134
46	135	0	136	137
47	138	0	139	140
48	141	0	142	143
49	144	0	145	146
50	147	0	148	149
51	150	0	151	152
52	153	0	154	155
53	156	0	157	158
54	159	0	160	161
55	162	0	163	164
56	165	0	166	167
57	168	0	169	170
58	171	0	172	173
59	174	0	175	176
60	177	0	178	179
61	0	0	0	0

TABLE 5.4.1 (contd.)

DISPLACEMENT AND STRESS RESULTANTS OF 'SEGMENT 1 SHELL TYPE 3 DZA= 0.10000D 01

ELM	S	U	V	W	β	N_ϕ	N_θ	$N_{\phi\theta}$	M_ϕ	M_θ	$M_{\phi\theta}$
1	0.400D-02	0.448D-09	0.000D 00	-0.333D-05	-0.479D-05	-0.603D 03	-0.603D 03	0.000D 00	0.239D-01	0.282D-01	0.000D 00
2	0.100D-01	0.187D-07	0.000D 00	-0.330D-05	-0.609D-05	-0.603D 03	-0.600D 03	0.000D 00	0.254D-02	0.107D-01	0.000D 00
3	0.140D-01	-0.258D-07	0.000D 00	-0.327D-05	-0.604D-05	-0.602D 03	-0.599D 03	0.000D 00	-0.450D-02	0.514D-02	0.000D 00
4	0.181D-01	0.756D-08	0.000D 00	-0.326D-05	-0.265D-05	-0.601D 03	-0.598D 03	0.000D 00	-0.168D-01	-0.407D-02	0.000D 00
5	0.222D-01	0.368D-07	0.000D 00	-0.325D-05	0.658D-06	-0.600D 03	-0.598D 03	0.000D 00	-0.833D-02	-0.363D-02	0.000D 00
6	0.263D-01	-0.140D-07	0.000D 00	-0.326D-05	0.163D-05	-0.600D 03	-0.599D 03	0.000D 00	-0.112D-01	-0.523D-02	0.000D 00
7	0.304D-01	0.826D-07	0.000D 00	-0.327D-05	0.445D-05	-0.600D 03	-0.600D 03	0.000D 00	0.183D-02	-0.156D-02	0.000D 00
8	0.347D-01	0.254D-07	0.000D 00	-0.328D-05	-0.107D-06	-0.600D 03	-0.602D 03	0.000D 00	0.252D-01	0.963D-02	0.000D 00
9	0.389D-01	-0.341D-07	0.000D 00	-0.327D-05	-0.428D-05	-0.600D 03	-0.600D 03	0.000D 00	0.620D-03	0.192D-02	0.000D 00
10	0.432D-01	0.403D-07	0.000D 00	-0.325D-05	-0.410D-06	-0.600D 03	-0.599D 03	0.000D 00	-0.188D-01	-0.699D-02	0.000D 00
11	0.477D-01	0.356D-07	0.000D 00	-0.326D-05	0.318D-05	-0.600D 03	-0.600D 03	0.000D 00	-0.140D-01	-0.630D-02	0.000D 00
12	0.522D-01	0.849D-07	0.000D 00	-0.328D-05	0.514D-05	-0.600D 03	-0.602D 03	0.000D 00	0.559D-02	0.698D-03	0.000D 00
13	0.569D-01	0.254D-08	0.000D 00	-0.329D-05	0.139D-05	-0.600D 03	-0.604D 03	0.000D 00	0.510D-02	0.158D-02	0.000D 00
14	0.616D-01	0.865D-07	0.000D 00	-0.330D-05	0.261D-05	-0.601D 03	-0.605D 03	0.000D 00	0.159D-02	0.130D-04	0.000D 00
15	0.666D-01	0.420D-07	0.000D 00	-0.330D-05	-0.122D-06	-0.601D 03	-0.605D 03	0.000D 00	0.690D-02	0.265D-02	0.000D 00
16	0.717D-01	0.846D-07	0.000D 00	-0.329D-05	-0.660D-06	-0.601D 03	-0.605D 03	0.000D 00	0.489D-02	0.198D-02	0.000D 00
17	0.771D-01	0.570D-07	0.000D 00	-0.328D-05	-0.259D-05	-0.601D 03	-0.603D 03	0.000D 00	-0.288D-03	0.325D-03	0.000D 00
18	0.825D-01	0.984D-07	0.000D 00	-0.327D-05	-0.684D-07	-0.601D 03	-0.602D 03	0.000D 00	-0.887D-02	-0.336D-02	0.000D 00
19	0.879D-01	0.112D-06	0.000D 00	-0.327D-05	0.168D-05	-0.601D 03	-0.603D 03	0.000D 00	-0.286D-02	-0.131D-02	0.000D 00
20	0.931D-01	0.998D-07	0.000D 00	-0.327D-05	0.219D-05	-0.602D 03	-0.604D 03	0.000D 00	-0.666D-02	-0.280D-02	0.000D 00
21	0.980D-01	0.161D-06	0.000D 00	-0.328D-05	0.445D-05	-0.602D 03	-0.606D 03	0.000D 00	0.412D-03	-0.320D-03	0.000D 00
22	0.103D 00	0.129D-06	0.000D 00	-0.329D-05	0.178D-05	-0.602D 03	-0.608D 03	0.000D 00	0.103D-01	0.374D-02	0.000D 00
23	0.107D 00	0.170D-06	0.000D 00	-0.329D-05	-0.218D-06	-0.602D 03	-0.608D 03	0.000D 00	0.113D-01	0.433D-02	0.000D 00
24	0.112D 00	0.158D-06	0.000D 00	-0.327D-05	-0.382D-05	-0.602D 03	-0.607D 03	0.000D 00	0.134D-01	0.541D-02	0.000D 00
25	0.116D 00	0.154D-06	0.000D 00	-0.324D-05	-0.558D-05	-0.602D 03	-0.604D 03	0.000D 00	-0.187D-02	-0.316D-03	0.000D 00
26	0.121D 00	0.226D-06	0.000D 00	-0.321D-05	-0.389D-05	-0.602D 03	-0.602D 03	0.000D 00	-0.483D-03	0.483D-04	0.000D 00
27	0.125D 00	0.164D-06	0.000D 00	-0.319D-05	-0.497D-05	-0.602D 03	-0.599D 03	0.000D 00	-0.468D-02	-0.151D-02	0.000D 00
28	0.129D 00	0.250D-06	0.000D 00	-0.316D-05	-0.172D-05	-0.602D 03	-0.598D 03	0.000D 00	-0.956D-02	-0.356D-02	0.000D 00
29	0.133D 00	0.195D-06	0.000D 00	-0.315D-05	-0.847D-06	-0.602D 03	-0.597D 03	0.000D 00	-0.116D-01	-0.438D-02	0.000D 00
30	0.137D 00	0.292D-06	0.000D 00	-0.314D-05	0.304D-05	-0.602D 03	-0.598D 03	0.000D 00	-0.730D-02	-0.286D-02	0.000D 00
31	0.141D 00	0.242D-06	0.000D 00	-0.314D-05	0.186D-05	-0.602D 03	-0.599D 03	0.000D 00	0.588D-02	0.219D-02	0.000D 00
32	0.145D 00	0.271D-06	0.000D 00	-0.313D-05	0.151D-05	-0.602D 03	-0.600D 03	0.000D 00	0.253D-03	0.723D-04	0.000D 00
33	0.149D 00	0.301D-06	0.000D 00	-0.313D-05	0.109D-05	-0.602D 03	-0.600D 03	0.000D 00	0.676D-02	0.256D-02	0.000D 00
34	0.153D 00	0.255D-06	0.000D 00	-0.312D-05	-0.462D-06	-0.602D 03	-0.600D 03	0.000D 00	-0.411D-02	-0.156D-02	0.000D 00
35	0.157D 00	0.363D-06	0.000D 00	-0.310D-05	0.218D-05	-0.602D 03	-0.601D 03	0.000D 00	-0.251D-02	-0.938D-03	0.000D 00
36	0.161D 00	0.315D-06	0.000D 00	-0.310D-05	-0.878D-08	-0.602D 03	-0.601D 03	0.000D 00	0.107D-01	0.406D-02	0.000D 00
37	0.166D 00	0.343D-06	0.000D 00	-0.308D-05	-0.134D-05	-0.602D 03	-0.601D 03	0.000D 00	0.390D-02	0.146D-02	0.000D 00
38	0.170D 00	0.367D-06	0.000D 00	-0.305D-05	-0.211D-05	-0.602D 03	-0.600D 03	0.000D 00	0.469D-02	0.172D-02	0.000D 00
39	0.174D 00	0.386D-06	0.000D 00	-0.302D-05	-0.362D-05	-0.602D 03	-0.599D 03	0.000D 00	0.959D-02	0.351D-02	0.000D 00
40	0.178D 00	0.329D-06	0.000D 00	-0.300D-05	-0.472D-05	-0.602D 03	-0.597D 03	0.000D 00	-0.149D-01	-0.585D-02	0.000D 00
41	0.182D 00	0.471D-06	0.000D 00	-0.295D-05	0.173D-05	-0.602D 03	-0.596D 03	0.000D 00	-0.163D-01	-0.608D-02	0.000D 00
42	0.186D 00	0.403D-06	0.000D 00	-0.295D-05	0.166D-05	-0.602D 03	-0.597D 03	0.000D 00	-0.133D-02	-0.405D-03	0.000D 00
43	0.191D 00	0.456D-06	0.000D 00	-0.293D-05	0.355D-05	-0.602D 03	-0.598D 03	0.000D 00	-0.702D-02	-0.242D-02	0.000D 00
44	0.195D 00	0.437D-06	0.000D 00	-0.293D-05	0.511D-05	-0.602D 03	-0.600D 03	0.000D 00	-0.145D-01	-0.510D-02	0.000D 00
45	0.200D 00	0.520D-06	0.000D 00	-0.293D-05	0.990D-05	-0.602D 03	-0.604D 03	0.000D 00	-0.959D-02	-0.273D-02	0.000D 00
46	0.204D 00	0.480D-06	0.000D 00	-0.296D-05	0.101D-04	-0.602D 03	-0.610D 03	0.000D 00	-0.699D-02	-0.163D-02	0.000D 00
47	0.209D 00	0.575D-06	0.000D 00	-0.298D-05	0.135D-04	-0.602D 03	-0.617D 03	0.000D 00	-0.117D-02	0.114D-02	0.000D 00
48	0.214D 00	0.595D-06	0.000D 00	-0.301D-05	0.902D-05	-0.601D 03	-0.625D 03	0.000D 00	0.316D-01	0.132D-01	0.000D 00
49	0.220D 00	0.630D-06	0.000D 00	-0.298D-05	-0.405D-05	-0.599D 03	-0.628D 03	0.000D 00	0.563D-01	0.208D-01	0.000D 00
50	0.225D 00	0.618D-06	0.000D 00	-0.288D-05	-0.214D-04	-0.598D 03	-0.621D 03	0.000D 00	0.444D-01	0.134D-01	0.000D 00
51	0.231D 00	0.709D-06	0.000D 00	-0.268D-05	-0.306D-04	-0.597D 03	-0.608D 03	0.000D 00	0.331D-01	0.686D-02	0.000D 00
52	0.235D 00	0.686D-06	0.000D 00	-0.247D-05	-0.412D-04	-0.597D 03	-0.589D 03	0.000D 00	0.276D-01	0.204D-02	0.000D 00
53	0.240D 00	0.751D-06	0.000D 00	-0.220D-05	-0.470D-04	-0.598D 03	-0.567D 03	0.000D 00	0.255D-01	-0.117D-02	0.000D 00
54	0.245D 00	0.728D-06	0.000D 00	-0.192D-05	-0.573D-04	-0.601D 03	-0.543D 03	0.000D 00	0.433D-01	0.183D-02	0.000D 00
55	0.249D 00	0.692D-06	0.000D 00	-0.160D-05	-0.693D-04	-0.607D 03	-0.513D 03	0.000D 00	0.390D-01	-0.475D-02	0.000D 00
56	0.253D 00	0.636D-06	0.000D 00	-0.124D-05	-0.784D-04	-0.616D 03	-0.478D 03	0.000D 00	0.173D-01	-0.180D-01	0.000D 00
57	0.257D 00	0.553D-06	0.000D 00	-0.863D-06	-0.812D-04	-0.631D 03	-0.441D 03	0.000D 00	-0.187D-01	-0.354D-01	0.000D 00
58	0.262D 00	0.438D-06	0.000D 00	-0.504D-06	-0.750D-04	-0.653D 03	-0.401D 03	0.000D 00	-0.674D-01	-0.547D-01	0.000D 00
59	0.266D 00	0.287D-06	0.000D 00	-0.206D-06	-0.568D-04	-0.683D 03	-0.360D 03	0.000D 00	-0.129D 00	-0.736D-01	0.000D 00
60	0.270D 00	0.101D-06	0.000D 00	-0.243D-07	-0.235D-04	-0.727D 03	-0.314D 03	0.000D 00	-0.203D 00	-0.886D-01	0.000D 00

TABLE 5.4.1 (contd.)

AXISYMMETRIC STRESSES AND EQUIVALENT STRESSES

SEGMENT NO. 1

ELEMENT	STATION	σ_θ(IN)	σ_θ(OUT)	σ_ϕ (IN)	σ_ϕ (OUT)	σ_{EQ}(IN)	σ_{EQ}(OUT)
1	0.4002812D-02	-0.1400810D 06	-0.1612570D 06	-0.1419117D 06	-0.1598256D 06	0.1410052D 06	0.1489347D 06
2	0.1001560D-01	-0.1461252D 06	-0.1541231D 06	-0.1497321D 06	-0.1516358D 06	0.1479616D 06	0.1412801D 06
3	0.1404114D-01	-0.1477368D 06	-0.1515894D 06	-0.1521558D 06	-0.1487842D 06	0.1499952D 06	0.1385871D 06
4	0.1808710D-01	-0.1509259D 06	-0.1478726D 06	-0.1566016D 06	-0.1439882D 06	0.1538423D 06	0.1343445D 06
5	0.2216749D-01	-0.1508252D 06	-0.1481011D 06	-0.1532446D 06	-0.1469971D 06	0.1520493D 06	0.1359152D 06
6	0.2627904D-01	-0.1516426D 06	-0.1477185D 06	-0.1542435D 06	-0.1458361D 06	0.1529597D 06	0.1351388D 06
7	0.3044357D-01	-0.1506854D 06	-0.1495145D 06	-0.1492879D 06	-0.1506619D 06	0.1499915D 06	0.1384301D 06
8	0.3466551D-01	-0.1467726D 06	-0.1539961D 06	-0.1405334D 06	-0.1594486D 06	0.1437546D 06	0.1451219D 06
9	0.3892047D-01	-0.1493223D 06	-0.1507638D 06	-0.1498015D 06	-0.1502668D 06	0.1495625D 06	0.1388218D 06
10	0.4324964D-01	-0.1523798D 06	-0.1471407D 06	-0.1571139D 06	-0.1430460D 06	0.1548012D 06	0.1334272D 06
11	0.4767888D-01	-0.1523345D 06	-0.1476131D 06	-0.1552876D 06	-0.1448169D 06	0.1538323D 06	0.1345014D 06
12	0.5222200D-01	-0.1502923D 06	-0.1508159D 06	-0.1479243D 06	-0.1521145D 06	0.1491224D 06	0.1397094D 06
13	0.5686144D-01	-0.1503441D 06	-0.1515323D 06	-0.1481639D 06	-0.1519863D 06	0.1492660D 06	0.1399722D 06
14	0.6163513D-01	-0.1511470D 06	-0.1511568D 06	-0.1495563D 06	-0.1507480D 06	0.1503580D 06	0.1391350D 06
15	0.6657644D-01	-0.1502834D 06	-0.1522693D 06	-0.1476278D 06	-0.1528043D 06	0.1489733D 06	0.1406861D 06
16	0.7170132D-01	-0.1504124D 06	-0.1518984D 06	-0.1484452D 06	-0.1521152D 06	0.1494385D 06	0.1401183D 06
17	0.7705063D-01	-0.1507198D 06	-0.1509638D 06	-0.1504341D 06	-0.1502180D 06	0.1505771D 06	0.1386626D 06
18	0.8253368D-01	-0.1518476D 06	-0.1493276D 06	-0.1536884D 06	-0.1470366D 06	0.1527763D 06	0.1362220D 06
19	0.8791718D-01	-0.1512141D 06	-0.1502291D 06	-0.1514415D 06	-0.1492936D 06	0.1513279D 06	0.1377428D 06
20	0.9307238D-01	-0.1520336D 06	-0.1499365D 06	-0.1528904D 06	-0.1478975D 06	0.1524638D 06	0.1368610D 06
21	0.9801568D-01	-0.1515771D 06	-0.1513374D 06	-0.1502496D 06	-0.1505585D 06	0.1509177D 06	0.1388357D 06
22	0.1027872D 00	-0.1504811D 06	-0.1532845D 06	-0.1465703D 06	-0.1542933D 06	0.1485643D 06	0.1416312D 06
23	0.1074282D 00	-0.1503270D 06	-0.1535739D 06	-0.1462151D 06	-0.1547220D 06	0.1483138D 06	0.1419446D 06
24	0.1119468D 00	-0.1496232D 06	-0.1536806D 06	-0.1454554D 06	-0.1555380D 06	0.1475834D 06	0.1423652D 06
25	0.1163760D 00	-0.1511081D 06	-0.1508710D 06	-0.1512427D 06	-0.1498380D 06	0.1511754D 06	0.1380578D 06
26	0.1207052D 00	-0.1503626D 06	-0.1503988D 06	-0.1507194D 06	-0.1503571D 06	0.1505413D 06	0.1380319D 06
27	0.1249602D 00	-0.1503832D 06	-0.1492541D 06	-0.1522918D 06	-0.1487786D 06	0.1513465D 06	0.1366245D 06
28	0.1291534D 00	-0.1507434D 06	-0.1480765D 06	-0.1541234D 06	-0.1469568D 06	0.1524615D 06	0.1350812D 06
29	0.1332892D 00	-0.1509053D 06	-0.1476173D 06	-0.1548888D 06	-0.1461698D 06	0.1529360D 06	0.1344140D 06
30	0.1373811D 00	-0.1504813D 06	-0.1483369D 06	-0.1532395D 06	-0.1477638D 06	0.1518792D 06	0.1355194D 06
31	0.1414339D 00	-0.1489139D 06	-0.1505569D 06	-0.1482767D 06	-0.1526879D 06	0.1485963D 06	0.1390563D 06
32	0.1454663D 00	-0.1499072D 06	-0.1499614D 06	-0.1503984D 06	-0.1505884D 06	0.1501534D 06	0.1376512D 06
33	0.1494787D 00	-0.1491387D 06	-0.1510587D 06	-0.1479399D 06	-0.1530096D 06	0.1485429D 06	0.1393732D 06
34	0.1534819D 00	-0.1507037D 06	-0.1495325D 06	-0.1520287D 06	-0.1489432D 06	0.1513705D 06	0.1365212D 06
35	0.1574850D 00	-0.1505733D 06	-0.1498699D 06	-0.1514218D 06	-0.1495363D 06	0.1509993D 06	0.1369394D 06
36	0.1614931D 00	-0.1488503D 06	-0.1518981D 06	-0.1464546D 06	-0.1544755D 06	0.1476670D 06	0.1403941D 06
37	0.1655143D 00	-0.1497387D 06	-0.1508315D 06	-0.1490188D 06	-0.1519473D 06	0.1493801D 06	0.1385359D 06
38	0.1695602D 00	-0.1494307D 06	-0.1507226D 06	-0.1487249D 06	-0.1522402D 06	0.1490791D 06	0.1385843D 06
39	0.1736406D 00	-0.1484131D 06	-0.1510486D 06	-0.1468796D 06	-0.1540701D 06	0.1476523D 06	0.1396341D 06
40	0.1777522D 00	-0.1513587D 06	-0.1469688D 06	-0.1560985D 06	-0.1449168D 06	0.1537834D 06	0.1329584D 06
41	0.1819327D 00	-0.1512391D 06	-0.1466782D 06	-0.1566291D 06	-0.1444386D 06	0.1540049D 06	0.1325299D 06
42	0.1861877D 00	-0.1493330D 06	-0.1490290D 06	-0.1510413D 06	-0.1500468D 06	0.1501944D 06	0.1364516D 06
43	0.1905168D 00	-0.1503908D 06	-0.1485788D 06	-0.1532141D 06	-0.1479513D 06	0.1518221D 06	0.1351306D 06
44	0.1949461D 00	-0.1519297D 06	-0.1481034D 06	-0.1560418D 06	-0.1451811D 06	0.1540269D 06	0.1334843D 06
45	0.1995145D 00	-0.1520408D 06	-0.1499951D 06	-0.1541897D 06	-0.1469998D 06	0.1531266D 06	0.1352939D 06
46	0.2042323D 00	-0.1530388D 06	-0.1518141D 06	-0.1531763D 06	-0.1479367D 06	0.1531076D 06	0.1366418D 06
47	0.2091480D 00	-0.1537721D 06	-0.1546270D 06	-0.1508807D 06	-0.1499998D 06	0.1523469D 06	0.1390499D 06
48	0.2143358D 00	-0.1511945D 06	-0.1610788D 06	-0.1383328D 06	-0.1620100D 06	0.1451915D 06	0.1481788D 06
49	0.2198218D 00	-0.1491268D 06	-0.1647193D 06	-0.1286947D 06	-0.1709082D 06	0.1400332D 06	0.1544926D 06
50	0.2253409D 00	-0.1502964D 06	-0.1603169D 06	-0.1328193D 06	-0.1661150D 06	0.1423647D 06	0.1498418D 06
51	0.2305317D 00	-0.1493184D 06	-0.1544602D 06	-0.1368650D 06	-0.1616640D 06	0.1434976D 06	0.1447000D 06
52	0.2354446D 00	-0.1465406D 06	-0.1480703D 06	-0.1388962D 06	-0.1596113D 06	0.1428719D 06	0.1406667D 06
53	0.2401402D 00	-0.1422692D 06	-0.1413906D 06	-0.1400084D 06	-0.1591408D 06	0.1411524D 06	0.1375691D 06
54	0.2446369D 00	-0.1349459D 06	-0.1363166D 06	-0.1340436D 06	-0.1665388D 06	0.1344970D 06	0.1403085D 06
55	0.2490051D 00	-0.1299524D 06	-0.1263868D 06	-0.1370948D 06	-0.1663230D 06	0.1336668D 06	0.1371840D 06
56	0.2532621D 00	-0.1262864D 06	-0.1128057D 06	-0.1476146D 06	-0.1605617D 06	0.1381905D 06	0.1298282D 06
57	0.2574267D 00	-0.1233966D 06	-0.9685692D 05	-0.1647868D 06	-0.1507346D 06	0.1484832D 06	0.1196371D 06
58	0.2615186D 00	-0.1207967D 06	-0.7974787D 05	-0.1884409D 06	-0.1378639D 06	0.1653444D 06	0.1076532D 06
59	0.2655590D 00	-0.1175867D 06	-0.6242132D 05	-0.2190438D 06	-0.1226605D 06	0.1898689D 06	0.9458091D 05
60	0.2695696D 00	-0.1116926D 06	-0.4522192D 05	-0.2578475D 06	-0.1057205D 06	0.2239664D 06	0.8103532D 05

TABLE 5.4.1 (contd.)

DISPLACEMENT AND STRESS RESULTANTS OF 'SEGMENT 1 SHELL TYPE 3 DZA= 0.10000D 02

ELM	S	U	V	W	β	N_ϕ	N_θ	$N_{\phi\theta}$	M_ϕ	M_θ	$M_{\phi\theta}$
1	0.400D-02	0.361D-05	0.000D 00	-0.129D-03	-0.515D-04	-0.603D 04	-0.603D 04	0.000D 00	0.262D 00	0.305D 00	0.000D 00
2	0.100D-01	0.975D-05	0.000D 00	-0.129D-03	-0.699D-04	-0.603D 04	-0.600D 04	0.000D 00	0.484D-01	0.130D 00	0.000D 00
3	0.140D-01	0.117D-04	0.000D 00	-0.128D-03	-0.729D-04	-0.602D 04	-0.598D 04	0.000D 00	-0.220D-01	0.743D-01	0.000D 00
4	0.181D-01	0.166D-04	0.000D 00	-0.127D-03	-0.426D-04	-0.601D 04	-0.596D 04	0.000D 00	-0.145D 00	-0.179D-01	0.000D 00
5	0.222D-01	0.214D-04	0.000D 00	-0.126D-03	-0.132D-04	-0.600D 04	-0.596D 04	0.000D 00	-0.608D-01	-0.137D-01	0.000D 00
6	0.263D-01	0.231D-04	0.000D 00	-0.125D-03	-0.702D-05	-0.599D 04	-0.596D 04	0.000D 00	-0.897D-01	-0.299D-01	0.000D 00
7	0.304D-01	0.305D-04	0.000D 00	-0.124D-03	0.176D-04	-0.599D 04	-0.597D 04	0.000D 00	0.401D-01	0.630D-02	0.000D 00
8	0.347D-01	0.319D-04	0.000D 00	-0.123D-03	-0.314D-04	-0.598D 04	-0.597D 04	0.000D 00	0.273D 00	0.118D 00	0.000D 00
9	0.389D-01	0.333D-04	0.000D 00	-0.122D-03	-0.765D-04	-0.598D 04	-0.594D 04	0.000D 00	0.275D-01	0.406D-01	0.000D 00
10	0.432D-01	0.397D-04	0.000D 00	-0.119D-03	-0.415D-04	-0.598D 04	-0.592D 04	0.000D 00	-0.166D 00	-0.487D-01	0.000D 00
11	0.477D-01	0.432D-04	0.000D 00	-0.118D-03	-0.916D-05	-0.597D 04	-0.591D 04	0.000D 00	-0.119D 00	-0.423D-01	0.000D 00
12	0.522D-01	0.485D-04	0.000D 00	-0.115D-03	0.684D-05	-0.597D 04	-0.592D 04	0.000D 00	0.749D-01	0.266D-01	0.000D 00
13	0.569D-01	0.492D-04	0.000D 00	-0.114D-03	-0.338D-04	-0.596D 04	-0.591D 04	0.000D 00	0.695D-01	0.350D-01	0.000D 00
14	0.616D-01	0.556D-04	0.000D 00	-0.110D-03	-0.250D-04	-0.596D 04	-0.590D 04	0.000D 00	0.338D-01	0.185D-01	0.000D 00
15	0.666D-01	0.577D-04	0.000D 00	-0.108D-03	-0.555D-04	-0.595D 04	-0.588D 04	0.000D 00	0.858D-01	0.440D-01	0.000D 00
16	0.717D-01	0.625D-04	0.000D 00	-0.104D-03	-0.641D-04	-0.595D 04	-0.585D 04	0.000D 00	0.649D-01	0.365D-01	0.000D 00
17	0.771D-01	0.652D-04	0.000D 00	-0.101D-03	-0.865D-04	-0.594D 04	-0.581D 04	0.000D 00	0.128D-01	0.194D-01	0.000D 00
18	0.825D-01	0.698D-04	0.000D 00	-0.962D-04	-0.647D-04	-0.594D 04	-0.577D 04	0.000D 00	-0.729D-01	-0.181D-01	0.000D 00
19	0.879D-01	0.734D-04	0.000D 00	-0.919D-04	-0.503D-04	-0.593D 04	-0.574D 04	0.000D 00	-0.147D-01	0.115D-02	0.000D 00
20	0.931D-01	0.758D-04	0.000D 00	-0.883D-04	-0.478D-04	-0.592D 04	-0.572D 04	0.000D 00	-0.529D-01	-0.143D-01	0.000D 00
21	0.980D-01	0.799D-04	0.000D 00	-0.830D-04	-0.279D-04	-0.591D 04	-0.571D 04	0.000D 00	0.155D-01	0.886D-02	0.000D 00
22	0.103D 00	0.813D-04	0.000D 00	-0.800D-04	-0.560D-04	-0.590D 04	-0.569D 04	0.000D 00	0.112D 00	0.479D-01	0.000D 00
23	0.107D 00	0.841D-04	0.000D 00	-0.751D-04	-0.774D-04	-0.590D 04	-0.566D 04	0.000D 00	0.121D 00	0.526D-01	0.000D 00
24	0.112D 00	0.856D-04	0.000D 00	-0.713D-04	-0.114D-03	-0.589D 04	-0.561D 04	0.000D 00	0.140D 00	0.622D-01	0.000D 00
25	0.116D 00	0.870D-04	0.000D 00	-0.672D-04	-0.133D-03	-0.589D 04	-0.555D 04	0.000D 00	-0.111D-01	0.518D-02	0.000D 00
26	0.121D 00	0.898D-04	0.000D 00	-0.610D-04	-0.117D-03	-0.588D 04	-0.550D 04	0.000D 00	0.104D-02	0.737D-02	0.000D 00
27	0.125D 00	0.898D-04	0.000D 00	-0.585D-04	-0.128D-03	-0.587D 04	-0.544D 04	0.000D 00	-0.413D-01	-0.860D-02	0.000D 00
28	0.129D 00	0.922D-04	0.000D 00	-0.518D-04	-0.970D-04	-0.587D 04	-0.539D 04	0.000D 00	-0.904D-01	-0.301D-01	0.000D 00
29	0.133D 00	0.921D-04	0.000D 00	-0.493D-04	-0.886D-04	-0.586D 04	-0.535D 04	0.000D 00	-0.112D 00	-0.390D-01	0.000D 00
30	0.137D 00	0.940D-04	0.000D 00	-0.423D-04	-0.506D-04	-0.586D 04	-0.532D 04	0.000D 00	-0.714D-01	-0.257D-01	0.000D 00
31	0.141D 00	0.937D-04	0.000D 00	-0.398D-04	-0.617D-04	-0.585D 04	-0.530D 04	0.000D 00	0.557D-01	0.226D-01	0.000D 00
32	0.145D 00	0.943D-04	0.000D 00	-0.349D-04	-0.646D-04	-0.585D 04	-0.527D 04	0.000D 00	0.111D-03	0.107D-02	0.000D 00
33	0.149D 00	0.946D-04	0.000D 00	-0.300D-04	-0.680D-04	-0.585D 04	-0.524D 04	0.000D 00	0.632D-01	0.246D-01	0.000D 00
34	0.153D 00	0.940D-04	0.000D 00	-0.273D-04	-0.825D-04	-0.585D 04	-0.520D 04	0.000D 00	-0.416D-01	-0.155D-01	0.000D 00
35	0.157D 00	0.945D-04	0.000D 00	-0.200D-04	-0.567D-04	-0.585D 04	-0.517D 04	0.000D 00	-0.236D-01	-0.940D-02	0.000D 00
36	0.161D 00	0.937D-04	0.000D 00	-0.174D-04	-0.787D-04	-0.585D 04	-0.514D 04	0.000D 00	0.109D 00	0.406D-01	0.000D 00
37	0.166D 00	0.931D-04	0.000D 00	-0.124D-04	-0.936D-04	-0.585D 04	-0.509D 04	0.000D 00	0.507D-01	0.174D-01	0.000D 00
38	0.170D 00	0.923D-04	0.000D 00	-0.745D-05	-0.105D-03	-0.586D 04	-0.504D 04	0.000D 00	0.690D-01	0.233D-01	0.000D 00
39	0.174D 00	0.912D-04	0.000D 00	-0.258D-05	-0.127D-03	-0.587D 04	-0.498D 04	0.000D 00	0.131D 00	0.454D-01	0.000D 00
40	0.178D 00	0.898D-04	0.000D 00	0.147D-06	-0.148D-03	-0.588D 04	-0.490D 04	0.000D 00	-0.888D-01	-0.397D-01	0.000D 00
41	0.182D 00	0.880D-04	0.000D 00	0.883D-05	-0.101D-03	-0.589D 04	-0.482D 04	0.000D 00	-0.788D-01	-0.355D-01	0.000D 00
42	0.186D 00	0.863D-04	0.000D 00	0.112D-04	-0.123D-03	-0.590D 04	-0.475D 04	0.000D 00	0.926D-01	0.279D-01	0.000D 00
43	0.191D 00	0.839D-04	0.000D 00	0.172D-04	-0.133D-03	-0.593D 04	-0.467D 04	0.000D 00	0.625D-01	0.143D-01	0.000D 00
44	0.195D 00	0.816D-04	0.000D 00	0.212D-04	-0.153D-03	-0.595D 04	-0.456D 04	0.000D 00	0.892D-02	-0.863D-02	0.000D 00
45	0.200D 00	0.781D-04	0.000D 00	0.281D-04	-0.147D-03	-0.599D 04	-0.445D 04	0.000D 00	0.640D-01	0.108D-01	0.000D 00
46	0.204D 00	0.753D-04	0.000D 00	0.316D-04	-0.185D-03	-0.603D 04	-0.431D 04	0.000D 00	0.659D-01	0.636D-02	0.000D 00
47	0.209D 00	0.706D-04	0.000D 00	0.386D-04	-0.183D-03	-0.610D 04	-0.415D 04	0.000D 00	0.501D-01	-0.244D-02	0.000D 00
48	0.214D 00	0.661D-04	0.000D 00	0.437D-04	-0.232D-03	-0.618D 04	-0.394D 04	0.000D 00	0.221D 00	0.534D-01	0.000D 00
49	0.220D 00	0.609D-04	0.000D 00	0.494D-04	-0.303D-03	-0.630D 04	-0.366D 04	0.000D 00	0.179D 00	0.230D-01	0.000D 00
50	0.225D 00	0.558D-04	0.000D 00	0.542D-04	-0.307D-03	-0.645D 04	-0.331D 04	0.000D 00	-0.406D 00	-0.204D 00	0.000D 00
51	0.231D 00	0.484D-04	0.000D 00	0.600D-04	-0.840D-04	-0.666D 04	-0.300D 04	0.000D 00	-0.109D 01	-0.431D 00	0.000D 00
52	0.235D 00	0.430D-04	0.000D 00	0.619D-04	0.272D-03	-0.690D 04	-0.281D 04	0.000D 00	-0.177D 01	-0.617D 00	0.000D 00
53	0.240D 00	0.350D-04	0.000D 00	0.629D-04	0.828D-03	-0.718D 04	-0.279D 04	0.000D 00	-0.234D 01	-0.699D 00	0.000D 00
54	0.245D 00	0.290D-04	0.000D 00	0.597D-04	0.144D-02	-0.748D 04	-0.296D 04	0.000D 00	-0.247D 01	-0.572D 00	0.000D 00
55	0.249D 00	0.230D-04	0.000D 00	0.537D-04	0.207D-02	-0.781D 04	-0.330D 04	0.000D 00	-0.247D 01	-0.353D 00	0.000D 00
56	0.253D 00	0.174D-04	0.000D 00	0.449D-04	0.267D-02	-0.815D 04	-0.375D 04	0.000D 00	-0.205D 01	0.573D-01	0.000D 00
57	0.257D 00	0.123D-04	0.000D 00	0.338D-04	0.312D-02	-0.851D 04	-0.423D 04	0.000D 00	-0.914D 00	0.739D 00	0.000D 00
58	0.262D 00	0.791D-05	0.000D 00	0.214D-04	0.323D-02	-0.887D 04	-0.458D 04	0.000D 00	0.127D 01	0.174D 01	0.000D 00
59	0.266D 00	0.433D-05	0.000D 00	0.953D-05	0.274D-02	-0.926D 04	-0.462D 04	0.000D 00	0.486D 01	0.304D 01	0.000D 00
60	0.270D 00	0.136D-05	0.000D 00	0.132D-05	0.126D-02	-0.976D 04	-0.420D 04	0.000D 00	0.103D 02	0.454D 01	0.000D 00

TABLE 5.4.1 (contd.)

AXISYMMETRIC STRESSES AND EQUIVALENT STRESSES

SEGMENT NO. 1

ELEMENT	STATION	σ_θ (IN)	σ_θ (OUT)	σ_ϕ (IN)	σ_ϕ (OUT)	σ_{EQ} (IN)	σ_{EQ} (OUT)
1	0.4002812D-02	-0.1391777D 07	-0.1620873D 07	-0.1410222D 07	-0.1606697D 07	0.1401091D 07	0.1497734D 07
2	0.1001560D-01	-0.1451558D 07	-0.1548815D 07	-0.1488227D 07	-0.1524548D 07	0.1470236D 07	0.1420731D 07
3	0.1404114D-01	-0.1466789D 07	-0.1522534D 07	-0.1512202D 07	-0.1495702D 07	0.1490015D 07	0.1393201D 07
4	0.1808710D-01	-0.1497513D 07	-0.1484097D 07	-0.1556304D 07	-0.1447309D 07	0.1527757D 07	0.1349961D 07
5	0.2216749D-01	-0.1494957D 07	-0.1484650D 07	-0.1522337D 07	-0.1476774D 07	0.1508834D 07	0.1364602D 07
6	0.2627904D-01	-0.1501272D 07	-0.1478836D 07	-0.1531766D 07	-0.1464459D 07	0.1516749D 07	0.1355566D 07
7	0.3044357D-01	-0.1489539D 07	-0.1494261D 07	-0.1481696D 07	-0.1511753D 07	0.1485633D 07	0.1386938D 07
8	0.3466551D-01	-0.1447850D 07	-0.1536141D 07	-0.1393601D 07	-0.1598425D 07	0.1421502D 07	0.1452118D 07
9	0.3892047D-01	-0.1470481D 07	-0.1500921D 07	-0.1485163D 07	-0.1505752D 07	0.1477877D 07	0.1387159D 07
10	0.4324964D-01	-0.1497843D 07	-0.1461289D 07	-0.1557139D 07	-0.1432518D 07	0.1528354D 07	0.1330934D 07
11	0.4767888D-01	-0.1493716D 07	-0.1461962D 07	-0.1537934D 07	-0.1448746D 07	0.1516309D 07	0.1339178D 07
12	0.5222200D-01	-0.1469284D 07	-0.1489206D 07	-0.1463583D 07	-0.1519755D 07	0.1466442D 07	0.1388482D 07
13	0.5686144D-01	-0.1465203D 07	-0.1491449D 07	-0.1464680D 07	-0.1516791D 07	0.1464941D 07	0.1388016D 07
14	0.6163513D-01	-0.1468272D 07	-0.1482143D 07	-0.1477210D 07	-0.1502557D 07	0.1472761D 07	0.1376156D 07
15	0.6657644D-01	-0.1454071D 07	-0.1487104D 07	-0.1456585D 07	-0.1520907D 07	0.1455329D 07	0.1387972D 07
16	0.7170132D-01	-0.1449236D 07	-0.1476623D 07	-0.1463109D 07	-0.1511805D 07	0.1456222D 07	0.1378173D 07
17	0.7705063D-01	-0.1445376D 07	-0.1459923D 07	-0.1481001D 07	-0.1490630D 07	0.1463514D 07	0.1359117D 07
18	0.8253368D-01	-0.1449103D 07	-0.1435557D 07	-0.1511313D 07	-0.1456609D 07	0.1481188D 07	0.1329744D 07
19	0.8791718D-01	-0.1435059D 07	-0.1435925D 07	-0.1487296D 07	-0.1476291D 07	0.1461878D 07	0.1340053D 07
20	0.9307238D-01	-0.1435280D 07	-0.1424546D 07	-0.1499746D 07	-0.1460038D 07	0.1468574D 07	0.1326091D 07
21	0.9801568D-01	-0.1423013D 07	-0.1429657D 07	-0.1472128D 07	-0.1483722D 07	0.1448195D 07	0.1340903D 07
22	0.1027872D 00	-0.1404292D 07	-0.1440215D 07	-0.1434381D 07	-0.1518040D 07	0.1419575D 07	0.1364143D 07
23	0.1074282D 00	-0.1394916D 07	-0.1434337D 07	-0.1429435D 07	-0.1519885D 07	0.1412492D 07	0.1362430D 07
24	0.1119468D 00	-0.1380028D 07	-0.1426681D 07	-0.1420618D 07	-0.1525618D 07	0.1400765D 07	0.1362104D 07
25	0.1163760D 00	-0.1386547D 07	-0.1390435D 07	-0.1475964D 07	-0.1467640D 07	0.1433349D 07	0.1313950D 07
26	0.1207052D 00	-0.1371296D 07	-0.1376826D 07	-0.1469789D 07	-0.1470573D 07	0.1423101D 07	0.1309383D 07
27	0.1249602D 00	-0.1363259D 07	-0.1356808D 07	-0.1484138D 07	-0.1453174D 07	0.1427542D 07	0.1290809D 07
28	0.1291534D 00	-0.1358726D 07	-0.1336158D 07	-0.1501231D 07	-0.1433466D 07	0.1435294D 07	0.1270681D 07
29	0.1332892D 00	-0.1351952D 07	-0.1322681D 07	-0.1508012D 07	-0.1424055D 07	0.1436355D 07	0.1259456D 07
30	0.1373811D 00	-0.1339641D 07	-0.1320365D 07	-0.1491494D 07	-0.1437979D 07	0.1421663D 07	0.1266253D 07
31	0.1414339D 00	-0.1315843D 07	-0.1332824D 07	-0.1442821D 07	-0.1484601D 07	0.1383709D 07	0.1298315D 07
32	0.1454663D 00	-0.1316984D 07	-0.1317787D 07	-0.1463100D 07	-0.1463183D 07	0.1395790D 07	0.1279581D 07
33	0.1494787D 00	-0.1300757D 07	-0.1319171D 07	-0.1438837D 07	-0.1486226D 07	0.1375007D 07	0.1293652D 07
34	0.1534819D 00	-0.1306822D 07	-0.1295219D 07	-0.1478003D 07	-0.1446820D 07	0.1400282D 07	0.1260667D 07
35	0.1574850D 00	-0.1296154D 07	-0.1289100D 07	-0.1471256D 07	-0.1453586D 07	0.1391989D 07	0.1262153D 07
36	0.1614931D 00	-0.1269085D 07	-0.1299554D 07	-0.1421692D 07	-0.1503730D 07	0.1351864D 07	0.1296456D 07
37	0.1655143D 00	-0.1266643D 07	-0.1279695D 07	-0.1444592D 07	-0.1482614D 07	0.1364349D 07	0.1275967D 07
38	0.1695602D 00	-0.1251270D 07	-0.1268752D 07	-0.1438861D 07	-0.1490638D 07	0.1354841D 07	0.1276844D 07
39	0.1736406D 00	-0.1227252D 07	-0.1261312D 07	-0.1417046D 07	-0.1515606D 07	0.1332326D 07	0.1289957D 07
40	0.1777522D 00	-0.1239636D 07	-0.1209893D 07	-0.1502107D 07	-0.1435478D 07	0.1389589D 07	0.1220931D 07
41	0.1819327D 00	-0.1219488D 07	-0.1192866D 07	-0.1501498D 07	-0.1442364D 07	0.1382241D 07	0.1219379D 07
42	0.1861877D 00	-0.1177800D 07	-0.1198713D 07	-0.1441266D 07	-0.1510729D 07	0.1329262D 07	0.1266308D 07
43	0.1905168D 00	-0.1161166D 07	-0.1171917D 07	-0.1457969D 07	-0.1504811D 07	0.1334555D 07	0.1254319D 07
44	0.1949461D 00	-0.1144201D 07	-0.1137727D 07	-0.1484957D 07	-0.1491645D 07	0.1347295D 07	0.1235632D 07
45	0.1995145D 00	-0.1108104D 07	-0.1116170D 07	-0.1473070D 07	-0.1521045D 07	0.1328727D 07	0.1251031D 07
46	0.2042323D 00	-0.1075446D 07	-0.1080213D 07	-0.1483934D 07	-0.1533381D 07	0.1327687D 07	0.1252126D 07
47	0.2091480D 00	-0.1037322D 07	-0.1035495D 07	-0.1505335D 07	-0.1542906D 07	0.1334374D 07	0.1251100D 07
48	0.2143358D 00	-0.9653884D 06	-0.1005415D 07	-0.1462080D 07	-0.1627521D 07	0.1287703D 07	0.1314127D 07
49	0.2198218D 00	-0.9066436D 06	-0.9239103D 06	-0.1507021D 07	-0.1640898D 07	0.1314071D 07	0.1319702D 07
50	0.2253409D 00	-0.9047636D 06	-0.7514887D 06	-0.1765727D 07	-0.1461474D 07	0.1529321D 07	0.1164155D 07
51	0.2305317D 00	-0.9123762D 06	-0.5892359D 06	-0.2074016D 07	-0.1254581D 07	0.1800469D 07	0.9890920D 06
52	0.2354446D 00	-0.9342960D 06	-0.4715552D 06	-0.2387777D 07	-0.1059727D 07	0.2084105D 07	0.8239372D 06
53	0.2401402D 00	-0.9601531D 06	-0.4357998D 06	-0.2672808D 07	-0.9147795D 06	0.2345100D 07	0.6946829D 06
54	0.2446369D 00	-0.9540411D 06	-0.5250083D 06	-0.2796397D 07	-0.9436573D 06	0.2462144D 07	0.7150019D 06
55	0.2490051D 00	-0.9562773D 06	-0.6914857D 06	-0.2877604D 07	-0.1027355D 07	0.2538363D 07	0.7963504D 06
56	0.2532621D 00	-0.9165758D 06	-0.9595298D 06	-0.2807403D 07	-0.1269776D 07	0.2479602D 07	0.1032129D 07
57	0.2574267D 00	-0.7806384D 06	-0.1334929D 07	-0.2469371D 07	-0.1783682D 07	0.2186207D 07	0.1492663D 07
58	0.2615186D 00	-0.4944481D 06	-0.1796881D 07	-0.1741662D 07	-0.2692342D 07	0.1554575D 07	0.2263471D 07
59	0.2655590D 00	-0.1660518D 05	-0.2294311D 07	-0.4938724D 06	-0.4138344D 07	0.4857828D 06	0.3485558D 07
60	0.2695696D 00	0.6537322D 06	-0.2751479D 07	0.1430793D 07	-0.6308430D 07	0.1240636D 07	0.5380801D 07

TABLE 5.4.2

C******** TABLE 5.4.2 **

LINEAR STRESS ANALYSIS OF BAIG- ECHIDOME APRIL 1979

TYPE OF ANALYSIS 6

NUMBER OF SEGMENTS IN THE STRUCTURE 1

E= 0.80000D 10 NU= 0.36000D 00 RO= 0.10000D 05

ATMOSPHERIC PRESSURE 0.00000D 00
LIQUID DENSITY * G =-0.98100D 04
DEPTH OF END A 0.00000D 01
INCREMENT IN DEPTH 0.15250D 01
MAXIMUM DEPTH OF END A 0.15250D 02

SEGMENT THICKNESSES

0.25000D-02

ANALYSIS FOR N VARYING FROM 0 TO 0

BOUNDARY CONDITIONS

1	1	1	1
0	0	0	0

EXTERNAL LOADS AT ENDS 0.00000D 00 0.00000D 00 0.00000D 00 0.00000D 00
0.00000D 00 0.00000D 00 0.00000D 00 0.00000D 00

NODE DATA FOR THE SEGMENTS IN THE STRUCTURE

SEGMENT 1 IS A GENERAL CURVE

NO. OF ELEMENTS IN THE SEGMENT 65

NODE	X	R
1	0.00000000D 00	0.00000000D 00
2	0.10000000D-03	0.80000000D-02
3	0.50000000D-03	0.16000000D-01
4	0.12000000D-02	0.24000000D-01
5	0.22000000D-02	0.32000000D-01
6	0.34000000D-02	0.40000000D-01
7	0.50000000D-02	0.48000000D-01
8	0.68000000D-02	0.56000000D-01
9	0.89000000D-02	0.64000000D-01
10	0.11400000D-01	0.72000000D-01
11	0.14100000D-01	0.80000000D-01
12	0.17200000D-01	0.88000000D-01
13	0.20700000D-01	0.96000000D-01
14	0.24500000D-01	0.10400000D 00
15	0.28700000D-01	0.11200000D 00
16	0.33300000D-01	0.12000000D 00
17	0.38400000D-01	0.12800000D 00
18	0.44000000D-01	0.13600000D 00
19	0.50100000D-01	0.14400000D 00
20	0.56900000D-01	0.15200000D 00

TABLE 5.4.2. (contd.)

21	0.64300000D-01	0.16000000D 00
22	0.72500000D-01	0.16800000D 00
23	0.80500000D-01	0.17500000D 00
24	0.88500000D-01	0.18150000D 00
25	0.96500000D-01	0.18730000D 00
26	0.10450000D 00	0.19250000D 00
27	0.11250000D 00	0.19730000D 00
28	0.12050000D 00	0.20160000D 00
29	0.12850000D 00	0.20540000D 00
30	0.13650000D 00	0.20890000D 00
31	0.14450000D 00	0.21190000D 00
32	0.15250000D 00	0.21460000D 00
33	0.16050000D 00	0.21690000D 00
34	0.16850000D 00	0.21890000D 00
35	0.17650000D 00	0.22050000D 00
36	0.18450000D 00	0.22180000D 00
37	0.19250000D 00	0.22270000D 00
38	0.20050000D 00	0.22330000D 00
39	0.20850000D 00	0.22360000D 00
40	0.21650000D 00	0.22350000D 00
41	0.22450000D 00	0.22310000D 00
42	0.23250000D 00	0.22230000D 00
43	0.24050000D 00	0.22120000D 00
44	0.24850000D 00	0.21970000D 00
45	0.25650000D 00	0.21790000D 00
46	0.26450000D 00	0.21560000D 00
47	0.27250000D 00	0.21300000D 00
48	0.28050000D 00	0.20990000D 00
49	0.28850000D 00	0.20640000D 00
50	0.29650000D 00	0.20230000D 00
51	0.30450000D 00	0.19770000D 00
52	0.31250000D 00	0.19250000D 00
53	0.32050000D 00	0.18660000D 00
54	0.32850000D 00	0.17980000D 00
55	0.33650000D 00	0.17210000D 00
56	0.34370000D 00	0.16410000D 00
57	0.35000000D 00	0.15610000D 00
58	0.35550000D 00	0.14810000D 00
59	0.36020000D 00	0.14010000D 00
60	0.36430000D 00	0.13210000D 00
61	0.36770000D 00	0.12410000D 00
62	0.37050000D 00	0.11610000D 00
63	0.37270000D 00	0.10810000D 00
64	0.37430000D 00	0.10010000D 00
65	0.37530000D 00	0.92100000D-01
66	0.37570000D 00	0.84100000D-01

SEGMENT NO. 1

ELEMENT	R(1)	R(2)	PHI(1)	PHI(2)	SL
1	0.0000000D 00	0.8000000D-02	0.8928384D 02	0.8928384D 02	0.8000625D-02
2	0.8000000D-02	0.1600000D-01	0.8713759D 02	0.8713759D 02	0.8009994D-02
3	0.1600000D-01	0.2400000D-01	0.8499936D 02	0.8499936D 02	0.8030567D-02
4	0.2400000D-01	0.3200000D-01	0.8287498D 02	0.8287498D 02	0.8062258D-02
5	0.3200000D-01	0.4000000D-01	0.8146923D 02	0.8146923D 02	0.8089499D-02
6	0.4000000D-01	0.4800000D-01	0.7869007D 02	0.7869007D 02	0.8158431D-02
7	0.4800000D-01	0.5600000D-01	0.7731962D 02	0.7731962D 02	0.8200000D-02
8	0.5600000D-01	0.6400000D-01	0.7529170D 02	0.7529170D 02	0.8271034D-02
9	0.6400000D-01	0.7200000D-01	0.7264598D 02	0.7264598D 02	0.8381527D-02
10	0.7200000D-01	0.8000000D-01	0.7135046D 02	0.7135046D 02	0.8443341D-02
11	0.8000000D-01	0.8800000D-01	0.6881865D 02	0.6881865D 02	0.8579627D-02
12	0.8800000D-01	0.9600000D-01	0.6637062D 02	0.6637062D 02	0.8732125D-02
13	0.9600000D-01	0.1040000D 00	0.6459228D 02	0.6459228D 02	0.8856636D-02
14	0.1040000D 00	0.1120000D 00	0.6230053D 02	0.6230053D 02	0.9035486D-02
15	0.1120000D 00	0.1200000D 00	0.6010110D 02	0.6010110D 02	0.9228218D-02

TABLE 5.4.2 (contd.)

16	0.1200000D 00	0.1280000D 00	0.5748249D 02	0.5748249D 02	0.9487360D-02
17	0.1280000D 00	0.1360000D 00	0.5500798D 02	0.5500798D 02	0.9765244D-02
18	0.1360000D 00	0.1440000D 00	0.5267448D 02	0.5267448D 02	0.1006032D-01
19	0.1440000D 00	0.1520000D 00	0.4963546D 02	0.4963546D 02	0.1049952D-01
20	0.1520000D 00	0.1600000D 00	0.4723117D 02	0.4723117D 02	0.1089771D-01
21	0.1600000D 00	0.1680000D 00	0.4429268D 02	0.4429268D 02	0.1145600D-01
22	0.1680000D 00	0.1750000D 00	0.4118593D 02	0.4118593D 02	0.1063015D-01
23	0.1750000D 00	0.1815000D 00	0.3909386D 02	0.3909386D 02	0.1030776D-01
24	0.1815000D 00	0.1873000D 00	0.3594211D 02	0.3594211D 02	0.9881295D-02
25	0.1873000D 00	0.1925000D 00	0.3302387D 02	0.3302387D 02	0.9541488D-02
26	0.1925000D 00	0.1973000D 00	0.3096376D 02	0.3096376D 02	0.9329523D-02
27	0.1973000D 00	0.2016000D 00	0.2825803D 02	0.2825803D 02	0.9082401D-02
28	0.2016000D 00	0.2054000D 00	0.2540772D 02	0.2540772D 02	0.8856636D-02
29	0.2054000D 00	0.2089000D 00	0.2362938D 02	0.2362938D 02	0.8732125D-02
30	0.2089000D 00	0.2119000D 00	0.2055605D 02	0.2055605D 02	0.8544004D-02
31	0.2119000D 00	0.2146000D 00	0.1864954D 02	0.1864954D 02	0.8443341D-02
32	0.2146000D 00	0.2169000D 00	0.1603994D 02	0.1603994D 02	0.8324062D-02
33	0.2169000D 00	0.2189000D 00	0.1403624D 02	0.1403624D 02	0.8246211D-02
34	0.2189000D 00	0.2205000D 00	0.1130993D 02	0.1130993D 02	0.8158431D-02
35	0.2205000D 00	0.2218000D 00	0.9229886D 01	0.9229886D 01	0.8104937D-02
36	0.2218000D 00	0.2227000D 00	0.6418787D 01	0.6418787D 01	0.8050466D-02
37	0.2227000D 00	0.2233000D 00	0.4289153D 01	0.4289153D 01	0.8022468D-02
38	0.2233000D 00	0.2236000D 00	0.2147585D 01	0.2147585D 01	0.8005623D-02
39	0.2236000D 00	0.2235000D 00	-0.7161599D 00	-0.7161599D 00	0.8000625D-02
40	0.2235000D 00	0.2231000D 00	-0.2862405D 01	-0.2862405D 01	0.8009994D-02
41	0.2231000D 00	0.2223000D 00	-0.5710593D 01	-0.5710593D 01	0.8039900D-02
42	0.2223000D 00	0.2212000D 00	-0.7829077D 01	-0.7829077D 01	0.8075271D-02
43	0.2212000D 00	0.2197000D 00	-0.1061966D 02	-0.1061966D 02	0.8139410D-02
44	0.2197000D 00	0.2179000D 00	-0.1268038D 02	-0.1268038D 02	0.8200000D-02
45	0.2179000D 00	0.2156000D 00	-0.1603994D 02	-0.1603994D 02	0.8324062D-02
46	0.2156000D 00	0.2130000D 00	-0.1800416D 02	-0.1800416D 02	0.8411896D-02
47	0.2130000D 00	0.2099000D 00	-0.2118135D 02	-0.2118135D 02	0.8579627D-02
48	0.2099000D 00	0.2064000D 00	-0.2362938D 02	-0.2362938D 02	0.8732125D-02
49	0.2064000D 00	0.2023000D 00	-0.2713514D 02	-0.2713514D 02	0.8989438D-02
50	0.2023000D 00	0.1977000D 00	-0.2989890D 02	-0.2989890D 02	0.9228218D-02
51	0.1977000D 00	0.1925000D 00	-0.3302387D 02	-0.3302387D 02	0.9541488D-02
52	0.1925000D 00	0.1866000D 00	-0.3640877D 02	-0.3640877D 02	0.9940322D-02
53	0.1866000D 00	0.1798000D 00	-0.4036454D 02	-0.4036454D 02	0.1049952D-01
54	0.1798000D 00	0.1721000D 00	-0.4390531D 02	-0.4390531D 02	0.1110360D-01
55	0.1721000D 00	0.1641000D 00	-0.4801279D 02	-0.4801279D 02	0.1076290D-01
56	0.1641000D 00	0.1561000D 00	-0.5177957D 02	-0.5177957D 02	0.1018283D-01
57	0.1561000D 00	0.1481000D 00	-0.5549148D 02	-0.5549148D 02	0.9708244D-02
58	0.1481000D 00	0.1401000D 00	-0.5956576D 02	-0.5956576D 02	0.9278470D-02
59	0.1401000D 00	0.1321000D 00	-0.6286486D 02	-0.6286486D 02	0.8989438D-02
60	0.1321000D 00	0.1241000D 00	-0.6697451D 02	-0.6697451D 02	0.8692526D-02
61	0.1241000D 00	0.1161000D 00	-0.7070995D 02	-0.7070995D 02	0.8475848D-02
62	0.1161000D 00	0.1081000D 00	-0.7462375D 02	-0.7462375D 02	0.8296987D-02
63	0.1081000D 00	0.1001000D 00	-0.7869007D 02	-0.7869007D 02	0.8158431D-02
64	0.1001000D 00	0.9210000D-01	-0.8287498D 02	-0.8287498D 02	0.8062258D-02
65	0.9210000D-01	0.8410000D-01	-0.8713759D 02	-0.8713759D 02	0.8009994D-02

DEGREES OF FREEDOM, IDF= 194 MAXIMUM BAND SIZE, IBAND= 6

DISPLACEMENT NUMBERS ASSIGNED TO THE NODES

NODE	U	V	W	BETA
1	1	0	2	0
2	3	0	4	5
3	6	0	7	8
4	9	0	10	11
5	12	0	13	14
6	15	0	16	17
7	18	0	19	20

TABLE 5.4.2 (contd.)

8	21	0	22	23
9	24	0	25	26
10	27	0	28	29
11	30	0	31	32
12	33	0	34	35
13	36	0	37	38
14	39	0	40	41
15	42	0	43	44
16	45	0	46	47
17	48	0	49	50
18	51	0	52	53
19	54	0	55	56
20	57	0	58	59
21	60	0	61	62
22	63	0	64	65
23	66	0	67	68
24	69	0	70	71
25	72	0	73	74
26	75	0	76	77
27	78	0	79	80
28	81	0	82	83
29	84	0	85	86
30	87	0	88	89
31	90	0	91	92
32	93	0	94	95
33	96	0	97	98
34	99	0	100	101
35	102	0	103	104
36	105	0	106	107
37	108	0	109	110
38	111	0	112	113
39	114	0	115	116
40	117	0	118	119
41	120	0	121	122
42	123	0	124	125
43	126	0	127	128
44	129	0	130	131
45	132	0	133	134
46	135	0	136	137
47	138	0	139	140
48	141	0	142	143
49	144	0	145	146
50	147	0	148	149
51	150	0	151	152
52	153	0	154	155
53	156	0	157	158
54	159	0	160	161
55	162	0	163	164
56	165	0	166	167
57	168	0	169	170
58	171	0	172	173
59	174	0	175	176
60	177	0	178	179
61	180	0	181	182
62	183	0	184	185
63	186	0	187	188
64	189	0	190	191
65	192	0	193	194
66	0	0	0	0

TABLE 5.4.2 (contd.)

DISPLACEMENT AND STRESS RESULTANTS OF 'SEGMENT 1 SHELL TYPE 3 DZA= 0.15250D 01

ELM	S	U	V	W	β	N_{ϕ}	N_{θ}	$N_{\phi\theta}$	M_{ϕ}	M_{θ}	$M_{\phi\theta}$
1	0.400D-02	-0.150D-06	0.000D 00	-0.582D-05	-0.841D-05	-0.174D 04	-0.174D 04	0.000D 00	0.153D-01	0.274D-01	0.000D 00
2	0.120D-01	-0.378D-06	0.000D 00	-0.579D-05	-0.274D-06	-0.174D 04	-0.174D 04	0.000D 00	-0.184D-01	-0.640D-02	0.000D 00
3	0.200D-01	-0.607D-06	0.000D 00	-0.587D-05	0.139D-04	-0.174D 04	-0.174D 04	0.000D 00	-0.178D-01	-0.136D-01	0.000D 00
4	0.281D-01	-0.834D-06	0.000D 00	-0.603D-05	0.156D-04	-0.174D 04	-0.175D 04	0.000D 00	0.148D-01	-0.439D-03	0.000D 00
5	0.361D-01	-0.113D-05	0.000D 00	-0.611D-05	0.877D-06	-0.174D 04	-0.175D 04	0.000D 00	-0.645D-02	-0.257D-02	0.000D 00
6	0.443D-01	-0.129D-05	0.000D 00	-0.625D-05	0.174D-04	-0.175D 04	-0.176D 04	0.000D 00	0.424D-02	-0.251D-02	0.000D 00
7	0.525D-01	-0.159D-05	0.000D 00	-0.633D-05	-0.814D-05	-0.175D 04	-0.176D 04	0.000D 00	0.298D-01	0.123D-01	0.000D 00
8	0.607D-01	-0.183D-05	0.000D 00	-0.629D-05	-0.105D-04	-0.175D 04	-0.175D 04	0.000D 00	-0.194D-01	-0.523D-02	0.000D 00
9	0.690D-01	-0.200D-05	0.000D 00	-0.638D-05	0.610D-05	-0.175D 04	-0.175D 04	0.000D 00	0.859D-02	0.220D-02	0.000D 00
10	0.774D-01	-0.233D-05	0.000D 00	-0.638D-05	-0.171D-04	-0.175D 04	-0.175D 04	0.000D 00	0.757D-02	0.494D-02	0.000D 00
11	0.859D-01	-0.252D-05	0.000D 00	-0.641D-05	0.384D-05	-0.175D 04	-0.174D 04	0.000D 00	-0.380D-01	-0.141D-01	0.000D 00
12	0.946D-01	-0.273D-05	0.000D 00	-0.666D-05	0.210D-04	-0.175D 04	-0.175D 04	0.000D 00	0.725D-02	0.430D-03	0.000D 00
13	0.103D 00	-0.301D-05	0.000D 00	-0.683D-05	-0.441D-05	-0.175D 04	-0.176D 04	0.000D 00	0.333D-01	0.124D-01	0.000D 00
14	0.112D 00	-0.324D-05	0.000D 00	-0.684D-05	-0.200D-04	-0.175D 04	-0.175D 04	0.000D 00	0.134D-01	0.653D-02	0.000D 00
15	0.121D 00	-0.349D-05	0.000D 00	-0.676D-05	-0.221D-04	-0.175D 04	-0.173D 04	0.000D 00	-0.139D-01	-0.327D-02	0.000D 00
16	0.131D 00	-0.371D-05	0.000D 00	-0.681D-05	-0.791D-06	-0.175D 04	-0.172D 04	0.000D 00	-0.241D-01	-0.863D-02	0.000D 00
17	0.140D 00	-0.395D-05	0.000D 00	-0.704D-05	0.102D-04	-0.175D 04	-0.173D 04	0.000D 00	-0.414D-02	-0.215D-02	0.000D 00
18	0.150D 00	-0.421D-05	0.000D 00	-0.731D-05	0.124D-04	-0.175D 04	-0.174D 04	0.000D 00	-0.184D-01	-0.736D-02	0.000D 00
19	0.161D 00	-0.439D-05	0.000D 00	-0.779D-05	0.339D-04	-0.175D 04	-0.176D 04	0.000D 00	-0.227D-02	-0.263D-02	0.000D 00
20	0.171D 00	-0.464D-05	0.000D 00	-0.825D-05	0.128D-04	-0.175D 04	-0.178D 04	0.000D 00	0.215D-01	0.710D-02	0.000D 00
21	0.183D 00	-0.483D-05	0.000D 00	-0.857D-05	0.156D-05	-0.175D 04	-0.179D 04	0.000D 00	0.938D-02	0.331D-02	0.000D 00
22	0.194D 00	-0.497D-05	0.000D 00	-0.881D-05	-0.860D-05	-0.175D 04	-0.179D 04	0.000D 00	0.347D-01	0.128D-01	0.000D 00
23	0.204D 00	-0.523D-05	0.000D 00	-0.867D-05	-0.494D-04	-0.175D 04	-0.176D 04	0.000D 00	0.839D-02	0.484D-02	0.000D 00
24	0.214D 00	-0.532D-05	0.000D 00	-0.859D-05	-0.164D-04	-0.175D 04	-0.172D 04	0.000D 00	-0.513D-01	-0.179D-01	0.000D 00
25	0.224D 00	-0.543D-05	0.000D 00	-0.888D-05	0.134D-04	-0.175D 04	-0.173D 04	0.000D 00	-0.243D-02	-0.127D-02	0.000D 00
26	0.233D 00	-0.564D-05	0.000D 00	-0.913D-05	-0.483D-05	-0.175D 04	-0.173D 04	0.000D 00	0.108D-01	0.402D-02	0.000D 00
27	0.242D 00	-0.572D-05	0.000D 00	-0.938D-05	0.628D-05	-0.175D 04	-0.173D 04	0.000D 00	-0.278D-01	-0.102D-01	0.000D 00
28	0.251D 00	-0.575D-05	0.000D 00	-0.982D-05	0.234D-04	-0.175D 04	-0.174D 04	0.000D 00	0.664D-02	0.188D-02	0.000D 00
29	0.260D 00	-0.593D-05	0.000D 00	-0.101D-04	0.355D-05	-0.175D 04	-0.175D 04	0.000D 00	0.282D-02	0.945D-03	0.000D 00
30	0.269D 00	-0.586D-05	0.000D 00	-0.105D-04	0.191D-04	-0.175D 04	-0.176D 04	0.000D 00	-0.191D-02	-0.102D-02	0.000D 00
31	0.277D 00	-0.598D-05	0.000D 00	-0.108D-04	0.117D-05	-0.175D 04	-0.177D 04	0.000D 00	0.220D-01	0.791D-02	0.000D 00
32	0.286D 00	-0.594D-05	0.000D 00	-0.111D-04	-0.718D-05	-0.175D 04	-0.177D 04	0.000D 00	0.182D-01	0.663D-02	0.000D 00
33	0.294D 00	-0.602D-05	0.000D 00	-0.111D-04	-0.222D-04	-0.175D 04	-0.176D 04	0.000D 00	0.313D-02	0.138D-02	0.000D 00
34	0.302D 00	-0.594D-05	0.000D 00	-0.113D-04	-0.117D-04	-0.175D 04	-0.174D 04	0.000D 00	-0.110D-01	-0.386D-02	0.000D 00
35	0.310D 00	-0.599D-05	0.000D 00	-0.114D-04	-0.856D-05	-0.175D 04	-0.174D 04	0.000D 00	-0.165D-01	-0.586D-02	0.000D 00
36	0.318D 00	-0.588D-05	0.000D 00	-0.117D-04	0.800D-05	-0.175D 04	-0.174D 04	0.000D 00	-0.629D-02	-0.231D-02	0.000D 00
37	0.326D 00	-0.589D-05	0.000D 00	-0.120D-04	-0.356D-05	-0.175D 04	-0.174D 04	0.000D 00	0.248D-01	0.894D-02	0.000D 00
38	0.335D 00	-0.589D-05	0.000D 00	-0.121D-04	-0.147D-04	-0.175D 04	-0.173D 04	0.000D 00	-0.771D-02	-0.275D-02	0.000D 00
39	0.343D 00	-0.573D-05	0.000D 00	-0.123D-04	0.323D-05	-0.175D 04	-0.173D 04	0.000D 00	-0.193D-01	-0.694D-02	0.000D 00
40	0.351D 00	-0.572D-05	0.000D 00	-0.126D-04	0.849D-05	-0.175D 04	-0.173D 04	0.000D 00	-0.151D-01	-0.541D-02	0.000D 00
41	0.359D 00	-0.553D-05	0.000D 00	-0.130D-04	0.219D-04	-0.175D 04	-0.174D 04	0.000D 00	-0.161D-02	-0.476D-03	0.000D 00
42	0.367D 00	-0.549D-05	0.000D 00	-0.133D-04	0.104D-04	-0.175D 04	-0.175D 04	0.000D 00	0.132D-01	0.482D-02	0.000D 00
43	0.375D 00	-0.529D-05	0.000D 00	-0.137D-04	0.583D-05	-0.175D 04	-0.176D 04	0.000D 00	0.175D-01	0.634D-02	0.000D 00
44	0.383D 00	-0.525D-05	0.000D 00	-0.138D-04	-0.876D-05	-0.175D 04	-0.176D 04	0.000D 00	-0.558D-02	-0.210D-02	0.000D 00
45	0.391D 00	-0.489D-05	0.000D 00	-0.141D-04	0.782D-05	-0.175D 04	-0.176D 04	0.000D 00	0.474D-02	0.181D-02	0.000D 00
46	0.400D 00	-0.487D-05	0.000D 00	-0.143D-04	-0.163D-04	-0.175D 04	-0.176D 04	0.000D 00	0.190D-01	0.660D-02	0.000D 00
47	0.408D 00	-0.455D-05	0.000D 00	-0.144D-04	-0.130D-04	-0.175D 04	-0.174D 04	0.000D 00	-0.836D-03	-0.533D-03	0.000D 00
48	0.417D 00	-0.442D-05	0.000D 00	-0.145D-04	-0.155D-04	-0.175D 04	-0.173D 04	0.000D 00	-0.191D-01	-0.720D-02	0.000D 00
49	0.426D 00	-0.403D-05	0.000D 00	-0.147D-04	0.110D-04	-0.175D 04	-0.173D 04	0.000D 00	-0.159D-01	-0.545D-02	0.000D 00
50	0.435D 00	-0.383D-05	0.000D 00	-0.150D-04	0.207D-05	-0.175D 04	-0.174D 04	0.000D 00	0.179D-01	0.648D-02	0.000D 00
51	0.444D 00	-0.353D-05	0.000D 00	-0.151D-04	-0.633D-05	-0.175D 04	-0.173D 04	0.000D 00	-0.285D-02	-0.121D-02	0.000D 00
52	0.454D 00	-0.318D-05	0.000D 00	-0.153D-04	0.945D-05	-0.175D 04	-0.173D 04	0.000D 00	-0.364D-01	-0.128D-01	0.000D 00
53	0.464D 00	-0.268D-05	0.000D 00	-0.158D-04	0.427D-04	-0.175D 04	-0.176D 04	0.000D 00	-0.128D-01	-0.304D-02	0.000D 00
54	0.475D 00	-0.228D-05	0.000D 00	-0.164D-04	0.284D-04	-0.175D 04	-0.179D 04	0.000D 00	0.218D-01	0.901D-02	0.000D 00
55	0.486D 00	-0.169D-05	0.000D 00	-0.168D-04	0.159D-04	-0.175D 04	-0.181D 04	0.000D 00	0.214D-01	0.845D-02	0.000D 00
56	0.496D 00	-0.116D-05	0.000D 00	-0.169D-04	-0.115D-04	-0.174D 04	-0.182D 04	0.000D 00	0.350D-01	0.120D-01	0.000D 00
57	0.506D 00	-0.608D-06	0.000D 00	-0.167D-04	-0.413D-04	-0.174D 04	-0.180D 04	0.000D 00	0.317D-01	0.908D-02	0.000D 00
58	0.516D 00	0.432D-07	0.000D 00	-0.162D-04	-0.732D-04	-0.174D 04	-0.177D 04	0.000D 00	0.709D-01	0.210D-01	0.000D 00
59	0.525D 00	0.443D-06	0.000D 00	-0.152D-04	-0.152D-03	-0.174D 04	-0.170D 04	0.000D 00	0.984D-01	0.251D-01	0.000D 00
60	0.534D 00	0.966D-06	0.000D 00	-0.135D-04	-0.222D-03	-0.174D 04	-0.159D 04	0.000D 00	0.958D-01	0.179D-01	0.000D 00
61	0.542D 00	0.126D-05	0.000D 00	-0.111D-04	-0.303D-03	-0.176D 04	-0.144D 04	0.000D 00	0.983D-01	0.106D-01	0.000D 00
62	0.551D 00	0.138D-05	0.000D 00	-0.822D-05	-0.366D-03	-0.179D 04	-0.127D 04	0.000D 00	0.443D-01	-0.168D-01	0.000D 00
63	0.559D 00	0.128D-05	0.000D 00	-0.502D-05	-0.376D-03	-0.183D 04	-0.109D 04	0.000D 00	-0.569D-01	-0.573D-01	0.000D 00
64	0.567D 00	0.926D-06	0.000D 00	-0.212D-05	-0.303D-03	-0.190D 04	-0.932D 03	0.000D 00	-0.194D 00	-0.102D 00	0.000D 00
65	0.575D 00	0.343D-06	0.000D 00	-0.267D-06	-0.129D-03	-0.200D 04	-0.801D 03	0.000D 00	-0.349D 00	-0.141D 00	0.000D 00

TABLE 5.4.2 (contd.)

AXISYMMETRIC STRESSES AND EQUIVALENT STRESSES

SEGMENT NO. 1

ELEMENT	STATION	σ_θ (IN)	σ_θ (OUT)	σ_ϕ (IN)	σ_ϕ (OUT)	σ_{EQ} (IN)	σ_{EQ} (OUT)
1	0.4000312D-02	-0.6690240D 06	-0.7217030D 06	-0.6809782D 06	-0.7104444D 06	0.6750805D 06	0.7011808D 06
2	0.1200562D-01	-0.7010173D 06	-0.6887306D 06	-0.7130219D 06	-0.6776237D 06	0.7070960D 06	0.6682832D 06
3	0.2002590D-01	-0.7099580D 06	-0.6838460D 06	-0.7125316D 06	-0.6784022D 06	0.7112483D 06	0.6661722D 06
4	0.2807231D-01	-0.7011552D 06	-0.7003122D 06	-0.6822804D 06	-0.7106015D 06	0.6919109D 06	0.6905374D 06
5	0.3614819D-01	-0.7039814D 06	-0.6990450D 06	-0.7036760D 06	-0.6913014D 06	0.7038287D 06	0.6802186D 06
6	0.4427216D-01	-0.7060156D 06	-0.7011891D 06	-0.6942858D 06	-0.7024234D 06	0.7002244D 06	0.6868056D 06
7	0.5245137D-01	-0.6925362D 06	-0.7162121D 06	-0.6706960D 06	-0.7279740D 06	0.6818785D 06	0.7071483D 06
8	0.6068689D-01	-0.7055916D 06	-0.6955428D 06	-0.7183349D 06	-0.6810248D 06	0.7120488D 06	0.6733639D 06
9	0.6901317D-01	-0.6983807D 06	-0.7026049D 06	-0.6915315D 06	-0.7080198D 06	0.6949815D 06	0.6902685D 06
10	0.7742561D-01	-0.6940939D 06	-0.7035830D 06	-0.6925378D 06	-0.7070725D 06	0.6933172D 06	0.6902490D 06
11	0.8593709D-01	-0.7100258D 06	-0.6829220D 06	-0.7359596D 06	-0.6630380D 06	0.7233415D 06	0.6580915D 06
12	0.9459296D-01	-0.7008177D 06	-0.7016428D 06	-0.6925277D 06	-0.7064516D 06	0.6967097D 06	0.6889136D 06
13	0.1033873D 00	-0.6920423D 06	-0.7158898D 06	-0.6678204D 06	-0.7318523D 06	0.6802549D 06	0.7088239D 06
14	0.1123334D 00	-0.6933968D 06	-0.7059338D 06	-0.6871016D 06	-0.7128204D 06	0.6902707D 06	0.6941815D 06
15	0.1214653D 00	-0.6959158D 06	-0.6896406D 06	-0.7129415D 06	-0.6863364D 06	0.7045830D 06	0.6727302D 06
16	0.1308230D 00	-0.6978295D 06	-0.6812526D 06	-0.7222080D 06	-0.6758620D 06	0.7103326D 06	0.6632618D 06
17	0.1404493D 00	-0.6942304D 06	-0.6901052D 06	-0.7025524D 06	-0.6945988D 06	0.6984286D 06	0.6769988D 06
18	0.1503621D 00	-0.7030763D 06	-0.6889427D 06	-0.7159405D 06	-0.6805921D 06	0.7095958D 06	0.6693847D 06
19	0.1606421D 00	-0.7073526D 06	-0.7022946D 06	-0.7005726D 06	-0.6962230D 06	0.7039871D 06	0.6837940D 06
20	0.1713407D 00	-0.7069050D 06	-0.7205276D 06	-0.6784828D 06	-0.7196675D 06	0.6931311D 06	0.7045432D 06
21	0.1825175D 00	-0.7123407D 06	-0.7186892D 06	-0.6907912D 06	-0.7087955D 06	0.7018142D 06	0.6981636D 06
22	0.1935606D 00	-0.7017978D 06	-0.7264209D 06	-0.6672678D 06	-0.7338304D 06	0.6851857D 06	0.7144438D 06
23	0.2040295D 00	-0.6977247D 06	-0.7070222D 06	-0.6927135D 06	-0.7088220D 06	0.6952326D 06	0.6921347D 06
24	0.2141241D 00	-0.7066111D 06	-0.6721844D 06	-0.7496853D 06	-0.6511594D 06	0.7291031D 06	0.6460609D 06
25	0.2238355D 00	-0.6916121D 06	-0.6891658D 06	-0.7025687D 06	-0.6979118D 06	0.6971550D 06	0.6776350D 06
26	0.2332710D 00	-0.6884584D 06	-0.6961695D 06	-0.6897079D 06	-0.7104185D 06	0.6890840D 06	0.6873802D 06
27	0.2424769D 00	-0.7016592D 06	-0.6821139D 06	-0.7265267D 06	-0.6730618D 06	0.7144176D 06	0.6615312D 06
28	0.2514465D 00	-0.6956051D 06	-0.6992057D 06	-0.6933807D 06	-0.7061218D 06	0.6944956D 06	0.6865083D 06
29	0.2602408D 00	-0.7006063D 06	-0.7024208D 06	-0.6970332D 06	-0.7024549D 06	0.6988266D 06	0.6861778D 06
30	0.2688789D 00	-0.7058101D 06	-0.7038564D 06	-0.7016139D 06	-0.6979550D 06	0.7037214D 06	0.6845862D 06
31	0.2773726D 00	-0.7006398D 06	-0.7158269D 06	-0.6787872D 06	-0.7210714D 06	0.6899731D 06	0.7020468D 06
32	0.2857563D 00	-0.7007994D 06	-0.7135308D 06	-0.6825866D 06	-0.7174405D 06	0.6918728D 06	0.6989983D 06
33	0.2940414D 00	-0.7011834D 06	-0.7038421D 06	-0.6970296D 06	-0.7030438D 06	0.6991157D 06	0.6868693D 06
34	0.3022437D 00	-0.7010247D 06	-0.6936204D 06	-0.7105922D 06	-0.6894458D 06	0.7058571D 06	0.6748903D 06
35	0.3103754D 00	-0.6999500D 06	-0.6886905D 06	-0.7157943D 06	-0.6841729D 06	0.7080051D 06	0.6697122D 06
36	0.3184531D 00	-0.6966520D 06	-0.6922220D 06	-0.7060048D 06	-0.6939227D 06	0.7013752D 06	0.6762645D 06
37	0.3264896D 00	-0.6870195D 06	-0.7041927D 06	-0.6761745D 06	-0.7238115D 06	0.6816617D 06	0.6973212D 06
38	0.3345036D 00	-0.6951102D 06	-0.6898291D 06	-0.7073375D 06	-0.6925310D 06	0.7013038D 06	0.6742177D 06
39	0.3425068D 00	-0.6973105D 06	-0.6839922D 06	-0.7184230D 06	-0.6814178D 06	0.7081028D 06	0.6656639D 06
40	0.3505121D 00	-0.6975875D 06	-0.6871958D 06	-0.7144202D 06	-0.6854490D 06	0.7061543D 06	0.6692008D 06
41	0.3585370D 00	-0.6974165D 06	-0.6965026D 06	-0.7014995D 06	-0.6984174D 06	0.6994670D 06	0.6802602D 06
42	0.3665946D 00	-0.6972371D 06	-0.7064835D 06	-0.6873156D 06	-0.7126451D 06	0.6923297D 06	0.6923046D 06
43	0.3747019D 00	-0.6982197D 06	-0.7103987D 06	-0.6831846D 06	-0.7167444D 06	0.6908249D 06	0.6962345D 06
44	0.3828716D 00	-0.7055438D 06	-0.7015118D 06	-0.7052483D 06	-0.6945366D 06	0.7053961D 06	0.6806137D 06
45	0.3911337D 00	-0.7018681D 06	-0.7053461D 06	-0.6953118D 06	-0.7044194D 06	0.6986130D 06	0.6873675D 06
46	0.3995017D 00	-0.6963291D 06	-0.7089988D 06	-0.6815999D 06	-0.7180986D 06	0.6890825D 06	0.6959991D 06
47	0.4079974D 00	-0.6984349D 06	-0.6974114D 06	-0.7006247D 06	-0.6990192D 06	0.6995324D 06	0.6805440D 06
48	0.4166533D 00	-0.7000808D 06	-0.6862642D 06	-0.7182091D 06	-0.6814901D 06	0.7093187D 06	0.6661388D 06
49	0.4255141D 00	-0.6976464D 06	-0.6871818D 06	-0.7152041D 06	-0.6847679D 06	0.7065888D 06	0.6681484D 06
50	0.4346229D 00	-0.6888708D 06	-0.7013169D 06	-0.6830366D 06	-0.7173218D 06	0.6859723D 06	0.6915501D 06
51	0.4440078D 00	-0.6950322D 06	-0.6927060D 06	-0.7030079D 06	-0.6975288D 06	0.6990542D 06	0.6771436D 06
52	0.4537487D 00	-0.7057491D 06	-0.6811487D 06	-0.7353644D 06	-0.6653859D 06	0.7210131D 06	0.6553444D 06
53	0.4639686D 00	-0.7058721D 06	-0.7000257D 06	-0.7128131D 06	-0.6881873D 06	0.7093681D 06	0.6760406D 06
54	0.4747701D 00	-0.7084906D 06	-0.7257969D 06	-0.6792590D 06	-0.7211075D 06	0.6943364D 06	0.7052418D 06
55	0.4857034D 00	-0.7177577D 06	-0.7339848D 06	-0.6785783D 06	-0.7197515D 06	0.6989920D 06	0.7086787D 06
56	0.4961763D 00	-0.7167359D 06	-0.7398254D 06	-0.6641383D 06	-0.7313994D 06	0.6919380D 06	0.7172867D 06
57	0.5061218D 00	-0.7129601D 06	-0.7303848D 06	-0.6658790D 06	-0.7267102D 06	0.6906242D 06	0.7101339D 06
58	0.5156152D 00	-0.6875617D 06	-0.7277879D 06	-0.6272292D 06	-0.7632896D 06	0.6594685D 06	0.7277177D 06
59	0.5247491D 00	-0.6559201D 06	-0.7040475D 06	-0.6007894D 06	-0.7897717D 06	0.6301660D 06	0.7321692D 06
60	0.5335901D 00	-0.6182253D 06	-0.6525860D 06	-0.6055820D 06	-0.7895293D 06	0.6120016D 06	0.7124473D 06
61	0.5421743D 00	-0.5671319D 06	-0.5874391D 06	-0.6092179D 06	-0.7979120D 06	0.5893031D 06	0.6983033D 06
62	0.5505607D 00	-0.5241544D 06	-0.4918032D 06	-0.6725868D 06	-0.7575968D 06	0.6120224D 06	0.6483325D 06
63	0.5587884D 00	-0.4915330D 06	-0.3814768D 06	-0.7885871D 06	-0.6794014D 06	0.6898245D 06	0.5731699D 06
64	0.5668987D 00	-0.4710840D 06	-0.2743768D 06	-0.9481290D 06	-0.5756483D 06	0.8211093D 06	0.4829228D 06
65	0.5749349D 00	-0.4556947D 06	-0.1851073D 06	-0.1135572D 07	-0.4649496D 06	0.9898017D 06	0.3906473D 06

TABLE 5.4.2. (contd.)

DISPLACEMENT AND STRESS RESULTANTS OF 'SEGMENT 1 SHELL TYPE 3 DZA= 0.15250D 02

ELM	S	U	V	W	β	N_ϕ	N_θ	$N_{\phi\theta}$	M_ϕ	M_θ	$M_{\phi\theta}$
1	0.400D-02	0.102D-04	0.000D 00	-0.996D-03	-0.883D-04	-0.174D 05	-0.174D 05	0.000D 00	0.170D 00	0.291D 00	0.000D 00
2	0.120D-01	0.431D-04	0.000D 00	-0.995D-03	-0.152D-04	-0.174D 05	-0.174D 05	0.000D 00	-0.168D 00	-0.472D-01	0.000D 00
3	0.200D-01	0.757D-04	0.000D 00	-0.993D-03	0.118D-03	-0.174D 05	-0.174D 05	0.000D 00	-0.161D 00	-0.119D 00	0.000D 00
4	0.281D-01	0.108D-03	0.000D 00	-0.991D-03	0.127D-03	-0.174D 05	-0.175D 05	0.000D 00	0.164D 00	0.123D-01	0.000D 00
5	0.361D-01	0.128D-03	0.000D 00	-0.988D-03	-0.283D-04	-0.174D 05	-0.175D 05	0.000D 00	-0.478D-01	-0.910D-02	0.000D 00
6	0.443D-01	0.171D-03	0.000D 00	-0.981D-03	0.129D-03	-0.174D 05	-0.175D 05	0.000D 00	0.586D-01	-0.875D-02	0.000D 00
7	0.525D-01	0.190D-03	0.000D 00	-0.977D-03	-0.135D-03	-0.175D 05	-0.175D 05	0.000D 00	0.314D 00	0.139D 00	0.000D 00
8	0.607D-01	0.220D-03	0.000D 00	-0.968D-03	-0.166D-03	-0.175D 05	-0.174D 05	0.000D 00	-0.178D 00	-0.361D-01	0.000D 00
9	0.690D-01	0.260D-03	0.000D 00	-0.957D-03	-0.879D-05	-0.174D 05	-0.174D 05	0.000D 00	0.101D 00	0.377D-01	0.000D 00
10	0.774D-01	0.276D-03	0.000D 00	-0.950D-03	-0.248D-03	-0.174D 05	-0.173D 05	0.000D 00	0.910D-01	0.649D-01	0.000D 00
11	0.859D-01	0.313D-03	0.000D 00	-0.935D-03	-0.474D-04	-0.174D 05	-0.172D 05	0.000D 00	-0.363D 00	-0.125D 00	0.000D 00
12	0.946D-01	0.348D-03	0.000D 00	-0.922D-03	0.116D-03	-0.174D 05	-0.172D 05	0.000D 00	0.864D-01	0.191D-01	0.000D 00
13	0.103D 00	0.372D-03	0.000D 00	-0.910D-03	-0.144D-03	-0.174D 05	-0.173D 05	0.000D 00	0.345D 00	0.138D 00	0.000D 00
14	0.112D 00	0.403D-03	0.000D 00	-0.893D-03	-0.307D-03	-0.174D 05	-0.171D 05	0.000D 00	0.147D 00	0.790D-01	0.000D 00
15	0.121D 00	0.431D-03	0.000D 00	-0.874D-03	-0.335D-03	-0.173D 05	-0.168D 05	0.000D 00	-0.124D 00	-0.185D-01	0.000D 00
16	0.131D 00	0.466D-03	0.000D 00	-0.851D-03	-0.131D-03	-0.173D 05	-0.167D 05	0.000D 00	-0.226D 00	-0.720D-01	0.000D 00
17	0.140D 00	0.496D-03	0.000D 00	-0.830D-03	-0.305D-04	-0.173D 05	-0.167D 05	0.000D 00	-0.281D-01	-0.814D-02	0.000D 00
18	0.150D 00	0.524D-03	0.000D 00	-0.809D-03	-0.163D-04	-0.172D 05	-0.167D 05	0.000D 00	-0.169D 00	-0.598D-01	0.000D 00
19	0.161D 00	0.561D-03	0.000D 00	-0.781D-03	0.188D-03	-0.172D 05	-0.168D 05	0.000D 00	-0.105D-01	-0.139D-01	0.000D 00
20	0.171D 00	0.587D-03	0.000D 00	-0.758D-03	-0.271D-04	-0.172D 05	-0.169D 05	0.000D 00	0.222D 00	0.813D-01	0.000D 00
21	0.183D 00	0.619D-03	0.000D 00	-0.726D-03	-0.144D-03	-0.172D 05	-0.168D 05	0.000D 00	0.102D 00	0.431D-01	0.000D 00
22	0.194D 00	0.651D-03	0.000D 00	-0.689D-03	-0.250D-03	-0.172D 05	-0.167D 05	0.000D 00	0.348D 00	0.135D 00	0.000D 00
23	0.204D 00	0.670D-03	0.000D 00	-0.660D-03	-0.655D-03	-0.171D 05	-0.163D 05	0.000D 00	0.903D-01	0.566D-01	0.000D 00
24	0.214D 00	0.699D-03	0.000D 00	-0.617D-03	-0.336D-03	-0.171D 05	-0.158D 05	0.000D 00	-0.494D 00	-0.167D 00	0.000D 00
25	0.224D 00	0.724D-03	0.000D 00	-0.579D-03	-0.490D-04	-0.171D 05	-0.157D 05	0.000D 00	-0.172D-01	-0.473D-02	0.000D 00
26	0.233D 00	0.739D-03	0.000D 00	-0.552D-03	-0.230D-03	-0.170D 05	-0.157D 05	0.000D 00	0.112D 00	0.466D-01	0.000D 00
27	0.242D 00	0.759D-03	0.000D 00	-0.515D-03	-0.126D-03	-0.170D 05	-0.155D 05	0.000D 00	-0.264D 00	-0.919D-01	0.000D 00
28	0.251D 00	0.779D-03	0.000D 00	-0.476D-03	0.368D-04	-0.170D 05	-0.155D 05	0.000D 00	0.701D-01	0.244D-01	0.000D 00
29	0.260D 00	0.788D-03	0.000D 00	-0.451D-03	-0.158D-03	-0.169D 05	-0.155D 05	0.000D 00	0.331D-01	0.151D-01	0.000D 00
30	0.269D 00	0.806D-03	0.000D 00	-0.408D-03	-0.110D-04	-0.169D 05	-0.155D 05	0.000D 00	-0.137D-01	-0.474D-02	0.000D 00
31	0.277D 00	0.814D-03	0.000D 00	-0.380D-03	-0.186D-03	-0.169D 05	-0.154D 05	0.000D 00	0.217D 00	0.810D-01	0.000D 00
32	0.286D 00	0.826D-03	0.000D 00	-0.341D-03	-0.269D-03	-0.169D 05	-0.153D 05	0.000D 00	0.179D 00	0.679D-01	0.000D 00
33	0.294D 00	0.833D-03	0.000D 00	-0.309D-03	-0.415D-03	-0.169D 05	-0.150D 05	0.000D 00	0.336D-01	0.169D-01	0.000D 00
34	0.302D 00	0.842D-03	0.000D 00	-0.266D-03	-0.315D-03	-0.168D 05	-0.148D 05	0.000D 00	-0.103D 00	-0.342D-01	0.000D 00
35	0.310D 00	0.846D-03	0.000D 00	-0.233D-03	-0.287D-03	-0.168D 05	-0.146D 05	0.000D 00	-0.156D 00	-0.539D-01	0.000D 00
36	0.318D 00	0.852D-03	0.000D 00	-0.190D-03	-0.129D-03	-0.168D 05	-0.144D 05	0.000D 00	-0.580D-01	-0.202D-01	0.000D 00
37	0.326D 00	0.854D-03	0.000D 00	-0.156D-03	-0.242D-03	-0.168D 05	-0.143D 05	0.000D 00	0.241D 00	0.876D-01	0.000D 00
38	0.335D 00	0.854D-03	0.000D 00	-0.122D-03	-0.350D-03	-0.168D 05	-0.141D 05	0.000D 00	-0.711D-01	-0.250D-01	0.000D 00
39	0.343D 00	0.854D-03	0.000D 00	-0.772D-04	-0.180D-03	-0.168D 05	-0.139D 05	0.000D 00	-0.182D 00	-0.655D-01	0.000D 00
40	0.351D 00	0.852D-03	0.000D 00	-0.440D-04	-0.132D-03	-0.168D 05	-0.138D 05	0.000D 00	-0.141D 00	-0.511D-01	0.000D 00
41	0.359D 00	0.848D-03	0.000D 00	-0.121D-05	-0.644D-05	-0.168D 05	-0.137D 05	0.000D 00	-0.106D-01	-0.385D-02	0.000D 00
42	0.367D 00	0.843D-03	0.000D 00	0.305D-04	-0.121D-03	-0.168D 05	-0.137D 05	0.000D 00	0.133D 00	0.469D-01	0.000D 00
43	0.375D 00	0.836D-03	0.000D 00	0.725D-04	-0.169D-03	-0.168D 05	-0.136D 05	0.000D 00	0.175D 00	0.615D-01	0.000D 00
44	0.383D 00	0.827D-03	0.000D 00	0.105D-03	-0.315D-03	-0.169D 05	-0.134D 05	0.000D 00	-0.471D-01	-0.202D-01	0.000D 00
45	0.391D 00	0.815D-03	0.000D 00	0.155D-03	-0.161D-03	-0.169D 05	-0.131D 05	0.000D 00	0.525D-01	0.168D-01	0.000D 00
46	0.400D 00	0.804D-03	0.000D 00	0.185D-03	-0.399D-03	-0.169D 05	-0.129D 05	0.000D 00	0.187D 00	0.614D-01	0.000D 00
47	0.408D 00	0.787D-03	0.000D 00	0.232D-03	-0.369D-03	-0.170D 05	-0.125D 05	0.000D 00	-0.939D-02	-0.995D-02	0.000D 00
48	0.417D 00	0.771D-03	0.000D 00	0.269D-03	-0.392D-03	-0.171D 05	-0.122D 05	0.000D 00	-0.193D 00	-0.774D-01	0.000D 00
49	0.426D 00	0.747D-03	0.000D 00	0.318D-03	-0.129D-03	-0.172D 05	-0.119D 05	0.000D 00	-0.159D 00	-0.603D-01	0.000D 00
50	0.435D 00	0.725D-03	0.000D 00	0.354D-03	-0.220D-03	-0.173D 05	-0.116D 05	0.000D 00	0.191D 00	0.629D-01	0.000D 00
51	0.444D 00	0.699D-03	0.000D 00	0.396D-03	-0.335D-03	-0.174D 05	-0.113D 05	0.000D 00	0.446D-01	0.631D-02	0.000D 00
52	0.454D 00	0.667D-03	0.000D 00	0.439D-03	-0.286D-03	-0.176D 05	-0.108D 05	0.000D 00	-0.157D 00	-0.659D-01	0.000D 00
53	0.464D 00	0.628D-03	0.000D 00	0.486D-03	-0.231D-03	-0.179D 05	-0.104D 05	0.000D 00	0.332D 00	0.111D 00	0.000D 00
54	0.475D 00	0.589D-03	0.000D 00	0.530D-03	-0.963D-03	-0.182D 05	-0.958D 04	0.000D 00	0.104D 01	0.335D 00	0.000D 00
55	0.486D 00	0.541D-03	0.000D 00	0.586D-03	-0.203D-02	-0.186D 05	-0.785D 04	0.000D 00	0.133D 01	0.387D 00	0.000D 00
56	0.496D 00	0.491D-03	0.000D 00	0.649D-03	-0.334D-02	-0.192D 05	-0.504D 04	0.000D 00	0.133D 01	0.308D 00	0.000D 00
57	0.506D 00	0.438D-03	0.000D 00	0.717D-03	-0.420D-02	-0.201D 05	-0.129D 04	0.000D 00	0.230D 00	-0.155D 00	0.000D 00
58	0.516D 00	0.375D-03	0.000D 00	0.785D-03	-0.376D-02	-0.212D 05	0.270D 04	0.000D 00	-0.163D 01	-0.820D 00	0.000D 00
59	0.525D 00	0.317D-03	0.000D 00	0.831D-03	-0.159D-02	-0.227D 05	0.603D 04	0.000D 00	-0.490D 01	-0.187D 01	0.000D 00
60	0.534D 00	0.245D-03	0.000D 00	0.845D-03	0.367D-02	-0.246D 05	0.749D 04	0.000D 00	-0.909D 01	-0.300D 01	0.000D 00
61	0.542D 00	0.179D-03	0.000D 00	0.795D-03	0.116D-01	-0.267D 05	0.595D 04	0.000D 00	-0.121D 02	-0.342D 01	0.000D 00
62	0.551D 00	0.116D-03	0.000D 00	0.669D-03	0.209D-01	-0.289D 05	0.124D 04	0.000D 00	-0.124D 02	-0.257D 01	0.000D 00
63	0.559D 00	0.634D-04	0.000D 00	0.468D-03	0.288D-01	-0.309D 05	-0.543D 04	0.000D 00	-0.640D 01	0.516D 00	0.000D 00
64	0.567D 00	0.263D-04	0.000D 00	0.228D-03	0.296D-01	-0.327D 05	-0.113D 05	0.000D 00	0.932D 01	0.654D 01	0.000D 00
65	0.575D 00	0.590D-05	0.000D 00	0.336D-04	0.157D-01	-0.342D 05	-0.133D 05	0.000D 00	0.383D 02	0.156D 02	0.000D 00

TABLE 5.4.2. (contd.)

AXISYMMETRIC STRESSES AND EQUIVALENT STRESSES

SEGMENT NO. 1

ELEMENT	STATION	σ_θ (IN)	σ_θ (OUT)	σ_ϕ(IN)	σ_ϕ(OUT)	σ_{EQ}(IN)	σ_{EQ}(OUT)
1	0.4000312D-02	-0.6673756D 07	-0.7232867D 07	-0.6793438D 07	-0.7120425D 07	0.6734395D 07	0.7027718D 07
2	0.1200562D-01	-0.6992104D 07	-0.6901541D 07	-0.7113406D 07	-0.6791743D 07	0.7053537D 07	0.6697711D 07
3	0.2002590D-01	-0.7078102D 07	-0.6849207D 07	-0.7107453D 07	-0.6798378D 07	0.7092823D 07	0.6674327D 07
4	0.2807231D-01	-0.6984891D 07	-0.7008432D 07	-0.6803423D 07	-0.7118443D 07	0.6895948D 07	0.6914475D 07
5	0.3614819D-01	-0.7006431D 07	-0.6988955D 07	-0.7014991D 07	-0.6923194D 07	0.7010715D 07	0.6806683D 07
6	0.4427216D-01	-0.7018276D 07	-0.7001476D 07	-0.6918651D 07	-0.7031155D 07	0.6968998D 07	0.6866720D 07
7	0.5245137D-01	-0.6873287D 07	-0.7140888D 07	-0.6679737D 07	-0.7282716D 07	0.6778585D 07	0.7063210D 07
8	0.6068689D-01	-0.6991784D 07	-0.6922439D 07	-0.7151404D 07	-0.6809885D 07	0.7072945D 07	0.6717190D 07
9	0.6901317D-01	-0.6905925D 07	-0.6978316D 07	-0.6879876D 07	-0.7074104D 07	0.6892937D 07	0.6877008D 07
10	0.7742561D-01	-0.6847312D 07	-0.6971953D 07	-0.6884700D 07	-0.7059367D 07	0.6866082D 07	0.6866350D 07
11	0.8593709D-01	-0.6988494D 07	-0.6748010D 07	-0.7311758D 07	-0.6614518D 07	0.7155605D 07	0.6532531D 07
12	0.9459296D-01	-0.6876962D 07	-0.6913548D 07	-0.6873646D 07	-0.7039467D 07	0.6875305D 07	0.6827590D 07
13	0.1033873D 00	-0.6767785D 07	-0.7032603D 07	-0.6621233D 07	-0.7284451D 07	0.6695712D 07	0.7012096D 07
14	0.1123334D 00	-0.6757382D 07	-0.6909060D 07	-0.6805411D 07	-0.7087121D 07	0.6781524D 07	0.6849963D 07
15	0.1214653D 00	-0.6755970D 07	-0.6720438D 07	-0.7053457D 07	-0.6815700D 07	0.6909518D 07	0.6618677D 07
16	0.1308230D 00	-0.6745756D 07	-0.6607485D 07	-0.7136247D 07	-0.6702523D 07	0.6949235D 07	0.6505571D 07
17	0.1404493D 00	-0.6677838D 07	-0.6662219D 07	-0.6931811D 07	-0.6877904D 07	0.6808378D 07	0.6622689D 07
18	0.1503621D 00	-0.6729842D 07	-0.6614930D 07	-0.7053177D 07	-0.6728839D 07	0.6897196D 07	0.6522566D 07
19	0.1606421D 00	-0.6733267D 07	-0.6706597D 07	-0.6890779D 07	-0.6870533D 07	0.6813389D 07	0.6639956D 07
20	0.1713407D 00	-0.6686174D 07	-0.6842272D 07	-0.6661494D 07	-0.7088008D 07	0.6673868D 07	0.6818265D 07
21	0.1825175D 00	-0.6693080D 07	-0.6775794D 07	-0.6770472D 07	-0.6966063D 07	0.6732109D 07	0.6722675D 07
22	0.1935606D 00	-0.6541572D 07	-0.6801282D 07	-0.6527988D 07	-0.7196099D 07	0.6534791D 07	0.6856868D 07
23	0.2040295D 00	-0.6453785D 07	-0.6562541D 07	-0.6764317D 07	-0.6937775D 07	0.6614520D 07	0.6607722D 07
24	0.2141241D 00	-0.6492741D 07	-0.6172910D 07	-0.7308852D 07	-0.6361015D 07	0.6936895D 07	0.6118621D 07
25	0.2238355D 00	-0.6298270D 07	-0.6289179D 07	-0.6837587D 07	-0.6804514D 07	0.6584515D 07	0.6411809D 07
26	0.2332710D 00	-0.6219120D 07	-0.6308539D 07	-0.6700340D 07	-0.6914969D 07	0.6473159D 07	0.6482397D 07
27	0.2424769D 00	-0.6299041D 07	-0.6122656D 07	-0.7047100D 07	-0.6540535D 07	0.6704444D 07	0.6191436D 07
28	0.2514465D 00	-0.6191625D 07	-0.6238547D 07	-0.6715529D 07	-0.6850183D 07	0.6469506D 07	0.6415446D 07
29	0.2602408D 00	-0.6190796D 07	-0.6219816D 07	-0.6740967D 07	-0.6804551D 07	0.6483413D 07	0.6381405D 07
30	0.2688789D 00	-0.6191870D 07	-0.6182767D 07	-0.6776908D 07	-0.6750600D 07	0.6504153D 07	0.6334818D 07
31	0.2773726D 00	-0.6091771D 07	-0.6247288D 07	-0.6548229D 07	-0.6964714D 07	0.6332351D 07	0.6484774D 07
32	0.2857563D 00	-0.6042825D 07	-0.6173245D 07	-0.6577695D 07	-0.6920854D 07	0.6327238D 07	0.6428598D 07
33	0.2940414D 00	-0.5995104D 07	-0.6027535D 07	-0.6710032D 07	-0.6774462D 07	0.6382669D 07	0.6283168D 07
34	0.3022437D 00	-0.5941424D 07	-0.5875779D 07	-0.6834689D 07	-0.6636699D 07	0.6434727D 07	0.6140406D 07
35	0.3103754D 00	-0.5877702D 07	-0.5774273D 07	-0.6879367D 07	-0.6580509D 07	0.6437251D 07	0.6066334D 07
36	0.3184531D 00	-0.5791591D 07	-0.5752827D 07	-0.6781066D 07	-0.6669782D 07	0.6344464D 07	0.6111662D 07
37	0.3264896D 00	-0.5643086D 07	-0.5811257D 07	-0.6491277D 07	-0.6953917D 07	0.6111486D 07	0.6309146D 07
38	0.3345036D 00	-0.5663331D 07	-0.5615350D 07	-0.6788338D 07	-0.6651800D 07	0.6301607D 07	0.6048933D 07
39	0.3425068D 00	-0.5625109D 07	-0.5499268D 07	-0.6893990D 07	-0.6544991D 07	0.6355274D 07	0.5939886D 07
40	0.3505121D 00	-0.5565885D 07	-0.5467813D 07	-0.6856262D 07	-0.6585482D 07	0.6310803D 07	0.5954085D 07
41	0.3585370D 00	-0.5499547D 07	-0.5492155D 07	-0.6734444D 07	-0.6714071D 07	0.6209780D 07	0.6044618D 07
42	0.3665946D 00	-0.5429612D 07	-0.5519733D 07	-0.6602542D 07	-0.6856998D 07	0.6101230D 07	0.6146531D 07
43	0.3747019D 00	-0.5366907D 07	-0.5484955D 07	-0.6569472D 07	-0.6905248D 07	0.6058375D 07	0.6167008D 07
44	0.3828716D 00	-0.5360844D 07	-0.5321987D 07	-0.6792361D 07	-0.6701992D 07	0.6201776D 07	0.5980540D 07
45	0.3911337D 00	-0.5243470D 07	-0.5275636D 07	-0.6710595D 07	-0.6811333D 07	0.6110586D 07	0.6039578D 07
46	0.3995017D 00	-0.5102477D 07	-0.5220331D 07	-0.6599250D 07	-0.6958553D 07	0.5992733D 07	0.6125069D 07
47	0.4079974D 00	-0.5029462D 07	-0.5010367D 07	-0.6810485D 07	-0.6792465D 07	0.6117607D 07	0.5952652D 07
48	0.4166533D 00	-0.4945487D 07	-0.4796926D 07	-0.7015913D 07	-0.6645125D 07	0.6243698D 07	0.5794095D 07
49	0.4255141D 00	-0.4811571D 07	-0.4695751D 07	-0.7021027D 07	-0.6715252D 07	0.6218028D 07	0.5821935D 07
50	0.4346229D 00	-0.4598711D 07	-0.4719411D 07	-0.6732899D 07	-0.7098681D 07	0.5959649D 07	0.6114159D 07
51	0.4440078D 00	-0.4501922D 07	-0.4514044D 07	-0.6931175D 07	-0.7016806D 07	0.6091378D 07	0.6016755D 07
52	0.4537487D 00	-0.4388877D 07	-0.4262424D 07	-0.7199104D 07	-0.6897593D 07	0.6284374D 07	0.5887587D 07
53	0.4639686D 00	-0.4046290D 07	-0.4259215D 07	-0.6828487D 07	-0.7465325D 07	0.5947324D 07	0.6348824D 07
54	0.4747701D 00	-0.3511267D 07	-0.4154889D 07	-0.6279468D 07	-0.8278238D 07	0.5450856D 07	0.7037041D 07
55	0.4857034D 00	-0.2766930D 07	-0.3510319D 07	-0.6174215D 07	-0.8736736D 07	0.5356605D 07	0.7492296D 07
56	0.4961763D 00	-0.1719638D 07	-0.2311035D 07	-0.6420404D 07	-0.8974207D 07	0.5756559D 07	0.7964564D 07
57	0.5061218D 00	-0.6639386D 06	-0.3672630D 06	-0.7812156D 07	-0.8253136D 07	0.7502253D 07	0.7995126D 07
58	0.5156152D 00	0.2911069D 06	0.1865858D 07	-0.1005679D 08	-0.6930711D 07	0.1020546D 08	0.7980968D 07
59	0.5247491D 00	0.6148252D 06	0.4209611D 07	-0.1379916D 08	-0.4392092D 07	0.1411662D 08	0.7449551D 07
60	0.5335901D 00	0.1181773D 06	0.5872944D 07	-0.1856538D 08	-0.1115524D 07	0.1862475D 08	0.6560453D 07
61	0.5421743D 00	-0.9064506D 06	0.5665151D 07	-0.2233552D 08	0.9670509D 06	0.2189637D 08	0.5346992D 07
62	0.5505607D 00	-0.1975954D 07	0.2966697D 07	-0.2342255D 08	0.3101779D 06	0.2249974D 08	0.2915983D 07
63	0.5587884D 00	-0.1677550D 07	-0.2667509D 07	-0.1851830D 08	-0.6221557D 07	0.1773911D 08	0.5280948D 07
64	0.5668987D 00	0.1756518D 07	-0.1080269D 08	-0.4128999D 07	-0.2201501D 08	0.5233225D 07	0.1893491D 08
65	0.5749349D 00	0.9696615D 07	-0.2032467D 08	0.2305009D 08	-0.5044574D 08	0.2004552D 08	0.4383774D 08

Section (5.5) Comparison of Results Obtained by Finite Element Method and Membrane Analysis

In Section (4.5) of chapter four it was suggested that the stress resultants obtained by membrane analysis should be compared with those obtained by finite element method for the same problem. Having discussed finite element method in this chapter this comparison could now be considered. For the comparison the typical problem used is the hypothetical araldite drop shaped shell (refer to sections (4.3) and (5.4.0) of chapters four and five). The stress resultants for heads 1m, 2m, 3m and 4m obtained by both methods are given in Tables 5.5.1, 5.5.2, 5.5.3 and 5.5.4. It can be deduced from the tables that there is some agreement between these results (correct to 1 decimal place in most cases). It appears that the area of disagreement is mainly towards the bottom of the shell, suggesting that the membrane approach is also applicable to this type of problem. This deduction could be useful when the available computer is small in storage size since finite element method requires a lot of storage for its execution, and this is only possible with large machines.

x (mm)	z (mm)	F E M N_ϕ (N/mm)	MEMBRANE N_ϕ (N/mm)	F E M N_θ (N/mm)	MEMBRANE N_θ (N/mm)
44.3	10.0	-0.60	-0.60	-0.60	-0.60
60.9	20.0	-0.60	-0.60	-0.61	-0.60
72.5	30.0	-0.60	-0.60	-0.60	-0.60
81.1	40.0	-0.60	-0.60	-0.60	-0.60
87.7	50.0	-0.60	-0.60	-0.61	-0.60
92.7	60.0	-0.60	-0.60	-0.60	-0.60
96.3	70.0	-0.60	-0.60	-0.60	-0.60
98.7	80.0	-0.60	-0.60	-0.60	-0.60
99.9	90.0	-0.60	-0.60	-0.60	-0.60
100.0	100.0	-0.60	-0.60	-0.60	-0.60
99.0	110.0	-0.60	-0.60	-0.60	-0.60
96.8	120.0	-0.60	-0.60	-0.60	-0.60
93.2	130.0	-0.60	-0.60	-0.60	-0.60
88.1	140.0	-0.60	-0.60	-0.60	-0.60
81.0	150.0	-0.60	-0.60	-0.63	-0.60
70.9	160.0	-0.60	-0.60	-0.62	-0.60
54.9	170.0	-0.61	-0.60	-0.51	-0.60
33.2	175.0	-0.68	-0.80	-0.36	6.70

TABLE 5.5.1

Stress resultants obtained by F E and membrane methods for head = 1 m.

x (mm)	z (mm)	F E M N_ϕ (N/mm)	MEMBRANE N_ϕ (N/mm)	F E M N_θ (N/mm)	MEMBRANE N_θ (N/mm)
44.3	10.0	-1.20	-1.20	-1.19	-1.19
60.9	20.0	-1.20	-1.20	-1.19	-1.18
72.5	30.0	-1.19	-1.19	-1.18	-1.17
81.1	40.0	-1.19	-1.19	-1.17	-1.16
87.7	50.0	-1.19	-1.19	-1.17	-1.15
92.7	60.0	-1.19	-1.19	-1.15	-1.14
96.3	70.0	-1.19	-1.18	-1.13	-1.13
98.7	80.0	-1.19	-1.18	-1.12	-1.12
99.9	90.0	-1.19	-1.18	-1.12	-1.11
100.0	100.0	-1.19	-1.18	-1.11	-1.10
99.0	110.0	-1.19	-1.18	-1.09	-1.09
96.8	120.0	-1.19	-1.19	-1.07	-1.08
93.2	130.0	-1.19	-1.19	-1.05	-1.06
88.1	140.0	-1.20	-1.20	-1.03	-1.03
81.0	150.0	-1.22	-1.22	-0.99	-1.00
70.9	160.0	-1.25	-1.25	-0.92	-0.91
54.9	170.0	-1.41	-1.39	-0.82	-0.54
33.2	175.0	-1.64	-11.3	-0.83	399.50

TABLE 5.5.2

Stress resultants obtained by F E and membrane methods for head = 2 m.

x (mm)	z (mm)	F E M N_ϕ (N/mm)	MEMBRANE N_ϕ (N/mm)	F E M N_θ (N/mm)	MEMBRANE N_θ (N/mm)
44.3	10.0	-1.79	-1.80	-1.78	-1.78
60.9	20.0	-1.79	-1.79	-1.78	-1.76
72.5	30.0	-1.79	-1.79	-1.75	-1.74
81.1	40.0	-1.78	-1.78	-1.74	-1.73
87.7	50.0	-1.78	-1.78	-1.73	-1.71
92.7	60.0	-1.78	-1.77	-1.70	-1.69
96.3	70.0	-1.77	-1.77	-1.66	-1.67
98.7	80.0	-1.77	-1.77	-1.65	-1.65
99.9	90.0	-1.77	-1.76	-1.64	-1.63
100.0	100.0	-1.77	-1.76	-1.62	-1.61
99.0	110.0	-1.77	-1.77	-1.59	-1.58
96.8	120.0	-1.77	-1.77	-1.55	-1.55
93.2	130.0	-1.79	-1.78	-1.50	-1.52
88.1	140.0	-1.80	-1.80	-1.46	-1.47
81.0	150.0	-1.84	-1.83	-1.36	-1.39
70.9	160.0	-1.90	-1.90	-1.22	-1.23
54.9	170.0	-2.21	-2.18	-1.13	-0.49
33.2	175.0	-2.59	-21.8	-1.31	792.4

TABLE 5.5.3

Stress resultants obtained by F E and membrane methods for head = 3 m.

x (mm)	z (mm)	F E M N_ϕ (N/mm)	MEMBRANE N_ϕ (N/mm)	F E M N_θ (N/mm)	MEMBRANE N_θ (N/mm)
44.3	10.0	-2.39	-2.40	-2.37	-2.37
60.9	20.0	-2.39	-2.39	-2.36	-2.34
72.5	30.0	-2.38	-2.38	-2.32	-2.31
81.1	40.0	-2.37	-2.37	-2.31	-2.29
87.7	50.0	-2.37	-2.36	-2.29	-2.26
92.7	60.0	-2.36	-2.36	-2.25	-2.23
96.3	70.0	-2.36	-2.35	-2.20	-2.21
98.7	80.0	-2.35	-2.35	-2.17	-2.18
99.9	90.0	-2.35	-2.35	-2.16	-2.15
100.0	100.0	-2.35	-2.35	-2.12	-2.11
99.0	110.0	-2.35	-2.35	-2.08	-2.07
96.8	120.0	-2.36	-2.36	-2.03	-2.03
93.2	130.0	-2.38	-2.37	-1.95	-1.98
88.1	140.0	-2.40	-2.39	-1.89	-1.91
81.0	150.0	-2.46	-2.44	-1.73	-1.79
70.9	160.0	-2.55	-2.55	-1.52	-1.55
54.9	170.0	-3.01	-2.97	-1.44	-0.44
33.2	175.0	-3.54	-32.3	-1.78	1185.3

TABLE 5.5.4

Stress resultants obtained by F E and membrane methods for head = 4 m.

Section (5.6) Summary and Conclusions

In this chapter, the behaviour of the drop shaped tank due to varying pressure head is investigated using finite element method. After a brief explanation of shell analysis, a summary of finite element method is given. The version implemented in this work is then described with two problems used for illustration. Results obtained by this method are then compared with the previous chapter's. From the comparative analysis it will appear that there is some agreement between the obtained results. The prediction of the computer program for the problems considered is encouraging and needs to be verified experimentally.

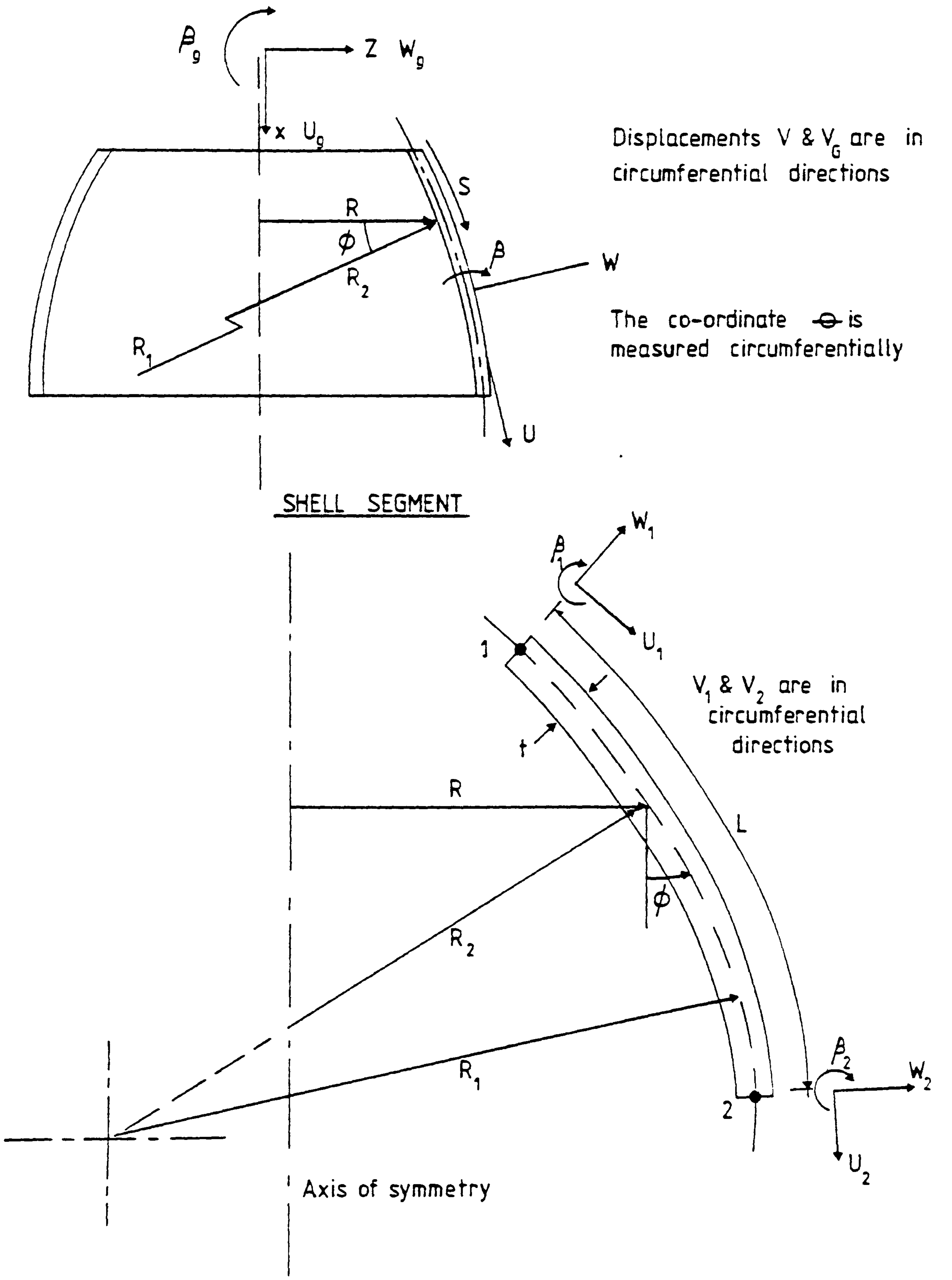

Fig. 5.3.1 SHELL ELEMENT

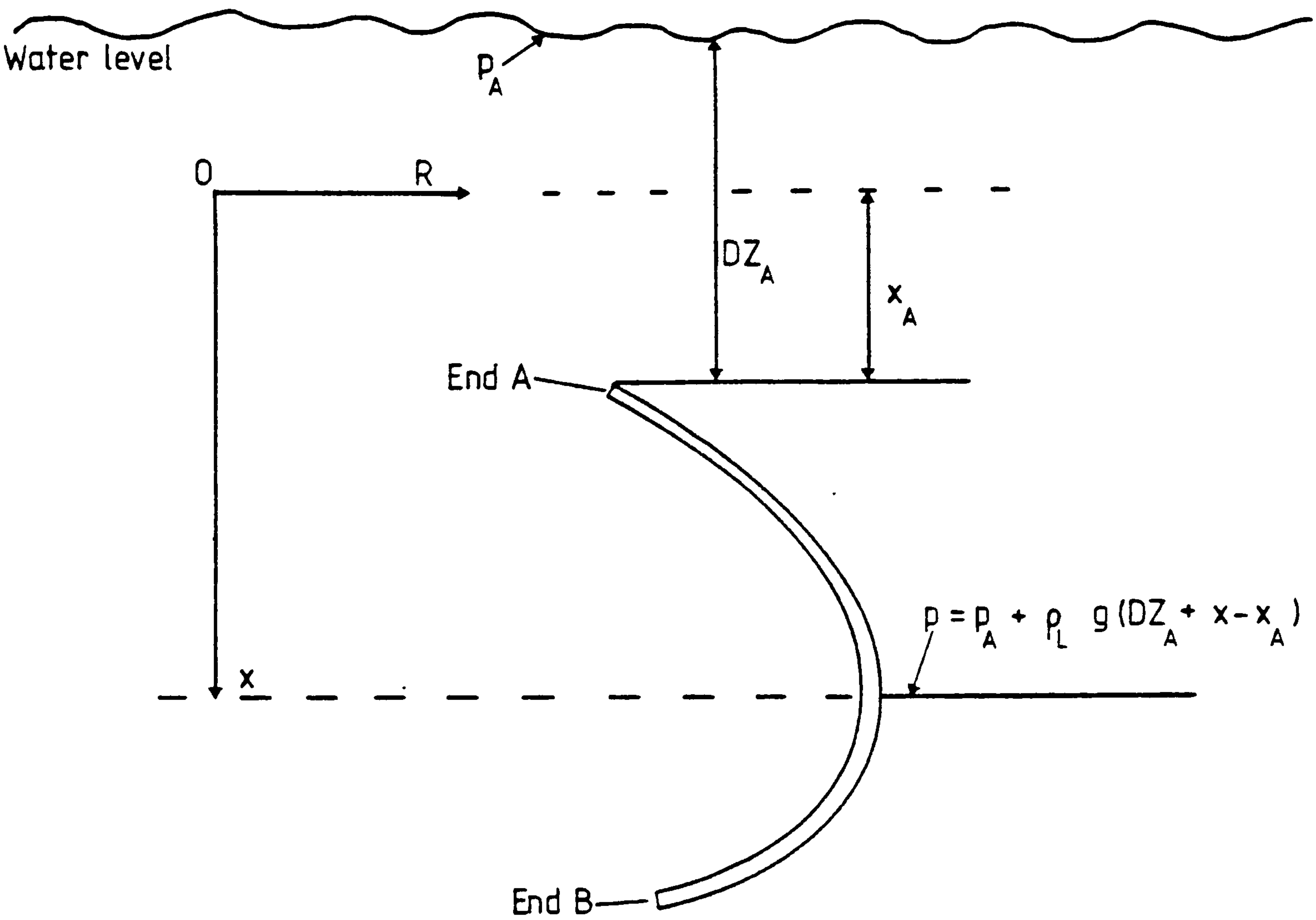

Fig 5.3.2

Drop shaped profile under hydrostatic force

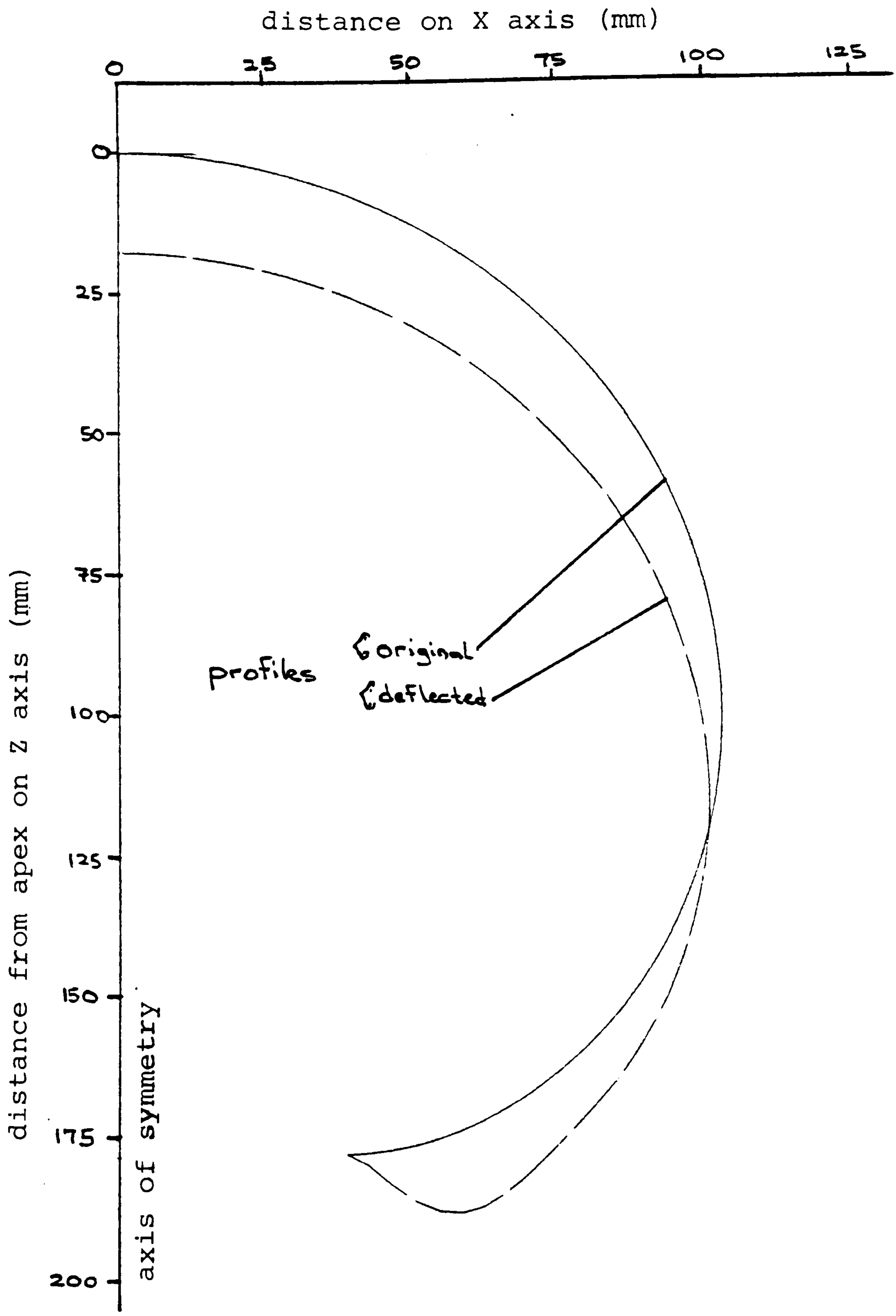

Figure 5.4.1

Deformation of shell at 2 × design head

(displacement × 1000)

CHAPTER SIX: EXPERIMENTAL STUDY OF THE DROP SHAPED TANK UNDER HYDROSTATIC HEADS

CHAPTER SIX: EXPERIMENTAL STUDY OF THE DROP SHAPED TANK UNDER HYDROSTATIC HEADS

Section (6.0) Introduction

In this chapter a description is given of experiments to investigate the behaviour of the drop shaped tank due to varying hydrostatic pressure head. Earlier, in chapters four and five, two theoretical (numerical) methods that may be used for this purpose were reported. Results obtained by the theoretical approaches are encouraging but do lack experimental confirmation. It is hoped that this gap could be narrowed and that the results obtained from the theoretical and experimental investigations would provide some justification for the possible use of the drop shaped tank under the stated conditions.

Before embarking upon the main experiments, material control tests were carried out to obtain the elastic properties of the material used in constructing the prototype and dummy tanks. These tests are reported in Section (6.1). The main experiments are reported in Section (6.2) and Section (6.3) contains a summary and conclusions of this chapter.

Section (6.1) Material control tests

These tests were carried out to determine the elastic properties of the fibreglass that was used in the construction of the experimental tanks {Section (6.2)}.

Two small shouldered pieces having central uniform sections of (a) test specimen: length 100 mm, width 13.15 mm, thickness 2.75 mm and (b) dummy specimen: length 100 mm, width 12.85 mm, thickness 2.75 mm, were cut from the base of the prototype tank. The prototype and dummy tanks were constructed from the same batch of fibreglass. Four (350 ohms, G.F. 2.15) electric resistance foil strain gauges were glued onto the pieces after preparing their surfaces for this purpose. A pair was glued to each specimen longitudinally and laterally on either side. Wire leads from the gauges were connected to a strain measuring device (Transducer Meter Type C56 manufactured by Sangamo Weston Controls Ltd., Sussex, England). A half bridge circuit was used in recording the gauge signals with the bridge supply voltage of 5 volts r.m.s. giving a gauge current of 7.1 mA.

The test specimen was mounted in a Hounsfield-Tensometer and the dummy placed close by in the same environment. Five runs with load varying from 0 to 0.8 KN were carried out. From the obtained results, the mean incremental changes in strain, and cummulative mean strain were calculated. These were used in drawing the graphs shown in Figures 6.1.1 and 6.1.2. From these graphs and by calculations, it was deduced that the Young's modulus of the material was 0.8×10^4 MN/m^2 and Poisson ratio 0.36.

After these runs, two other runs were carried out to determine the failure load and stress of the specimens. The test specimen had a failure load of 1.96 KN and the dummy specimen a failure load of 2.1 KN. Consequently the failure stresses for the specimens were 54.20 and 59.40 MN/m^2 and the average failure stress was 56.80 MN/m^2.

The above tests were performed in a tensile mode. It was felt desirable to carry out some compression tests in order to obtain some indication of the compressive characteristics of the material. Unfortunately this was difficult due to the thickness of the test material. It is therefore assumed that this material has the same elastic properties both in tension and in compression.

Section (6.2) Hydrostatic tests

The main objective of the experiments described below (Section (6.2.3)) is to examine the development of surface strains and consequently stresses along the intersections of three equally spaced meridians and parallel circles on the prototype tank under symmetric hydrostatic loading. The outcome of the investigation could indicate the possibility of using such tank shape under the stated conditions.

Section (6.2.1) Pressure chamber arrangement

This arrangement consisted of an autoclave adapted to act as a pressure chamber. The autoclave made of copper had an internal diameter of 465 mm and an overall height of 750 mm. Through an outlet on the side of the chamber was a tube connecting it to a pressure measuring system. The measuring system consisted of manometers filled with water, bromoform, mercury and an electromanometer attached to a free end. The different fluids and electromanometer were to provide reasonable accuracy over a range of low pressures from zero to 20 m of water and offer a means of checking one system against another. Due to convenience, safety and cost the liquid used in pressurising the prototype tank which was placed inside the chamber was water. The prototype tank had dimensions: height 380 mm, maximum diameter 450 mm, thickness (mean) 2.5 mm ±10% and was designed for a head of 1.525 m (water) using a design stress of 0.7 MN/m^2 (refer to Figure 6.2.1.1 for a photograph of the tank).

At the intersections of three equally spaced meridians and parallel circles on the tank, electrical resistance foil strain gauge rosettes (350 ohms, G.F. 2.15) were bonded, adequate care being taken to prepare the shell surface for attaching the gauges. Similar gauges were bonded in similar positions on the surface of the dummy tank which was placed inside a water filled reservoir positioned close to the pressure chamber. The base of the prototype tank was bonded to a square tufnol plate and bolted to a circular dural table. This was attached to an aluminium cylindrical strut which acted as an antibuoyancy device for the tank.

The gauges on both tanks and leads from them were waterproofed using silicone rubber. The cable of leads from the prototype was passed through an opening in the wall of the chamber. This opening was sealed using silicone rubber. So as to allow for lifting of the tank from time to time for inspection without unsoldering, at the strain gauge terminal tags a sufficient length of leads was left inside the chamber. Care was taken to allow the same length of leads inside the reservoir containing the dummy. A removable lid was used in covering the pressure chamber. On this lid, there was a sealable bleed hole. This enabled one to inspect and ascertain that the chamber was completely filled with water before commencing a test.

The chamber was pressurized through an opening on its side directly from the water mains. The pressure chamber arrangement was as shown in Figure 6.2.1.2.

Section (6.2.2) Strain measuring arrangement

This arrangement consisted of an interface, a power supply unit, a voltmeter, a computer and a printing machine. The interface linked the plugs attached to the strain gauge wire leads to the voltmeter. The half bridge circuit of the gauges was excited by 2.5 volts D.C. supplied by the power unit (Farnell LT30-1 stabilised power supply unit 2 × 0 - 30v) giving a gauge current of 4.80 mA. The voltmeter used for measuring the out of balance voltages across the gauges was a Fluke 8500 digital multiplier. A computer (COMMODORE PET) was programmed to instruct the voltmeter to measure the voltages when required. The number of measurements per point for each averaged scan was 32. The total sampling time per point in each scan was (4 × 32 =) 128 milliseconds, and the voltmeter responded to the instruction of the computer as fast as necessary. A computer program written in BASIC language {see appendix (A 6.2.2.1)} was employed with PET for scanning, recording and storing the measured voltages. The photograph shown in Figure 6.2.2.1 depicts the strain measuring arrangement with the pressure chamber (lid closed) in the background.

Section (6.2.3) Test procedure

After setting up the arrangements described in Sections (6.2.1) and (6.2.2) the prototype tank, with its cylindrical antibuoyancy strut attached, was placed inside the pressure chamber. By running the BASIC computer program (appendix (A 6.2.2.1)) on the PET the initial out of balance voltages of the gauges were obtained. The chamber was closed with its lid and filled with water up to the bleed hole in the lid. After establishing that there were no leaks and bubbles in the system the bleed hole was sealed. Also the dummy tank was placed inside its reservoir which was full of water and positioned close to the pressure chamber. The initial head of water above the tank was registered on the scale attached to the manometers and recorded. By running the BASIC program again on the PET, the new out of balance voltages of the gauges for this new head of water was obtained. Continuing to increase the water head gradually scans of the voltages at specific levels were obtained.

Remembering that the prototype tank was designed for 1.525 m of water head two types of tests were carried out. In the first, the applied water head was rapidly increased with scans taken when the water head was close to the designed value. In the second test, the applied water loads were made to be much higher than the designed one. In this case, the tank was pressurised to a required level and voltages before and after pressurising obtained. Then the chamber was emptied completely to release the applied pressure. Two hours was allowed to elapse before pressurising again to another level.

Section (6.2.4) Results and discussion of results

The arrangement of the rosettes on the tanks was as illustrated in Figure 6.2.4.1. The out of balance voltages of the gauges due to an applied pressure load measured by FLUKE and recorded by PET are converted to their equivalent strain values using the expression

$$\varepsilon = \frac{4\ \Delta v}{Ku}$$

where ε is the strain, u the supply voltage in volts, K the gauge-factor and Δv the out of balance voltage due to straining in volts. For a specific head, using the obtained results, the principal strains, angle and stresses of each rosette are evaluated using a FORTRAN computer program (appendix (A 6.2.4.1)) written for this purpose and based on the usual technique of stress analysis [69]. The "averaged" principal strains, direction and magnitude of the principal stresses at each parallel circle on a meridian are evaluated for the considered heads. For this calculation, the change in strain values of rosettes lying on the same parallel circle are added together and averaged. The obtained averaged values are then used in calculating the "averaged" principal strains, angle and stresses. Also by choosing the X and Y directions of the coordinate axes at the point of fixture of each rosette so that they coincide with the directions of the parallel circle and the meridian on which this point lies, stresses in these directions are evaluated. The "averaged" stresses in these directions are also evaluated. Tables 6.2.4.1 and 6.2.4.2 give the results of these calculations for Test 1 and Test 2 respectively at the considered heads. Figures 6.2.4.2 to 6.2.4.5 are plots of the evaluated maximum principal stresses (in MN/m^2) at each parallel circle for each meridian in Test 1. Figures 6.2.4.6 to 6.2.4.8 give similar graphs for Test 2. The predicted values of stresses given by MISTRY's finite element program (refer to chapter five) at these positions on the tank for the considered heads are also displayed in Tables 6.2.4.1 and 6.2.4.2.

It is observed from Tables 6.2.4.1 and 6.2.4.2 that for rosette 1 lying on the intersection of parallel circle 1 and meridian 1 the stress in the meridional direction is tensile. (Notice that compressive stresses are positive and tensile negative in the tables). This appears illogical as it is suspected that if any part of the tank is to undergo tension it will not be close to its apex, rather it may be at or close to its base. Therefore it is suspected that this rosette was malfunctioning and as there were no spare rosettes from the batch used in instrumenting the tanks it was decided to leave the rosette on the tank and ignore the results given by it.

For Test 1 and from Table 6.2.4.1 and Figures 6.2.4.2 to 6.2.4.5, it is observed that the obtained strain values do not show any uniform pattern at the heads considered for the three meridians. This of course is expected for heads different from the design head which was the case in this test since it was not easy to achieve exactly the design head condition. However, this variation was also due to the practical difficulty of constructing the tank so as to have uniform thickness. Nevertheless, close to the design head, the average strain increment from one head to another appear to have a uniform tendency. This is encouraging as it seems to support the application of membrane shell theory to the drop shaped tank in compression.

In the case of the second test (refer to Table 6.2.4.2 and Figures 6.2.4.6 to 6.2.4.8), the strain/stress pattern of the first test is repeated. For the third head considered the value obtained from the rosette No. 9 at the intersection of meridian 3 and parallel circle 3 is low and possibly has been influenced by local distortion near the base.

Overall for both tests, the evaluated stresses are low compared with the material failure stress obtained in Section (6.1). This is very encouraging but does not eliminate a possibility of the tank becoming unstable due to buckling. This could not be checked visually as the chamber wall and strut were not transparent and the experiments were not carried out to investigate buckling. In future work, buckling tests could be worth carrying out for this tank shape. It will also be interesting to try and obtain, if possible, a mathematical relationship between the applied pressure load and maximum principal stress at a meridian and/or parallel circle. This is not investigated here since the tank does not appear to behave symmetrically according to the obtained results. This could be due to the variation in shell thickness and associated differences in the time dependent response of the material to stress. This situation also makes it difficult to compare meaningfully the obtained experimental results with the theoretical predicted ones. However, it is noticed that the finite element computer program predicted that there would be no axisymmetric failure of the tank up to about nine times the design head (see appendix A 6.2.4.2). The results of the second test indicate that failure did not occur for pressures up to five times the design head although some evidence of distortion at or near the base was apparent at this pressure level. When the pressure head was raised to more than six times the design head it was found that the shell became unbonded from its Tufnol base over part of the contact area and permitted the ingress of a little water. The break down in bond definitely had not occurred at the lower heads since there was no difficulty in maintaining a constant head. The distortions in the base region would influence the behaviour of the rosette No. 9 as indicated by the results obtained at five times the design head.

Referring specifically to the magnitude and direction of the principal stresses computed from the measured surface strains it could be expected that at the design head the principal stresses would be equal and their directions would coincide with the meridian and the parallel circle. The results shown in Table 6.2.4.2(a) indicate that at or near the theoretical design head of 1.525 m no uniformity existed in the magnitude and distribution of the principal stresses. This in part could be attributed to the time dependent nature of the shell material's response to applied force, and additionally to the shell wall thickness not being uniform.

Referring specifically to the magnitude and direction of the principal stresses computed from the measured surface strains it could be expected that at the design head the principal stresses would be equal and their directions would coincide with the meridian and the parallel circle. The results shown in Table 6.2.4.2(a) indicate that at or near the theoretical design head of 1.525 m no uniformity existed in the magnitude and distribution of the principal stresses. This in part could be attributed to the time dependent nature of the shell material's response to applied force, and additionally to the shell wall thickness not being uniform.

Section (6.3) Summary and Conclusions

The experiments carried out to investigate the behaviour of the drop shaped tank under hydrostatic loadings were reported here. These were carried out with two tanks (a prototype and a dummy) constructed from fibreglass, a pressure chamber, a pressure measuring system, electrical resistance foil strain gauge rosettes and a strain measuring system. From the results of the experiments, the manner in which the surface strains and consequently stresses developed along the intersections of three equally spaced meridians and parallel circles on the prototype under this loading condition was obtained.

Results of the investigation are encouraging and give some credibility to those obtained theoretically in chapter five. Theoretically, it was predicted that the tank will not undergo axi-symmetric failure for heads up to about nine times the design head. Results from the experiments do support this for heads up to about five times the design head. This was the maximum head for which the experiments were conducted. It may be possible to improve upon this by considering higher heads. It is also suggested that other experiments and theoretical investigations involving different loading conditions, e.g., buckling, hydrodynamic and nondeterministic loading will have to be investigated before one can attest to the safety of such tanks in an underwater environment.

TABLE 6.2.4.1 (a)

Results for Test 1

	Parallel Circle	Rosette Number	Max. Prin. Strain (μ strain)	Min. Prin. Strain (μ strain)	2 × Angle (radians)	Max. Prin. Stress (MN/m²)	Min. Prin. Stress (MN/m²)
HEAD = 1210.5 mm	1	1	103.9	-140.3	0.7897	0.5	-0.9
		2	255.3	-23.8	-1.4236	2.3	0.6
		3	175.5	-79.5	1.0144	1.4	-0.2
	2	4	207.0	5.9	0.9643	1.9	0.7
		5	235.8	104.3	-0.5016	2.5	1.7
		6	239.6	-75.9	-1.0903	2.0	0.1
	3	7	284.6	-86.6	-0.4170	2.3	0.1
		8	342.6	-37.4	0.8048	3.0	0.8
		9	303.9	-56.1	1.5108	2.6	0.5
HEAD = 1510.5 mm	1	1	158.0	-179.6	0.8571	0.9	-1.1
		2	298.0	7.9	-1.4499	2.8	1.1
		3	237.5	-45.5	1.1189	2.0	0.4
	2	4	259.7	61.8	0.6360	2.6	1.4
		5	275.0	135.8	-0.7249	3.0	2.2
		6	275.9	-45.9	-1.0978	2.4	0.5
	3	7	295.9	-55.5	-0.3888	2.5	0.5
		8	432.6	-20.3	0.7482	3.9	1.2
		9	347.0	-73.2	-1.4429	2.9	0.5
HEAD = 1515.5 mm	1	1	151.5	-173.8	0.8080	0.8	-1.1
		2	349.8	6.7	-1.5643	3.2	1.2
		3	259.6	-86.2	1.1139	2.1	0.1
	2	4	255.2	64.8	0.6857	2.6	1.4
		5	266.9	140.9	-0.6175	2.9	2.2
		6	258.0	-31.7	-1.0313	2.3	0.6
	3	7	309.0	-44.8	-0.4740	2.7	0.6
		8	453.6	-21.2	0.7588	4.1	1.3
		9	346.4	-48.0	-1.4631	3.0	0.7
HEAD = 1540.5 mm	1	1	157.0	-193.5	0.8034	0.8	-1.3
		2	338.0	19.9	-1.5263	3.2	1.3
		3	256.4	-45.8	1.0642	2.2	0.4
	2	4	242.2	38.4	0.8681	2.4	1.2
		5	280.1	134.4	-0.6404	3.0	2.2
		6	288.2	-35.2	-1.0454	2.5	0.6
	3	7	335.2	-51.6	-0.3887	2.9	0.6
		8	461.2	-4.3	0.7447	4.2	1.5
		9	356.5	-0.8	-1.4118	3.3	1.2

TABLE 6.2.4.1 (b)

Results for Test 1

	Parallel Circle	AVERAGED: Averaged Maximum Principal Strain (μ strain)	Averaged Minimum Principal Strain (μ strain)	2 × Angle (radians)	Averaged Maximum Principal Stress (MN/m^2)	Averaged Minimum Principal Stress (MN/m^2)
HEAD = 1210.5 mm	1	* 128.2	35.5	-0.3268	1.3	0.8
	2	188.7	50.1	-0.4420	1.9	1.1
	3	144.7	105.6	-0.5862	1.7	1.4
HEAD = 1510.5 mm	1	* 165.0	84.0	-0.2082	1.8	1.3
	2	241.4	79.3	-0.5672	2.5	1.5
	3	165.3	143.6	0.3869	2.0	1.9
HEAD = 1515.5 mm	1	* 172.0	92.9	-0.2085	1.9	1.4
	2	233.4	84.6	-0.4666	2.4	1.5
	3	185.8	145.8	0.9261	2.2	2.0
HEAD = 1540.5 mm	1	* 184.5	99.8	-0.3218	2.0	1.5
	2	235.0	81.0	-0.4752	2.4	1.5
	3	202.2	163.2	1.2128	2.4	2.2

* The averaged values of parallel circle 1 for each head are the averages obtained from two rosettes, i.e. rosette numbers 2 and 3.

TABLE 6.2.4.1 (c)

Results for Test 1

Head	Parallel Circle	Rosette Number	Stress in Parallel Circle Direction (MN/m^2)	Stress in Meridional Direction (MN/m^2)	AVERAGED: Av. Stress in Parallel Circle Direction (MN/m^2)	AVERAGED: Av. Stress in Meridional Direction (MN/m^2)	MISTRY: Stress in Parallel Circle Direction (MN/m^2)	MISTRY: Stress in Meridional Direction (MN/m^2)
HEAD = 1210.5 mm	1	1	0.3	-0.7				
		2	1.6	1.3	* 1.3	0.8	0.6	0.6
		3	1.0	0.2				
	2	4	1.7	1.0				
		5	2.5	1.8	1.9	1.1	0.5	0.6
		6	1.5	0.6				
	3	7	2.2	0.2				
		8	1.1	2.7	1.7	1.5	0.5	0.6
		9	1.6	1.5				
HEAD = 1510.5 mm	1	1	0.5	-0.8				
		2	2.0	1.8	* 1.8	1.3	0.7	0.7
		3	1.6	0.8				
	2	4	2.5	1.5				
		5	2.9	2.3	2.4	1.6	0.7	0.7
		6	1.9	1.0				
	3	7	2.5	0.5				
		8	1.6	3.6	1.9	2.0	0.7	0.7
		9	1.6	1.9				
HEAD = 1515.5 mm	1	1	0.5	-0.8				
		2	2.2	2.2	* 1.9	1.4	0.7	0.7
		3	1.5	0.6				
	2	4	2.4	1.6				
		5	2.9	2.2	2.4	1.6	0.7	0.7
		6	1.9	1.0				
	3	7	2.6	0.7				
		8	1.7	3.7	2.0	2.1	0.7	0.7
		9	1.7	2.0				
HEAD = 1540.5 mm	1	1	0.5	-0.9				
		2	2.3	2.2	* 2.0	1.5	0.7	0.7
		3	1.7	0.9				
	2	4	2.1	1.4				
		5	2.9	2.2	2.4	1.6	0.7	0.7
		6	2.1	1.1				
	3	7	2.8	0.7				
		8	1.8	3.9	2.2	2.3	0.7	0.7
		9	2.1	2.4				

* The averaged values of parallel circle 1 for each head are the averages obtained from two rosettes, i.e. rosette numbers 2 and 3.

TABLE 6.2.4.2 (a)

Results for Test 2

	Parallel Circle	Rosette Number	Max. Prin. Strain (μ strain)	Min. Prin. Strain (μ strain)	2 × Angle (radians)	Max. Prin. Stress (MN/m²)	Min. Prin. Stress (MN/m²)
HEAD = 4.7905m (4790.5mm)	1	1	273.8	-305.1	0.7927	1.5	-1.9
		2	414.7	118.1	-1.5206	4.2	2.5
		3	384.6	21.0	1.4208	3.6	1.5
	2	4	305.0	135.6	0.8974	3.3	2.3
		5	317.3	237.9	-0.2268	3.7	3.2
		6	365.8	69.6	-1.2141	3.6	1.8
	3	7	408.6	56.6	-0.5346	3.9	1.9
		8	631.3	71.2	0.6951	6.0	2.7
		9	492.0	83.3	-1.0379	4.8	2.4
HEAD = 5.7425m (5742.5mm)	1	1	319.4	-369.3	0.8206	1.7	-2.3
		2	446.5	139.2	-1.5635	4.6	2.8
		3	438.5	46.7	1.2178	4.2	1.9
	2	4	325.8	161.6	0.7918	3.5	2.6
		5	338.3	262.3	-0.1474	4.0	3.5
		6	444.8	70.1	-1.2641	4.3	2.1
	3	7	463.1	60.8	-0.5315	4.5	2.1
		8	673.8	106.1	0.6739	6.5	3.2
		9	520.7	131.9	-1.0120	5.2	2.9
HEAD = 7.8m (7800.0mm)	1	1	371.2	-418.1	0.7814	2.0	-2.6
		2	547.0	182.3	-1.4768	5.6	3.5
		3	555.2	61.8	1.4148	5.3	2.4
	2	4	338.8	326.5	0.2450	4.2	4.1
		5	372.0	297.8	0.6716	4.4	4.0
		6	461.7	121.8	-1.2684	4.6	2.6
	3	7	532.5	117.2	-0.5527	5.3	2.8
		8	779.3	153.9	0.7585	7.7	4.0
		9	1094.0	-2755.1	-0.7993	0.9	-21.7

TABLE 6.2.4.2 (b)

Results for Test 2

	Parallel Circle	AVERAGED Averaged Maximum Principal Strain (μ strain)	Averaged Minimum Principal Strain (μ strain)	2 × Angle (radians)	Averaged Maximum Principal Stress (MN/m^2)	Averaged Minimum Principal Stress (MN/m^2)
HEAD = 4.7905m (4790.5mm)	1	* 258.0	211.2	0.7405	3.1	2.8
	2	293.4	183.6	-0.5172	3.3	2.7
	3	354.3	226.6	0.5071	4.0	3.3
HEAD = 5.7425m(5742.5mm)	1	* 305.3	230.1	0.4127	3.6	3.1
	2	332.9	201.4	-0.6918	3.7	3.0
	3	389.3	262.8	0.6462	4.4	3.7
HEAD = 7.8m (7800.0mm)	1	* 378.2	294.9	0.8423	4.5	4.0
	2	372.7	266.8	-1.0883	4.3	3.7
	3	** 559.7	231.7	1.4170	5.9	4.0

* The averaged values of parallel circle 1 for each head are the averages obtained from two rosettes, i.e. rosette numbers 2 and 3.

** The averaged values of parallel circle 3 for head = 7.8m are the averages obtained from rosette numbers 7 and 8.

TABLE 6.2.4.2 (c)

Results for Test 2

Head	Parallel Circle	Rosette Number	Stress in Parallel Circle Direction (MN/m^2)	Stress in Meridional Direction (MN/m^2)	A V E R A G E D Av.Stress in Parallel Circle Direction (MN/m^2)	 Av.Stress in Meridional Direction (MN/m^2)	M I S T R Y Stress in Parallel Circle Direction (MN/m^2)	 Stress in Meridional Direction (MN/m^2)
HEAD = 4.7905m (4790.5mm)	1	1	1.0	-1.4	* 3.0	2.8	2.2	2.1
		2	3.4	3.3				
		3	2.7	2.4				
	2	4	3.1	2.4	3.3	2.7	2.1	1.9
		5	3.7	3.2				
		6	3.0	2.4				
	3	7	3.8	2.0	3.3	4.0	2.1	1.5
		8	3.1	5.7				
		9	3.0	4.2				
HEAD = 5.7425m (5742.5mm)	1	1	1.1	-1.7	* 3.5	3.1	2.6	2.5
		2	3.7	3.7				
		3	3.4	2.6				
	2	4	3.4	2.7	3.6	3.0	2.5	2.3
		5	4.0	3.5				
		6	3.6	2.9				
	3	7	4.3	2.3	3.8	4.4	2.6	1.8
		8	3.6	6.2				
		9	3.5	4.7				
HEAD = 7.8m (7800.0mm)	1	1	1.4	-1.9	* 4.4	4.0	3.5	3.4
		2	4.7	4.5				
		3	4.1	3.6				
	2	4	4.1	4.2	4.1	3.9	3.5	3.1
		5	4.4	4.0				
		6	3.9	3.3				
	3	7	5.1	3.0	** 4.8	5.1	3.5	2.4
		8	4.5	7.2				
		9	-18.3	-2.5				

* The averaged values of parallel circle 1 for each head are the averages obtained from two rosettes, i.e. rosette numbers 2 and 3.

** The averaged values of parallel circle 3 for head = 7.8m are the averages obtained from rosette numbers 7 and 8.

Fig. 6.1.1 Load against cummulative mean longitudinal strain relationship.

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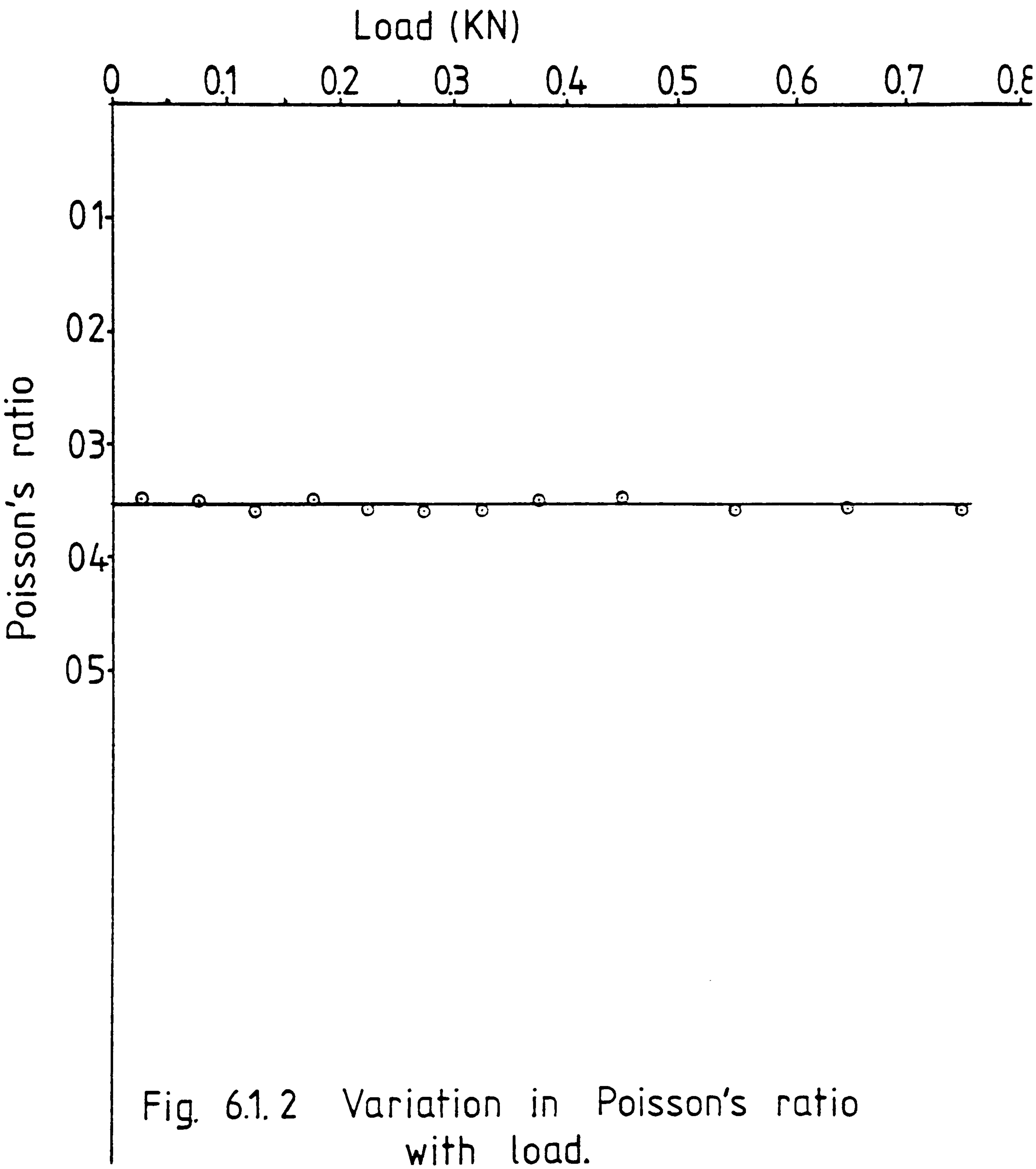


Fig. 6.1. 2 Variation in Poisson's ratio with load.

Figure 6.2.1.1 Prototype tank

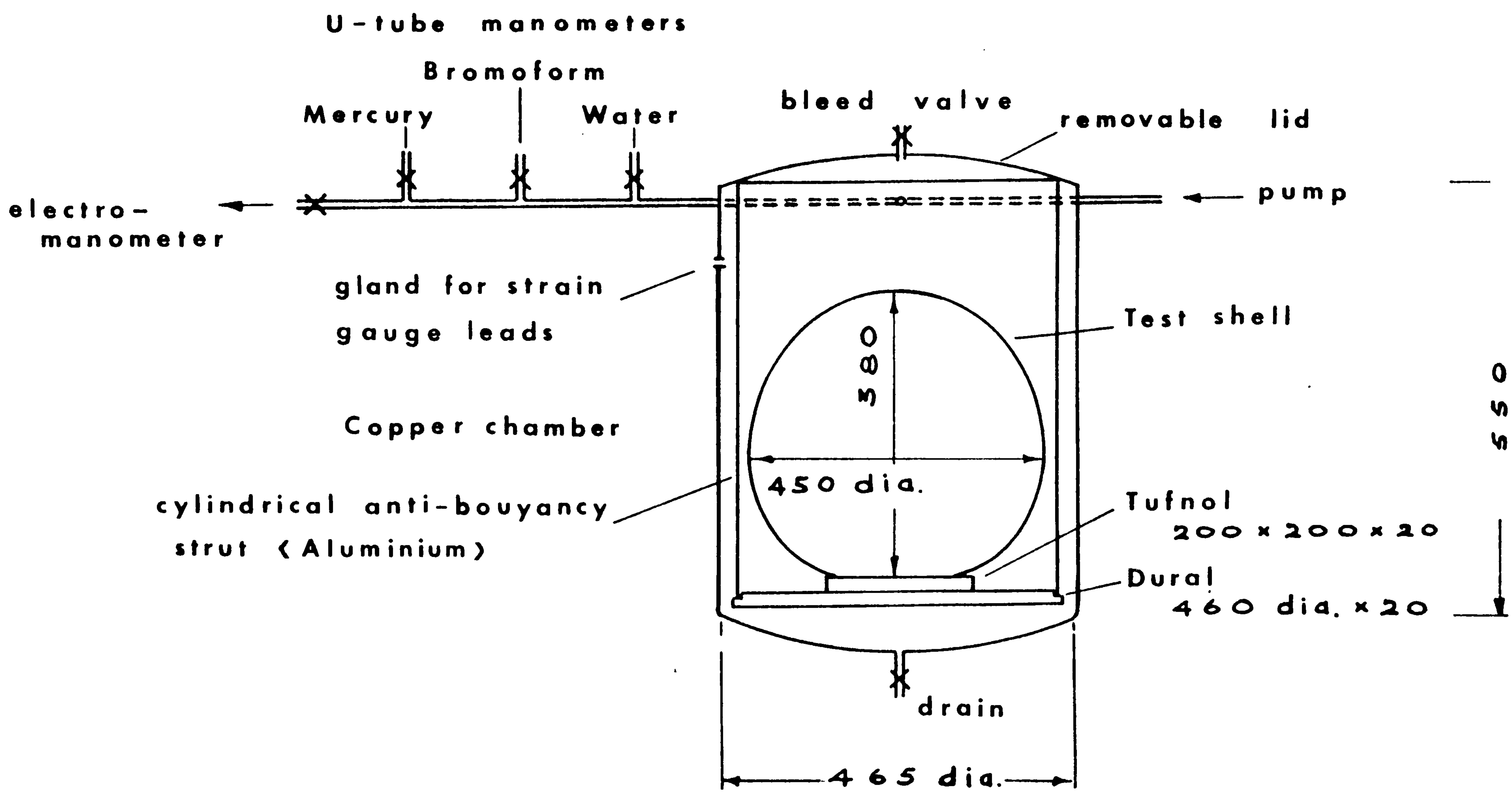
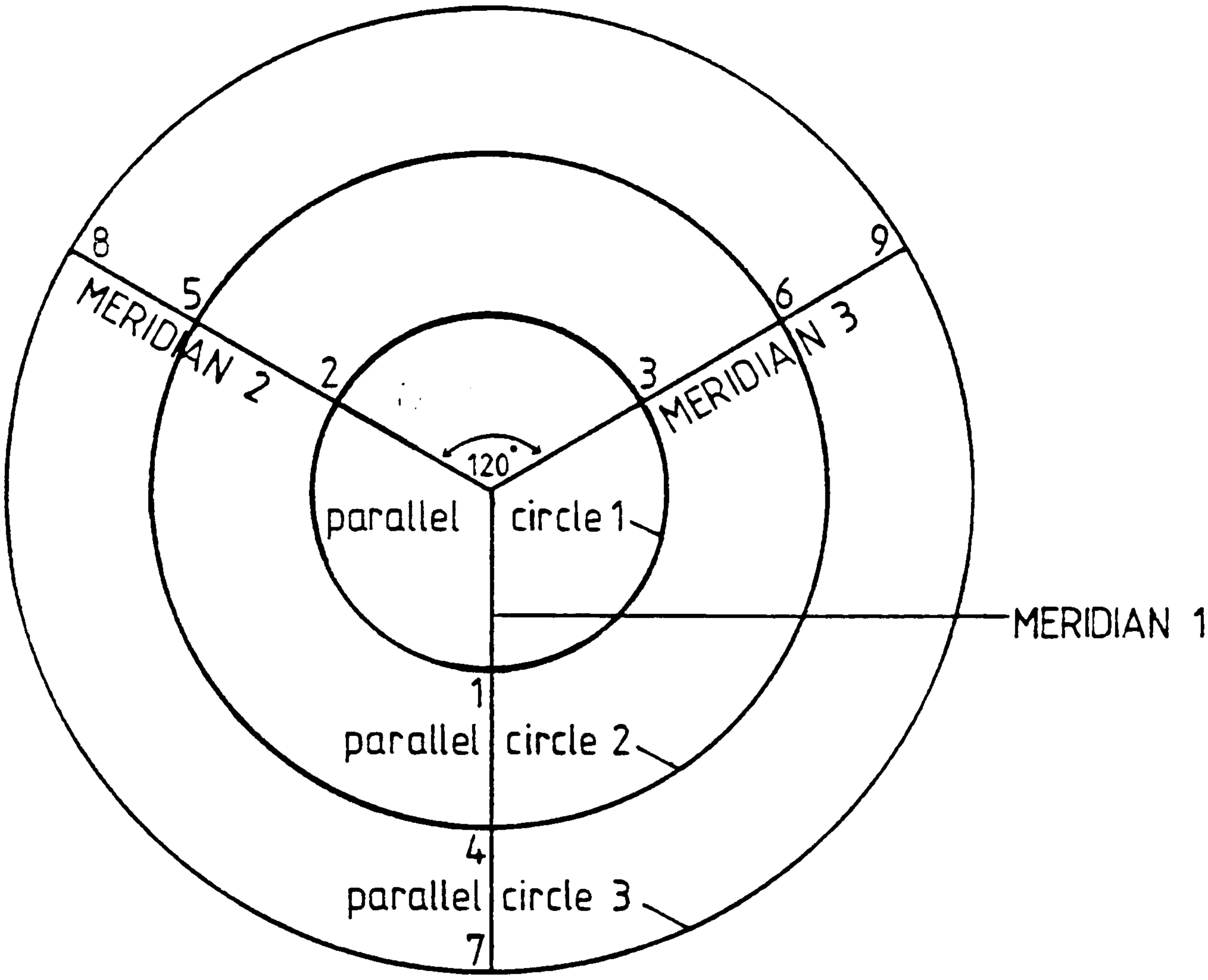
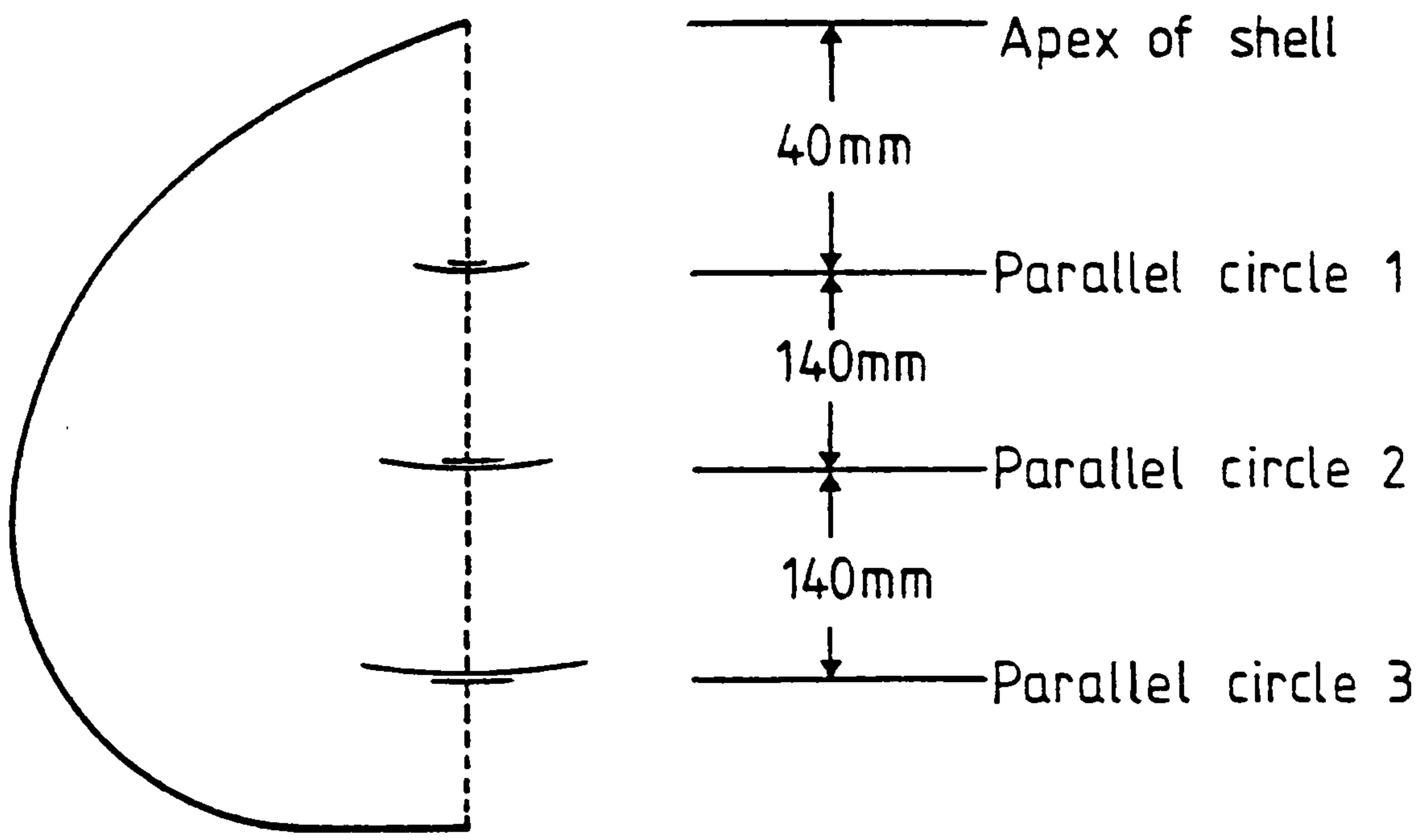


Figure 6.2.1.2 Pressure chamber test arrangement (dimensions in mm) (By courtesy of STRAIN)

Figure 6.2.2.1 General view of test equipment



a) Plan of tank with position of rosettes



b) Section of side view of tank with positions of parallel circles.

Fig. 6.2.4.1 Position of rosettes on tank.

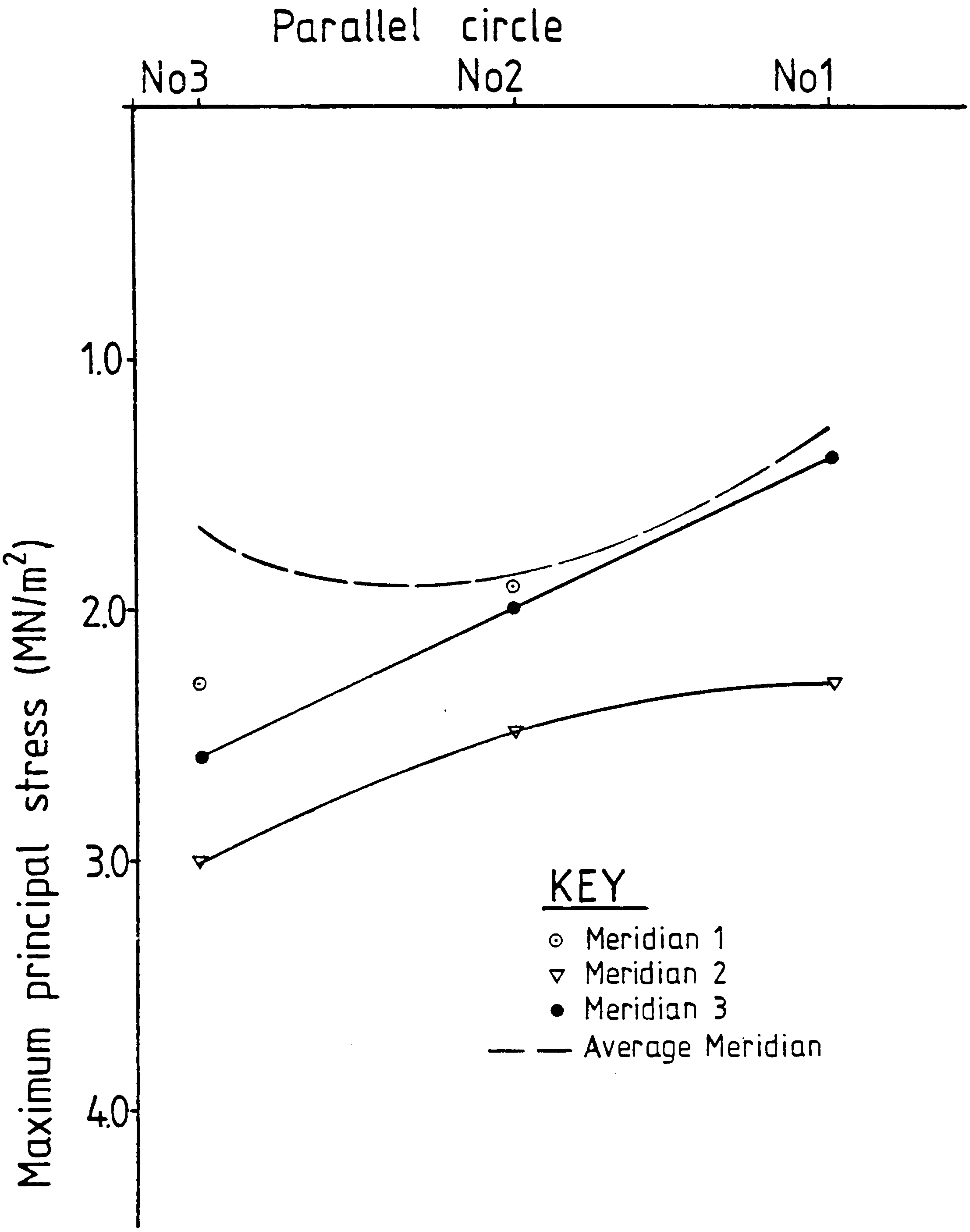


Fig. 6.2.4.2 Meridional variations in max. principal stress (head = 1210.5mm)

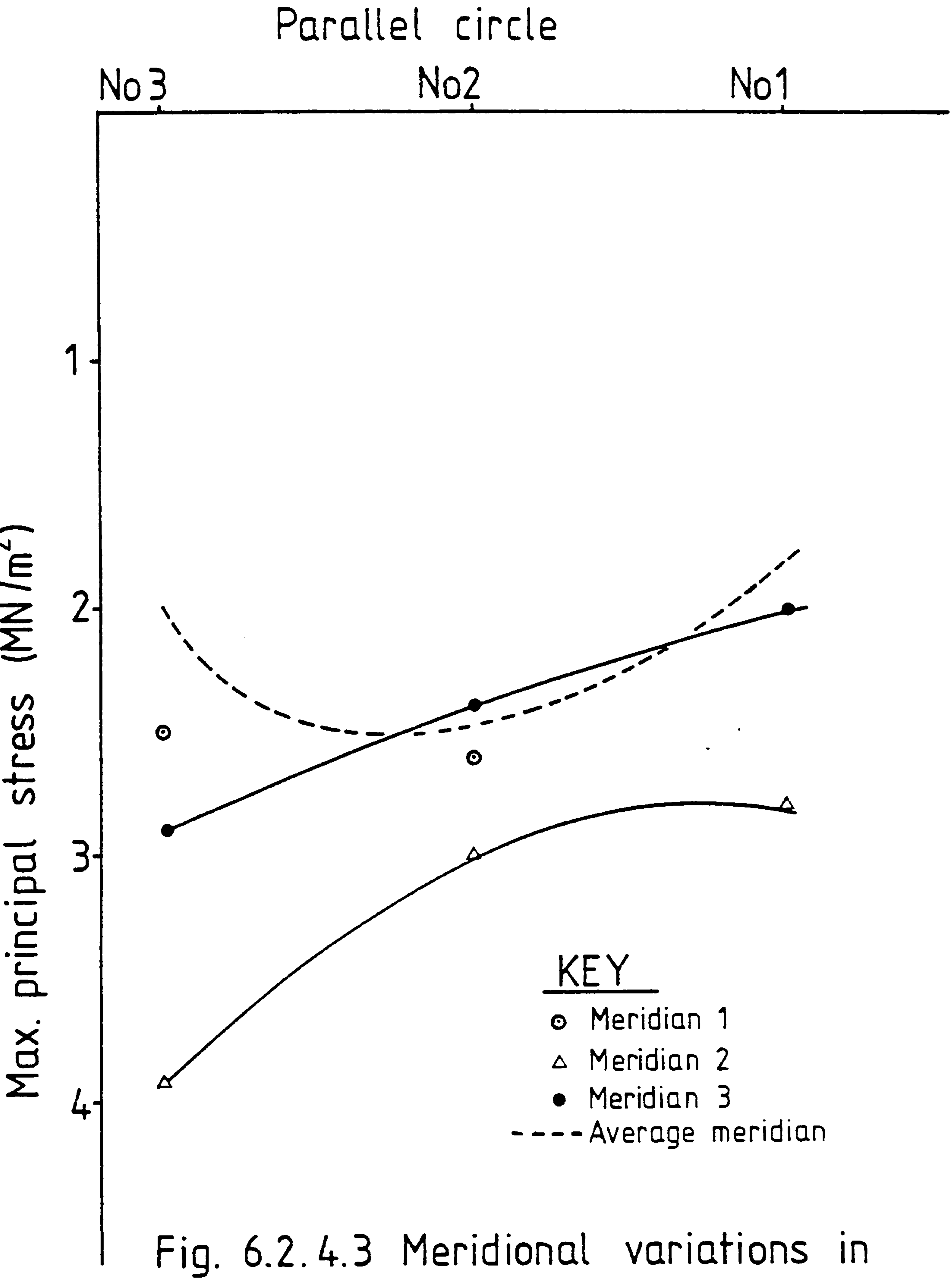


Fig. 6.2.4.3 Meridional variations in maximum principal stress (head = 1510.5mm)

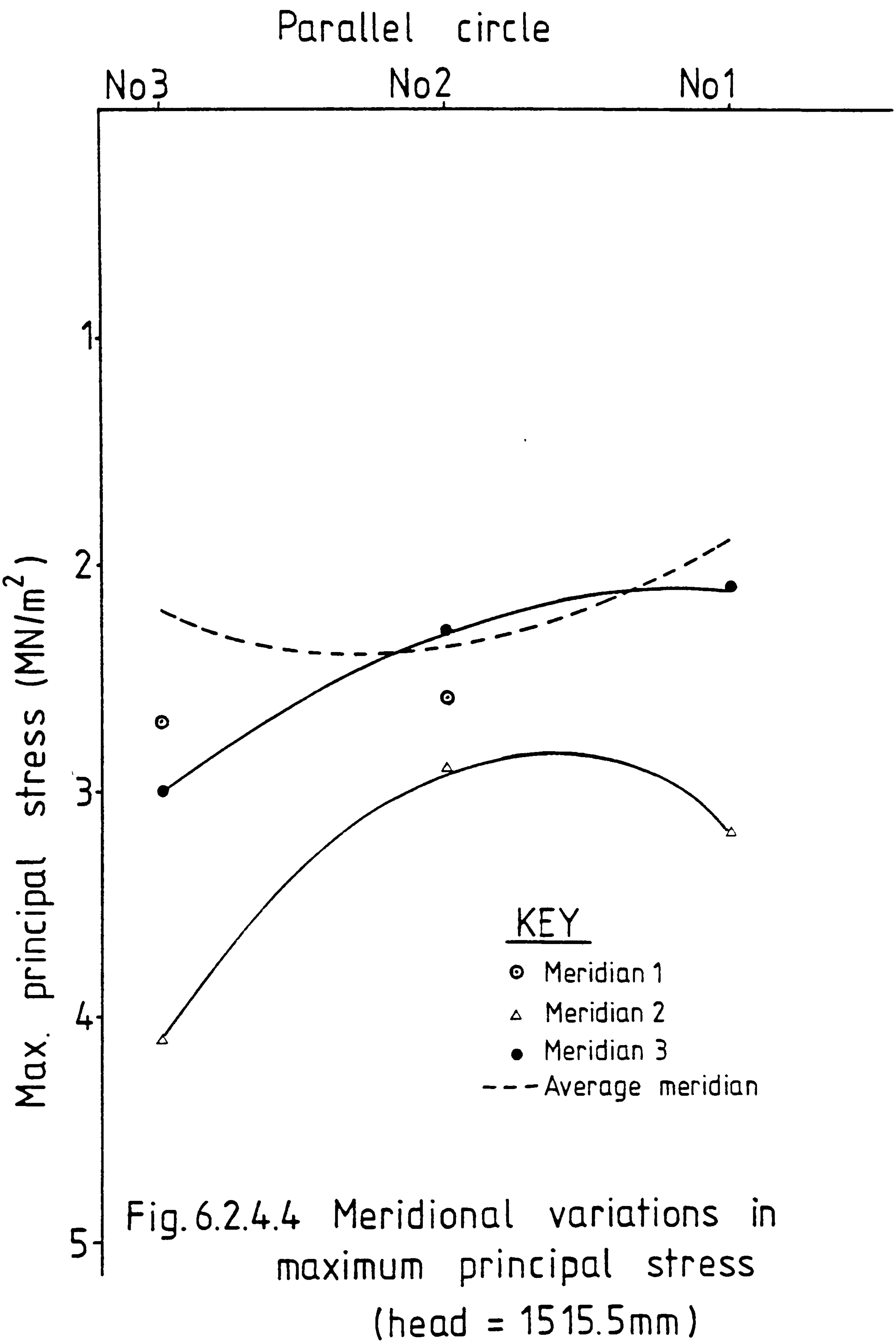


Fig. 6.2.4.4 Meridional variations in maximum principal stress (head = 1515.5mm)

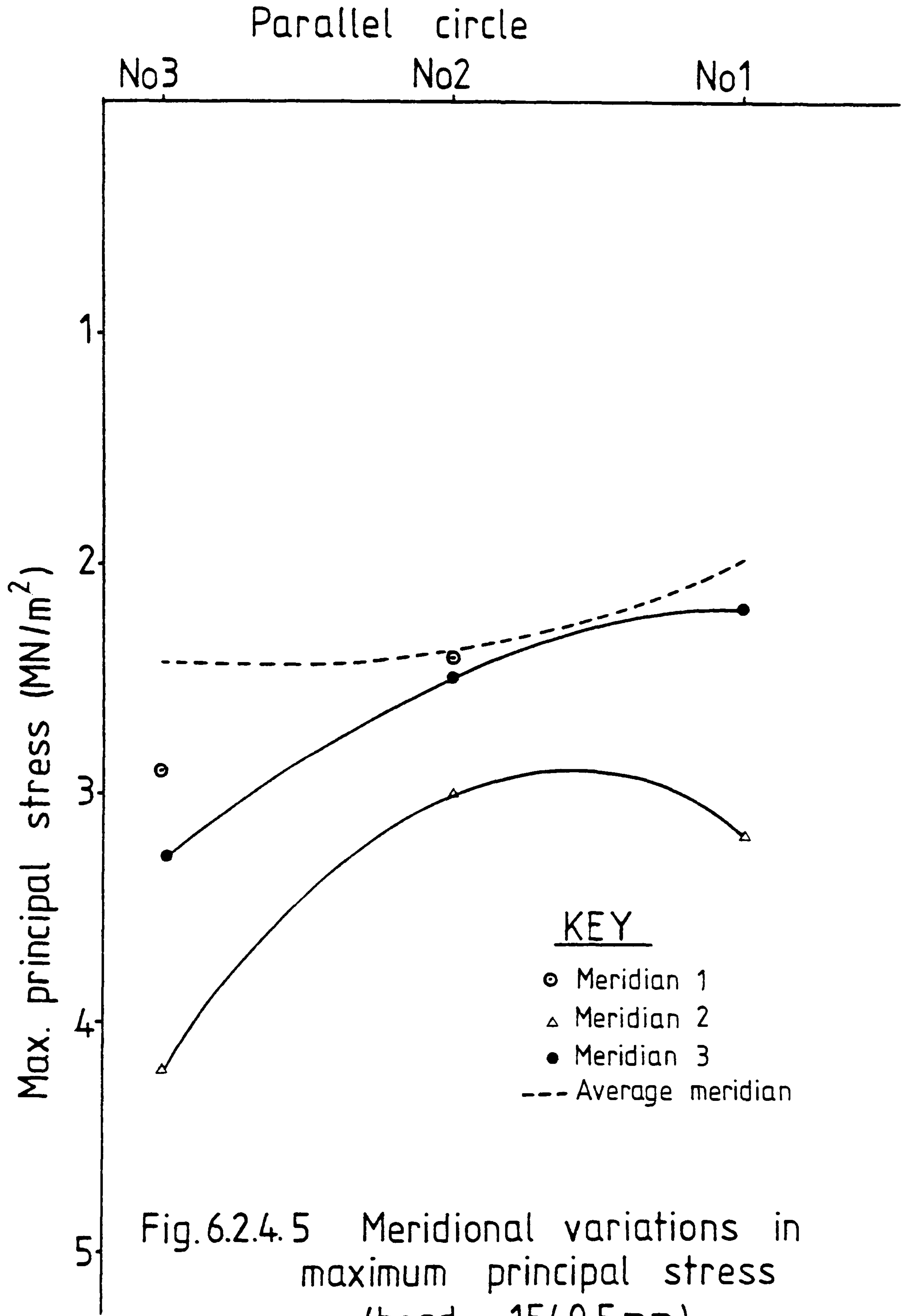


Fig. 6.2.4.5 Meridional variations in maximum principal stress (head = 1540.5 mm)

Fig. 6.2.4.6 Meridional variations in maximum principal stress (head = 4790.5mm)

Fig. 6.2.4.7 Meridional variations in maximum principal stress (head = 5742.5 mm)

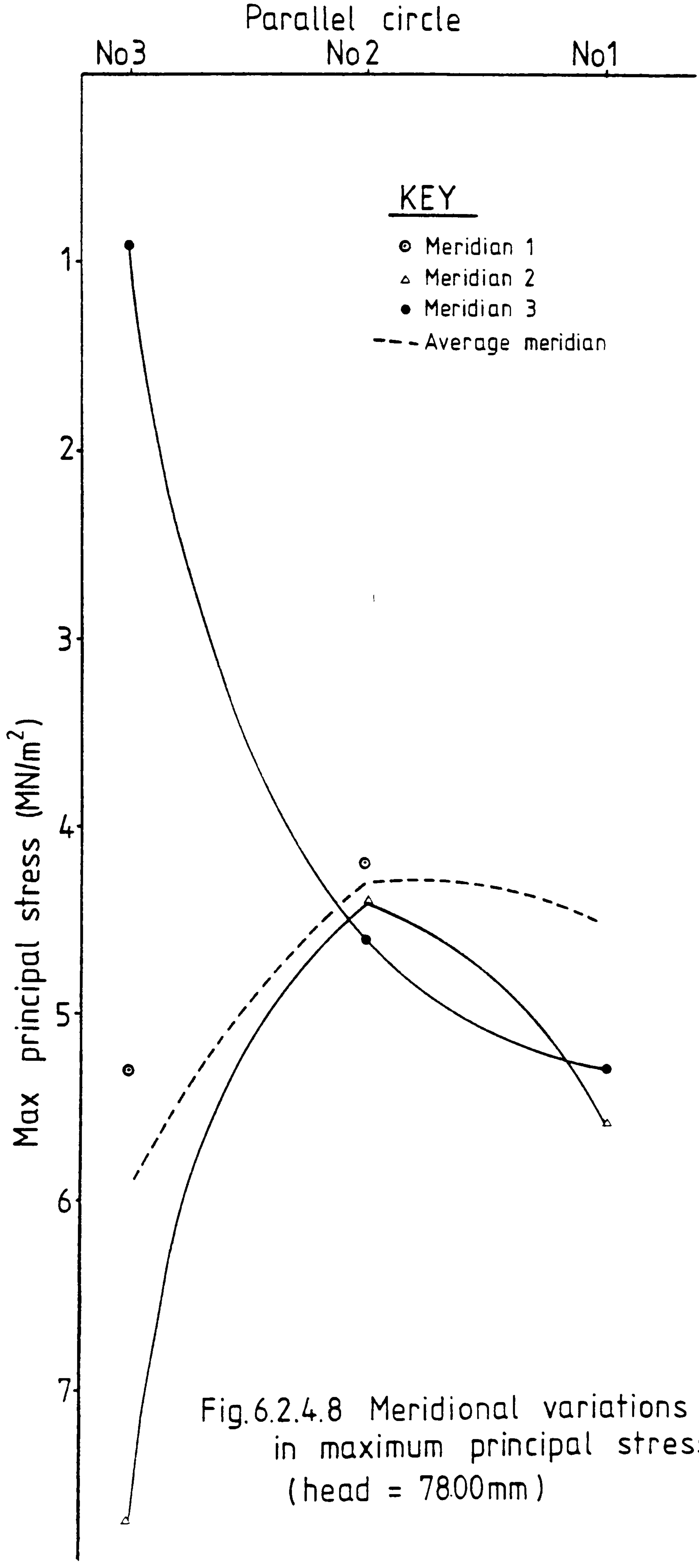


Fig. 6.2.4.8 Meridional variations in maximum principal stress (head = 78.00mm)

CHAPTER SEVEN: GENERAL SUMMARY, CONCLUSIONS AND DISCUSSION

CHAPTER SEVEN: GENERAL SUMMARY, CONCLUSIONS AND DISCUSSION

This thesis is concerned with the possibility of using the drop shaped tank in an underwater environment. The investigations had been prompted by

(a) the need to provide an alternative solution to the problem of off-shore storage which is encountered in oil and other sea explorations,

(b) the need to explore radical and new forms of possible habitat for the human species on earth as world population continues growing,

(c) the need of initiating investigations into structures that might be used in association with devices which are of interest in the search for alternative energy sources.

From the work carried out the following conclusions can be drawn:

(a) Explicit improved or modified Euler method should be used for generating the coordinates of the tank numerically. Compared with other possible methods, it is the most reliable shape prediction method in the present circumstance.

(b) A time and cost saving procedure is needed if such tanks are to be built in reality. A possible scheme for achieving this is presented and demonstrated.

(c) The response of the tank to varying hydrostatic pressure head is investigated theoretically and experimentally. Theoretically, membrane shell theory may be applied as a first approximant when evaluating the stresses developed by the tank due to the design load condition and to some extent for heads in excess of this level. The special purpose finite element computer program of MISTRY provided a more realistic numerical scheme for this analysis.

(d) MISTRY's program predicted that there would be no axisymmetric failure of the prototype tank considered for pressures up to nine times the design head. The experiment carried out indicated a safe stress state for the tank up to five times the design head. It may be possible to extend this limit to higher heads which were not investigated.

A few words are necessary and may suffice as a sketch of pertinent problems that must be looked into in future before this form is used underwater since this thesis does not pretend to have covered all relevant ground. The problem of shape generation could still be studied further especially by analytical and analogue computer approaches. These may yield and provide a more efficient method for this problem. It should be noted that the difficulty associated with shape generation stems from the nature of the mathematical relation describing it. This manifests itself as a system of non-linear ordinary differential equations. There is presently a lot of vigorous activity going on among interested mathematicians and other scientists to try and establish a firm basis for the study of non-linear ordinary differential equations. The equations of the drop shape and its generalized form could provide a positive stimulus in this direction.

The generalized form of the equations as proposed by the writer is

$$\frac{d^2u}{dx^2} + P_1(x)\frac{du}{dx} + \left[P_2(x) + \varepsilon P_3(u)\right]u = 0$$

with the initial conditions $u = \frac{du}{dx} = 0$ when $x = 0$,

where $P_i(\;)$ represents a function with respect to the indicated variable, $P_3(u)$ is discontinuous at a point or points within the domain of interest, ε is a small or large constant, and to avoid the discontinuities of P_1 and P_2 the initial conditions considered are perturbed forms of the original ones. Assuming the existence of solution of the differential equations, certain questions regarding the conditions that may be imposed on the functions P_1, P_2 and P_3 before this solution is unique must be answered. The conditions that must be satisfied before such equations have periodic or non-periodic solutions may also be established.

Returning to the drop shaped tank it is recalled that the loading condition investigated in this thesis is hydrostatic. Other loading conditions, i.e., hydrodynamic, drag, thermal and nondeterministic must be investigated before the shape can be employed under water.

It is encouraging that some of these conditions have been investigated for different forms by other workers. These investigations may guide and benefit future work on the drop shaped tank.

It is also important to note that only the general shape of the tank has been studied. The effect of openings or holes on the strain distribution in the tank wall, which will become necessary for any realistic structure of this form, will have to be studied.

As for the material(s) of construction, it would appear that the two most likely materials are steel and concrete. It may be possible to consider other materials provided their performance in the environment of interest is well studied, and understood. The author is of the opinion that there is a lot of scope regarding this aspect of drop shaped tank in future work. Another area which is worthy of consideration is the method of construction. It is believed that present day technology is capable of providing an appropriate method. The main factor of consideration may have to be cost. It cannot be overemphasised that stringent safety conditions would be imposed on such a structure more especially when human lives are involved as in a habitat. Appropriate ways of evacuation in case of an accident, and maintenance procedures when operational must be studied. The problems of launching, installation and operation have been briefly discussed elsewhere [70] and there is a need for an extensive study of these problems especially as a lot of money and time could be saved by a reasonable approach.

The need to continue analytical and experimental investigations of the drop shape under relevant conditions cannot be overemphasised. Mistry's computer program could help with numerical investigations as it has been updated to perform buckling and vibration analyses. Other computer packages when appropriate and available may be used. Ultimately the study of a full size form in the intended environment must be carried out. It is hoped that by the time this is done, some of the difficulties mankind is facing and to which this research has tried to address itself would have become amenable to this and other possible alternative solutions.

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UDC 624.074.4/621.642

Behaviour of underwater enclosures of optimum design

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First presented at the B.S.S.M. 15th Annual Conference, Bristol, 3-6 Sept. 1979

Experiments are described which support the application of the membrane theory of shell analysis in determining optimum shapes for underwater structures of constant strength. A relationship between marine life and such structures is considered and design procedures for shape selection are given. Proposals for the investigation of the behaviour of such structures under loadings other than the initial design forces are outlined both from an analytical/numerical and an experimental approach.

Key words: Underwater, structures, constant strength.

Notation

A list of the main symbols used in this work is given below:

D	maximum diameter of shell
H	height of shell
K	constant, $\frac{N}{\gamma}$
N	stress resultant at design head
N_φ, N_θ	stress resultants in meridional and circumferential directions
R	resultant of external forces above a parallel circle defined by φ
r_1, r_2	meridional and circumferential radii of curvature
r'	parallel circle radius of curvature
t	thickness of shell
X, Y, Z	load intensities in parallel circle, meridional and radial directions.
x, z	cartesian co-ords
z_0	design head above shell apex
γ	unit weight of fluid
φ	angle defining position on meridian
θ	angle defining azimuth position

Introduction

In the harnessing of marine resources to the support of the present civilisation as land based supplies diminish there is a need for underwater and sea-bed chambers for storage purposes or within which operations related to these activities could be undertaken.

It is evident that environmental conditions prevailing at the air-sea interface are much more demanding of a structure than those at or near the sea bed. Consequently surface structures, either floating or founded on the sea bed, are at a disadvantage compared with submerged ones although the former are more accessible in calm weather. Such conditions and needs are very apparent in the North Sea as the current drive to explore and exploit reserves of gas and oil under its bed gathers momentum. The search for other raw materials and their extraction both from the seas and on and under the sea bed will be affected in a similar way.

There are many complexities involved in the design and construction of an underwater structure[1, 2, 3] and its production, from minimum material consistent with safety requirements, function, building and maintenance costs, is of great importance.

Against this background it is interesting to examine the possibilities attaching to a shell of uniform strength and to take note of any hints or help that might be obtained from a study of the indigenous population of the oceans.

The shell of constant strength

The shell of revolution of constant strength is one of uniform or varying thickness in which the forces at all points are equal. Given uniform thickness this is synonymous with uniform stressing or strain under elastic conditions. Such shells can occur in the form of domes or enclosed vessels and it is the latter which are of interest here.

It has been shown[4, 5, 6] that in order to contain a liquid of unit weight γ in a tank such that the equivalent internal pressure head at the apex of the tank is z_0 the form necessary for uniform stressing to exist in all parts of the shell must be the shape taken up by a drop of liquid on a plane surface, Fig 1. The shape is very dependent on the pressure head at the apex and the stress in the membrane. For a given stress and head there is only one shell shape of

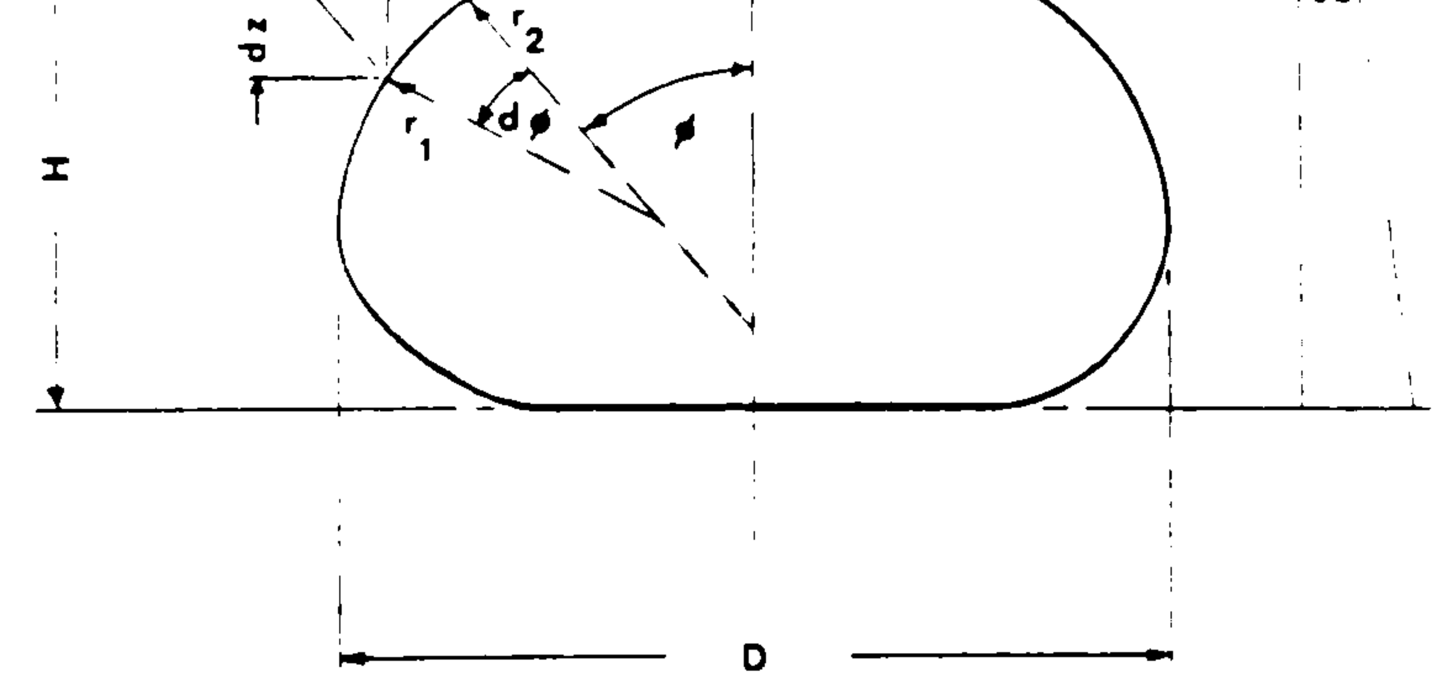

Fig 1. Shell of constant strength—drop shape.

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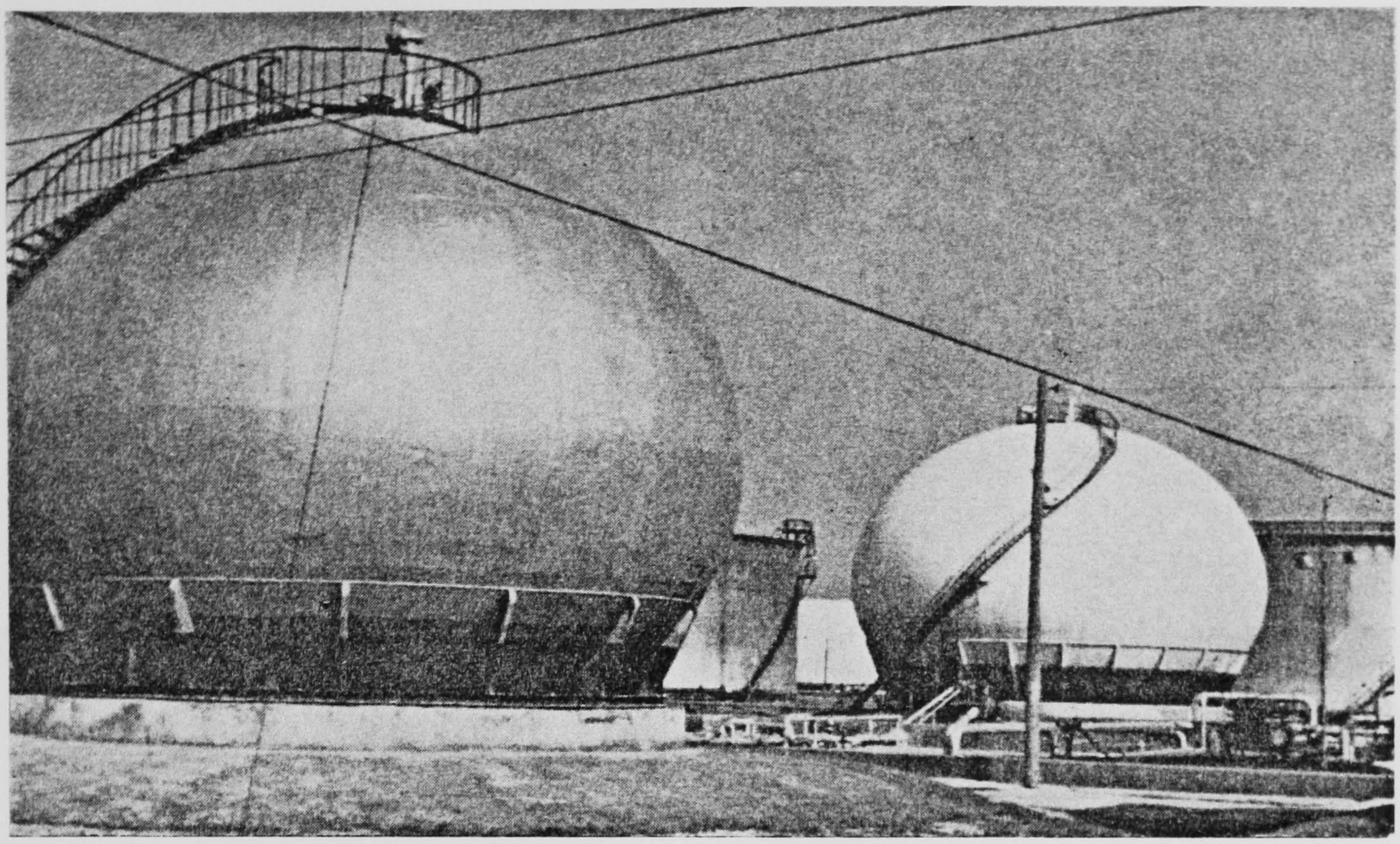

Fig 2. Tanks of constant strength—foreground 765 m^3, background 3970 m^3. (*Courtesy of Aramco*)

uniform strength. The analysis is based on the membrane theory and owes its origin to Laplace[7] in his study of capillarity and to developments by Lord Kelvin[8] and Milanković.[9] The basic equations for the prediction of the drop shape are given in appendix 1.

Steel containers for the storage of volatile liquids under pressure have been designed on the above principles and developed for use in the oil industry primarily by the Chicago Bridge and Iron Company (see Fig 2). This company built and tested the first one in 1928.[10] Some development has taken place also in Holland[11] and in France where a slightly modified design was introduced—the Caquot reservoir[12]—having an external girdle around the maximum horizontal diameter. Further applications of the drop shaped membrane as an individual container or in cellular groups have been suggested by the German architect Frei Otto.[13]

The foregoing relate to designs produced from considerations of skins in tension. Some indication that such a skin, of suitable material, might be able to withstand external pressure was given by the Chicago tests[10] in which a tensile designed steel tank of approximately 1600 m^3 capacity was subjected accidentally to an internal pressure of the order of nine times its design head (0·035 MN/m^2 apex pressure) followed by an equivalent external pressure approaching 0·08 MN/m^2 at the apex without disastrous effects. The shell thickness was nominally 4·75 mm and riveted construction was employed.

For an underwater closed shell structure subject to external pressure it is the compression state which is likely to predominate in the shell.

It is often valuable when seeking a solution to a structural problem to try and identify similar naturally occurring situations and study how nature has dealt with the challenge. In this case it is the marine animal kingdom which is worthy of attention.

Connections with marine life

As a result of some observations on the Sea Urchin (Common Echinus- Echinus Esculentis) it was suggested[14] that its shell or testa, Fig 3, conformed with those of the family of uniform strength and constituted a little more evidence of the ability of such a shell to exist under conditions of external pressure.

The Sea Urchin is a free flooding animal of the phylum Echinodermata[15] and it is unlikely that the variation of hydrostatic head over the depth of the shell would be the reason for it adopting a spheroidal shape of testa. The animal starts life in the sea without a shell structure as a swimming larva and gradually secretes a shell or skeleton—building it up from individual calcite plates connected together by suture type material. The cavity between the intestines or water vascular system of the animal and its body wall is fluid filled. The water vascular system, Fig 4, serves the needs of locomotion, nutrition, respiration, and sensory perception. This system,

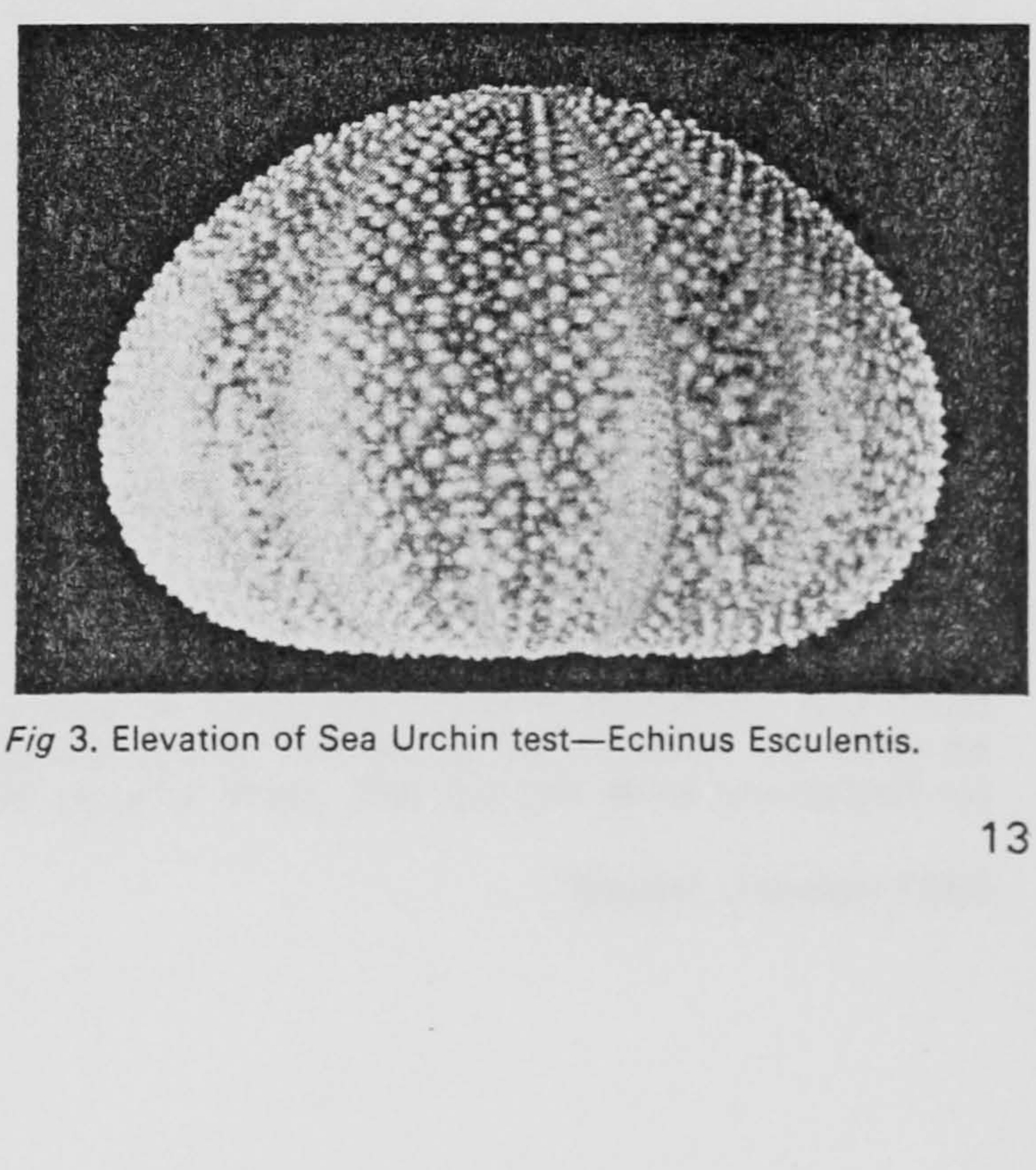

Fig 3. Elevation of Sea Urchin test—Echinus Esculentis.

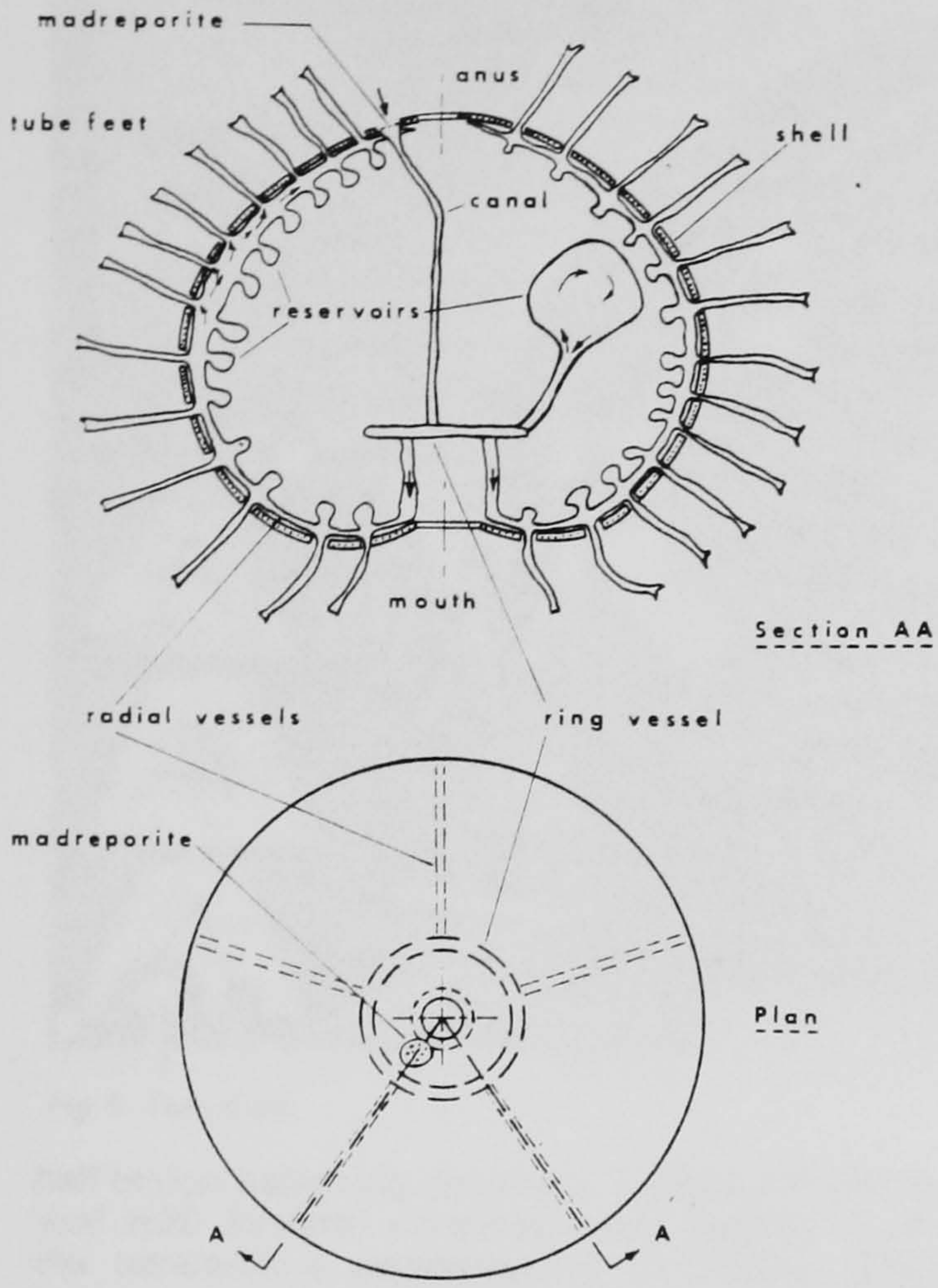

Fig 4. Water vascular system.

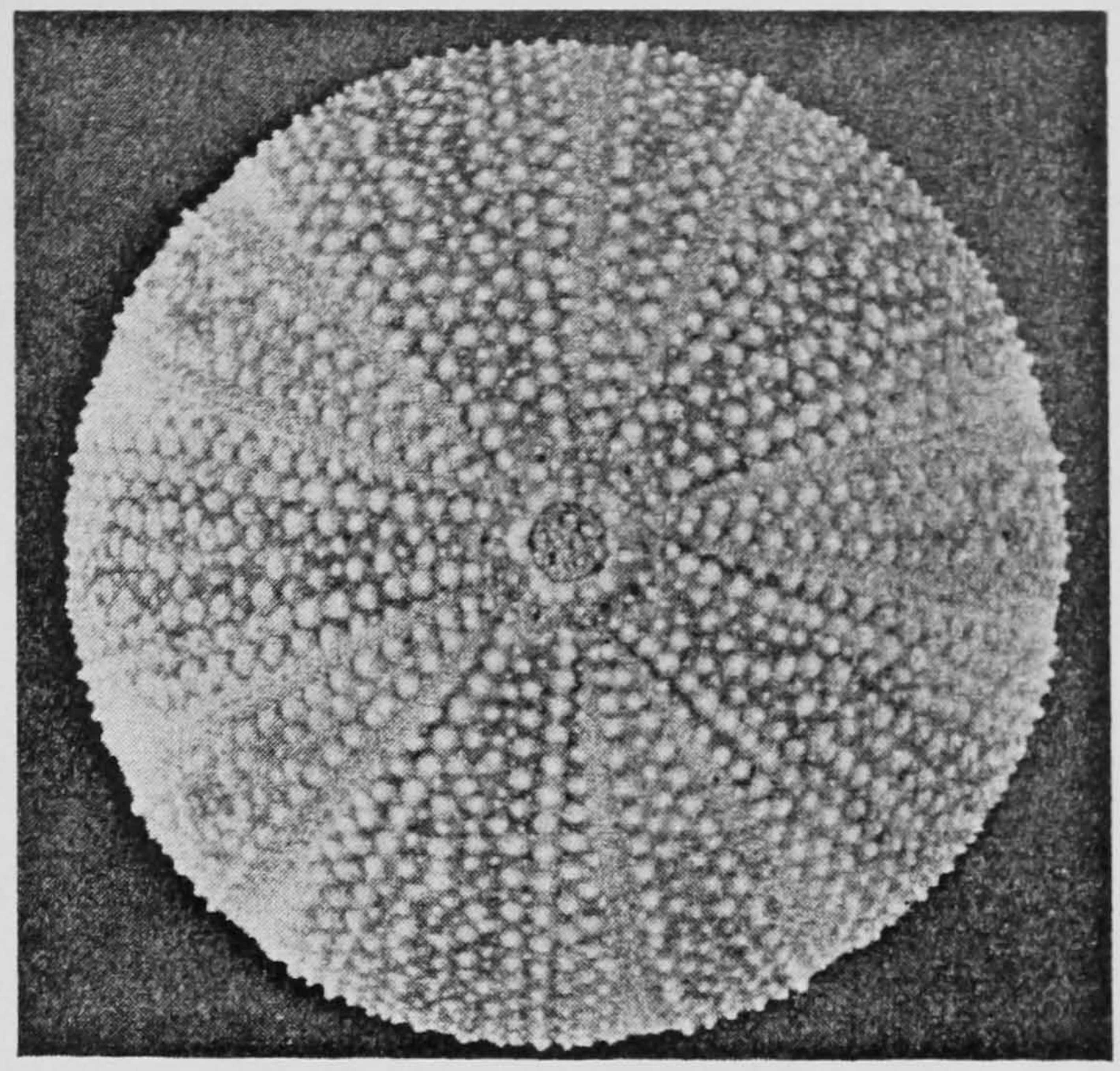

Fig 5. Plan of Sea Urchin test from above.

unique to echinoderms, is sealed and filled with water filtered through pores in a membrane, the madreporite, in the anal region at the top of the shell. A skin covers the shell from which protrude defensive spines that articulate about pivots at their bases (see small surface eruptions Figs 3 and 5). Retractable tube feet or tentacles forming part of the water vascular system can be pushed out through meridional bands of holes in the shell (see Figs 3-5). The extremities of the tentacles are bell mouthed and enable the animal to attach itself by means of suction to a rock surface for eating or safety purposes. When free the tube feet can be used to waft food towards the mouth at the bottom of the shell and waste matter is expelled through the anus at the top, Figs 4 and 5.

An explanation for the shape of the structure, in accord with recent evolutionary theory,[16] (appendix 3), is that it is formed in response to a nominally linear increase in external pressure over the depth of the shell caused by the tube feet when the animal attaches itself to a surface. Geological records indicate that echinoderms have existed for over six hundred million years and during that time several divergent structural patterns have evolved so that surviving members bear little resemblance to the original stock. Consequently it is possible that the Sea Urchin has secreted its shell more and more in response to the function of attaching itself to a surface, and the external pressure created, whilst at the same time using a minimum amount of material.

Sea Urchins living at greater depths tend to have thinner shells and larger height to maximum diameter ratios than those living in shallow waters. These facts are consistent with the calmer conditions prevailing at greater depths calling for less effort in attachment to a surface.

The shell of the Sea Urchin is secreted from the top and displays radial symmetry in plan, Fig 5, of a pentamerous nature (i.e. based on a system of five). The system of five is common to all echinoderms and it has been suggested by Nichols[17] that this arrangement is advantageous in stabilising the plates of the Sea Urchin's shell against displacement and provides a means of forming a circular plan shape from straight sided polygonal plates with a minimum of joints and lines of weakness.

Of course the shell of the Sea Urchin is not exactly of uniform thickness since spine seatings protrude from its surface and the perforated bands are slightly less thick than other parts, see Figs 3 and 5. However the shape developed by this animal for its life in the seas could be considered an encouragement to further investigation of the suitability of this type of structural form for human underwater purposes.

Experimental investigation

In order to determine firstly whether the membrane theory would be applicable to a compression structure of constant strength a small prototype was designed for a head of 1·525 m using a design stress of 0·7 MN/m^2 with a shell thickness of 2·5 mm.[18] The shell was constructed using fibre glass for which material control tests on specimens cut from the base of the test structure revealed a Young's modulus of 6900 MN/m^2.

The shell Fig 6 had a height of 380 mm and a maximum horizontal diameter of 450 mm. It was glued securely to a wooden base on which were mounted lead anti-buoyancy weights totalling some 600 N. Electric resistance strain gauges (120 ohm, gauge factor=2·07, Budd type Cb 141B) were bonded orthogonally in pairs to the shell at the intersection of three equi-spaced meridians and three parallel circles, one along the meridian the other on the parallel circle. The gauges were connected via

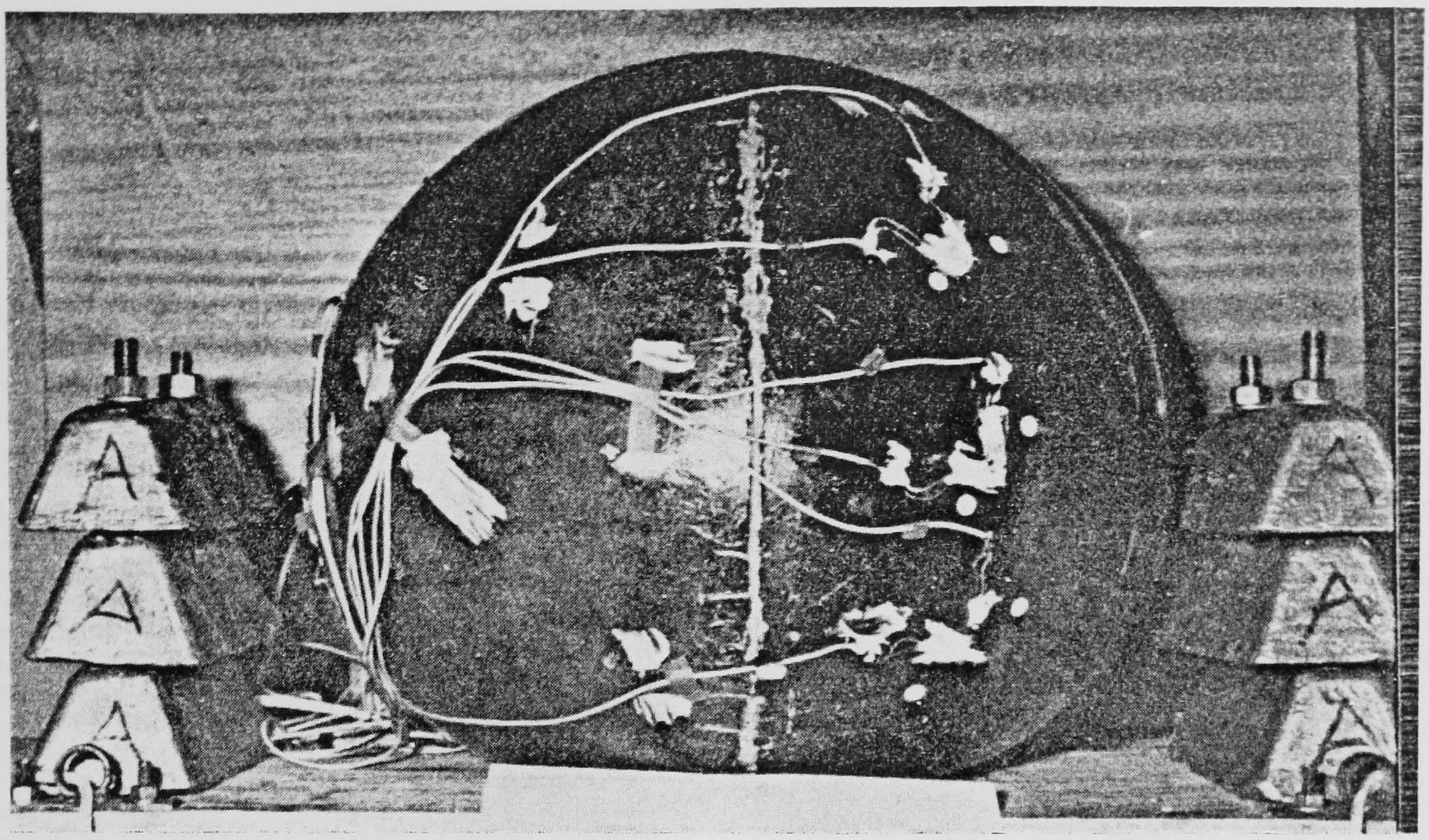

Fig 6. Test shell.

half bridge balancing circuits to a computer (Honeywell H20 System) controlled data logging system, the temperature compensating or dummy gauges being mounted on a similar piece of material under the same environmental conditions as that of the test shell. The insulated lead lengths were made the same for both dummy and active gauges, care being taken to see that the proportion of immersed lead was the same in each case, and all gauges were water proofed including terminal tags, see Fig 6. A bridge excitation of 7V, D.C., was adopted which was within the limits of the thermal capacity of the gauges bonded to a fibre glass surface. The power supply used was operating at a little below saturation level.

The test tank was 0·915 m dia., 3·660 m deep, in fibre glass. Due to flexibility in the shell base a test procedure of placing the shell on the bottom of the empty tank and gradually raising and lowering the water head was adopted. This was a slow process taking several hours, and because of the time dependent deformation characteristics of the fibre glass several sets of readings were obtained quickly at any one head (scanning rate=200 points/second). The maximum head achieved was 2·135 m (i.e. 1·4×design head) without any signs of distress.

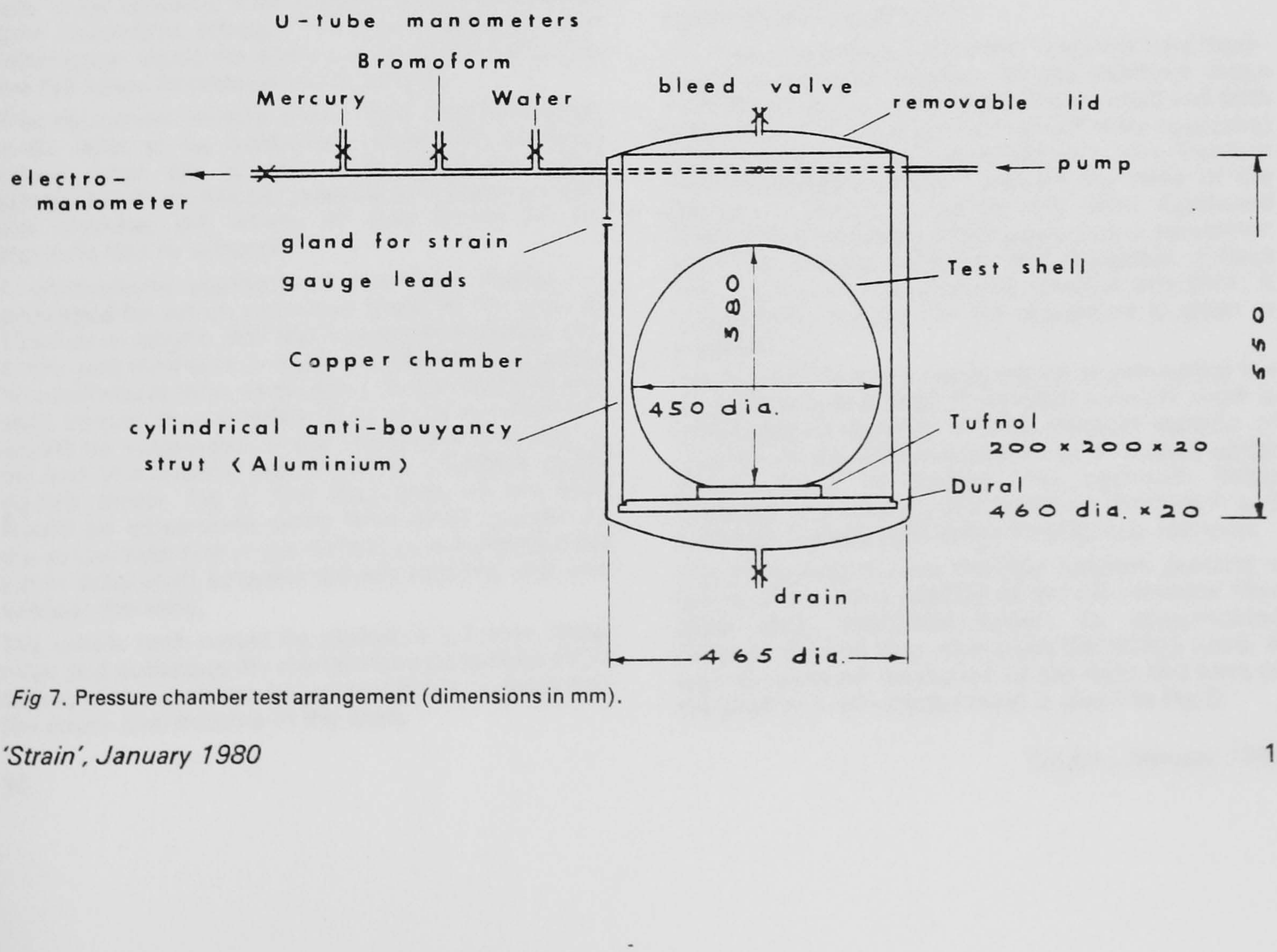

Fig 7. Pressure chamber test arrangement (dimensions in mm).

A certain amount of erratic behaviour was evident from the results and the total averaged strain pattern was not uniform at the design head. However at or near the design head there was a definite trend indicating uniform changes in strain with changes in head of 0·3 m.

Future developments

The experiments described above were aimed mainly at examining the validity of the membrane theory in compression structures of uniform strength and the results were of limited value at pressures other than the design head.

Preparations are being made for some further experiments on the same shell using 350 ohm foil strain gauge rosettes at the intersections of three meridians and three parallel circles as before. The tests will be carried out in a pressure chamber, Fig 7, adapted from an autoclave. The pressurising fluid will be water, for reasons of safety and cost, and heads up to 40 m should be attainable. The strain gauge leads will be waterproofed and pass through the wall of the chamber near the top via a silicone rubber bung, sufficient length of lead being left inside the chamber to permit the shell to be lifted out for inspection without unsoldering leads. The shell, glued to a tufnol base, is to be mounted on a dural disc and its buoyancy countered by a cylindrical strut between the lid of the chamber and the dural disc. A sealable bleed hole at the apex of the lid will give a simple indication of the chamber being full of water ready for test. A pressure measuring system employing three types of liquid manometer—water, bromoform, and mercury—and an electromanometer will be used to cover various ranges of pressure head, in the first instance up to 15 m of water, provided by hand pump.

By using the higher resistance foil strain gauges is it hoped that thermal effects will be reduced and the pressure chamber should permit a much faster loading rate to be achieved with a consequent reduction in time dependent effects. The rosettes should yield information about the strain pattern in the shell over the full range of pressure head utilised.

This equipment should allow static and fluctuating static tests to be performed, simulating to some extent mean sea conditions and tidal and wave effects. However it is not possible to investigate with this chamber the effects of drag forces on the structure due to currents.

A photoelastic approach to this latter problem is envisaged for which prototype shells of the order of 175 mm in height, 200 mm maximum diameter, and 4 mm wall thickness in araldite CT200 will be tested in a cylindrical tank nominally 1 m dia. and 400 mm deep containing a suitable oil or glycerine. The shell would be suspended in the liquid and mounted at the end of a rotating arm driven by a variable speed electric motor, Fig 8. The drag force on the shell would be determined either from strain gauges on the suspension arm or the difference in torque on the motor drive shaft between the arm rotating with and without the shell.

The whole tank would be placed in a frozen stress oven and subsequently meridional and parallel circle slicing of the shell would be carried out to determine the stress distributions in the shell.

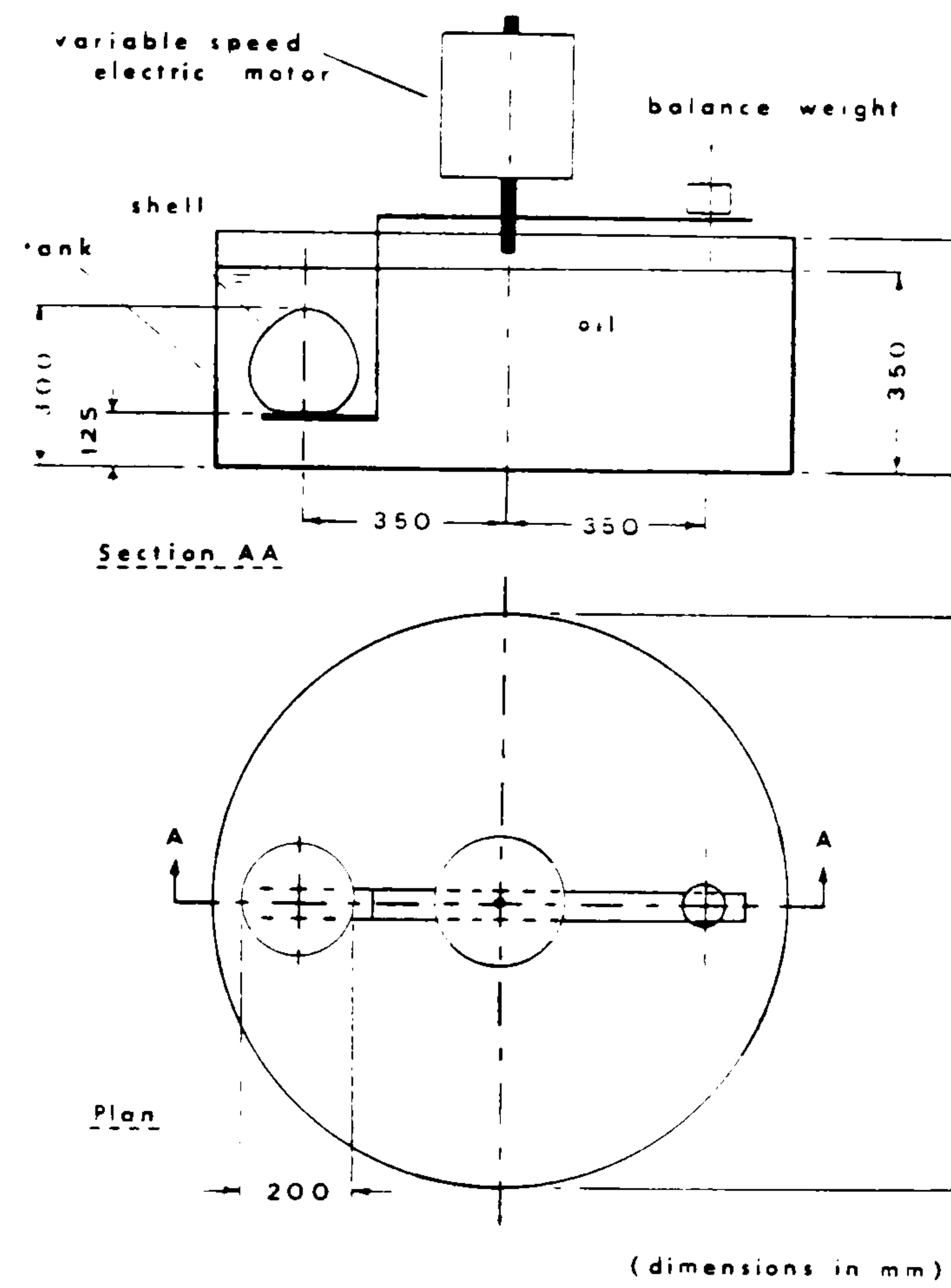

Fig 8. Tank for drag testing.

Using a smaller pressure chamber fitted with heating coils the effect of static head alone on these small shells be studied by means of the frozen stress method and the results from the drag and static head tests could be superimposed.

Numerical approach

The basic equations (appendix 1) do not lend themselves to an exact solution for the optimum shape corresponding to a particular pressure head and both a numerical[4] and a graphical method[5] were suggested for this purpose. Both these methods were found to give comparable results[18] and on the basis of the former a computer program has been developed which gives results for shape prediction in agreement with the manual methods and computes surface area, cross sectional area and capacity of a shell. A simple flow diagram for the procedure is given in appendix 2.

Having established a quick means of generating the co-ordinates of a shell of constant strength work is continuing to develop a finite element method of analysis of the deformations in such a shell under various types of loading. The approach being followed is based on the work of Bushnell[19] and Aylward, Galletly and Mistry[20] using ring elements.

It is interesting to note that the program predicts a failure under static loading of the experimental fibre glass shell, described earlier, by axisymmetric collapse at more than nine times the design head. A typical computer prediction of the deflected form of the shell at 1·40×design head is given in Fig 9.

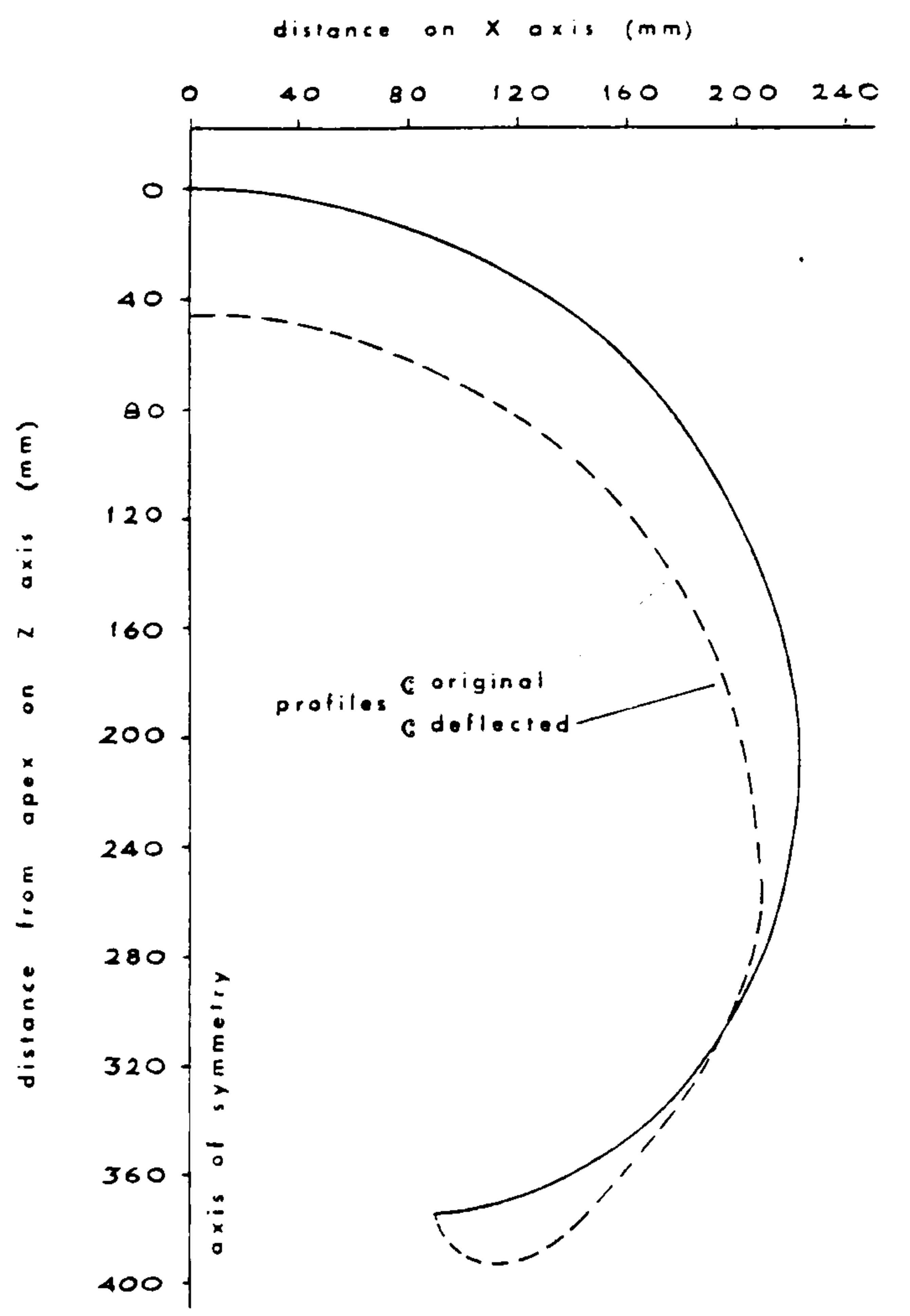

Fig 9. Deformation of shell at 1·4×design head (displacements×1000).

Design procedures

Using the shape prediction programme it is possible to generate a whole range of shapes and hence capacities relating to various design heads for a range of thicknesses and design stresses consistent with a particular construction material. This information can be stored readily in computer data banks. Additionally from the above information curves can be drawn up which would aid a designer in selecting a shell of constant strength[21] for his purpose.

For a particular material and thickness these curves take the form of

(i) volume *v* operating depth,
(ii) volume *v* design head,
(iii) maximum diameter *v* design head,
(iv) shell height/maximum diameter *v* design head, and
(v) volume of material *v* design head.

Initially a designer would know the operating depth and capacity required for his particular structure together with the mass density of the surrounding fluid. However the material of construction, design stress and shell thickness could be open to choice resulting in the possibility of more than one design fulfilling the specific requirements of capacity at a particular operating depth. The design curves would provide a quick visual aid to final selection in the following manner.

Firstly considering a particular material and the minimum practical thickness, graph (i) should be consulted. If the required volume and or operating depth be outside the scope of this plot the same graph for the first incremented thickness should be studied and so on until the minimum thickness for the desired values is found. This graph (i) gives the appropriate design stress and knowing it the design head can be established from graph (ii) for that thickness. The shell height is given by the difference between the operating depth (synonymous with the bottom of a shell) and the design head.

The maximum diameter of the shell can be determined from graph (iii) of the minimum thickness group and be checked from graph (iv) of that same group.

The amount of material used in this minimum thickness design which bears directly on the cost involved can be established from graph (v) of that same suite of curves. In the same way, further designs could be devised using thicknesses greater than the minimum and final choice would rest usually on the material costs.

A typical set of design curves are shown in Fig 10 a-e for reinforced concrete with a thickness of 150 mm.

Discussion

Some variation in the experimental results could be anticipated because of the slow rate of loading which allowed plenty of time for the development of time dependent deformation. Such deformation would be stress level dependent and only under equal stress conditions throughout the shell could it be uniform. Consequently prior strain history would dictate that by the time the design head was attained the total strain pattern over the shell would not be uniform. If the membrane theory were to be valid it might be expected that changes in strain at or near the design head would be uniform over the shell. A strong tendency for this to be true was detected.

Although the power supply used for the tests was sufficient to drive the eighteen circuits, in any future tests it would be advisable to use a supply of capacity large enough to avoid operating near saturation.

Reassuringly the experimental prototype shell was able to withstand a pressure head of at least 40% more than the design head without any signs of distress. The deformed state of the structure predicted by the finite element programme for 1·4× design head indicated little distortion. Indeed an axisymmetric collapse was not forecast below 9× design head. This may be optimistic or indicative of a very conservative design and needs verification by further tests. The deflected form of the shell shown in Fig 9 indicates that the zone of greatest distortion was near the base and corresponded with the region of highest stress (tensile). The axisymmetric type of failure in the higher levels of the shell (compressive zone) might well occur before tensile rupture near the base. Local bending effects are inevitable near the shell base because of shear at the bottom.

The finite element method of analysing such shells under general loading should prove most useful to designers but the approach requires further experimental verification in order for it to be acceptable in design.

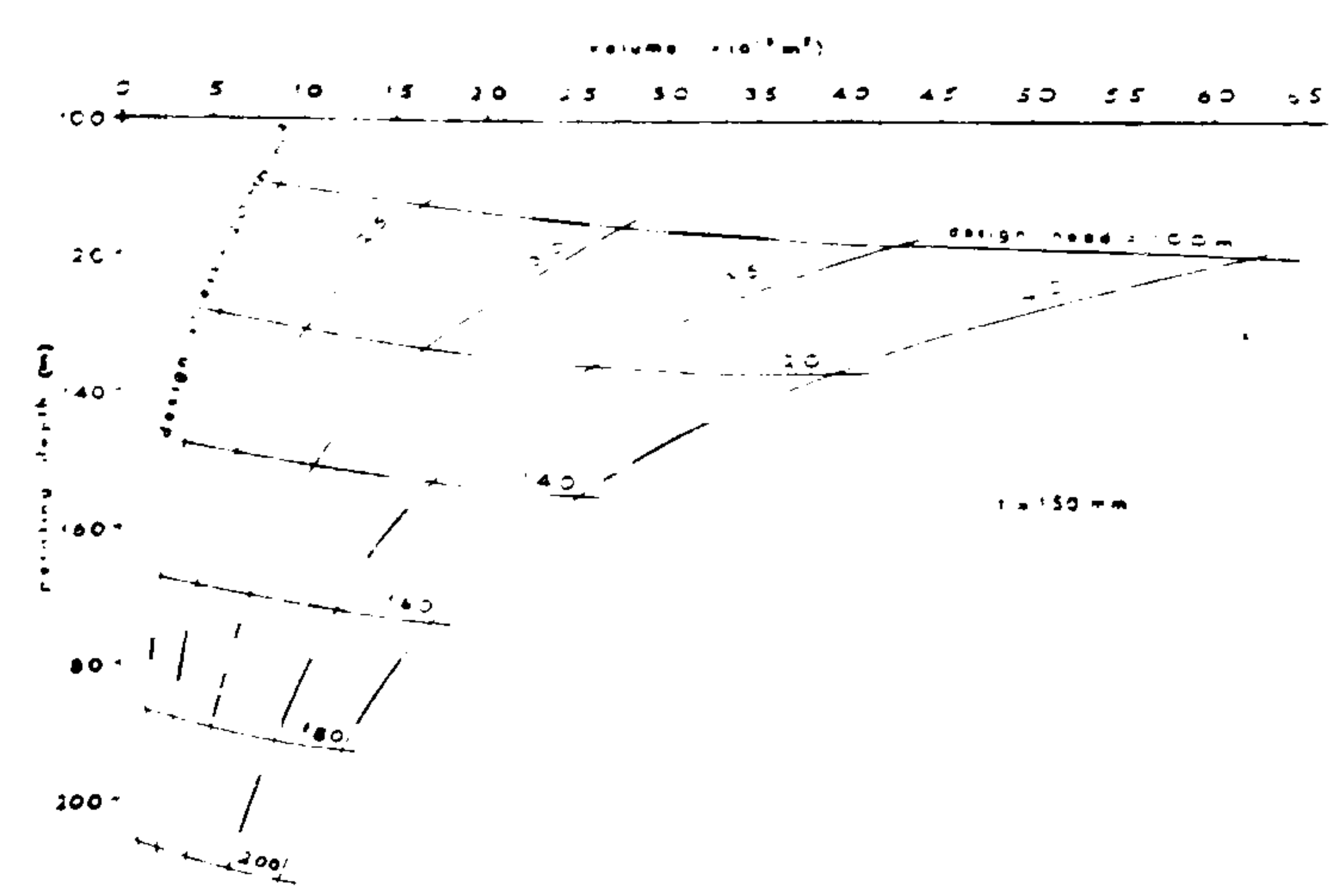

Fig 10(a) Volume *v*. depth.

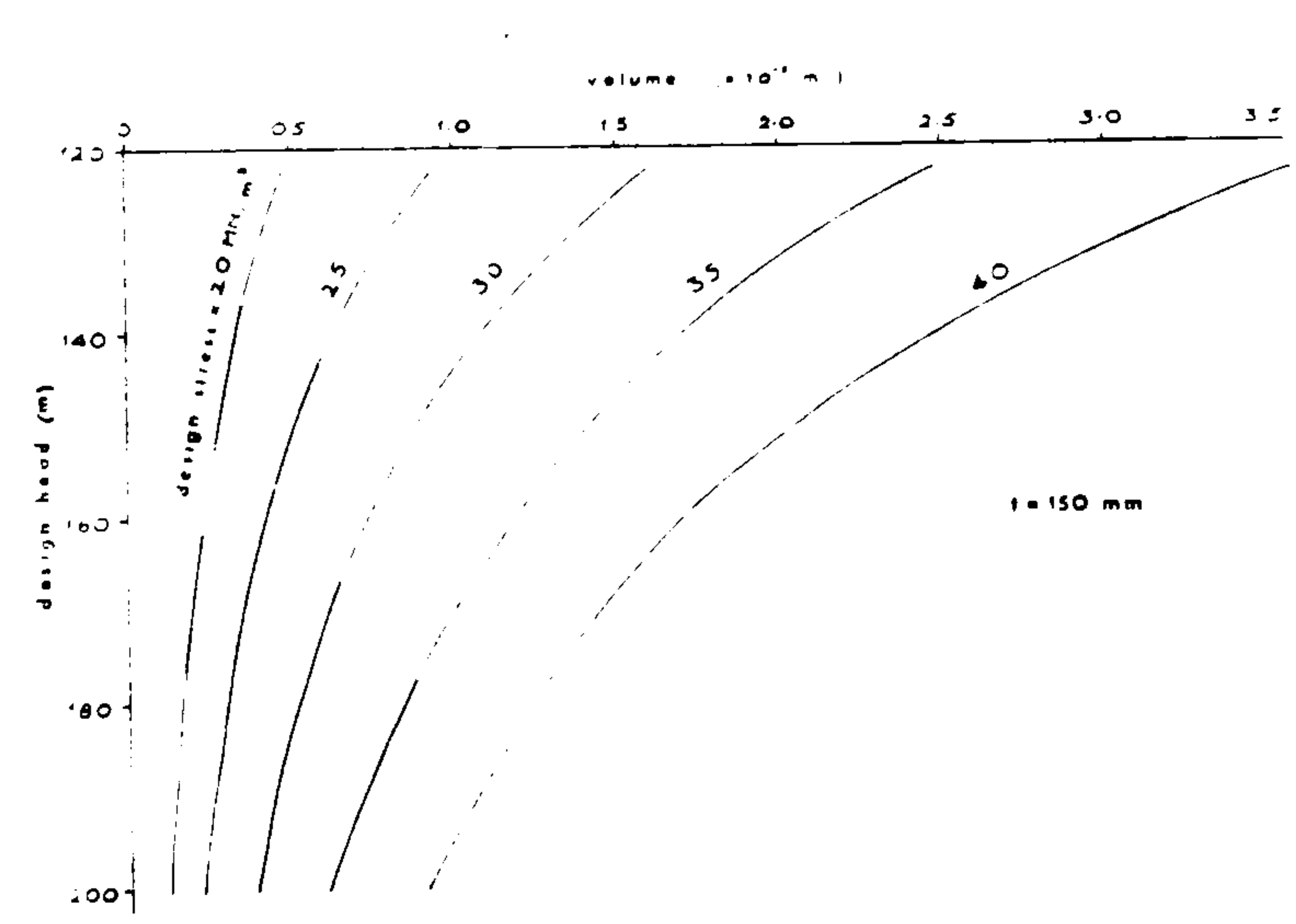

Fig 10(b) Volume *v*. design head.

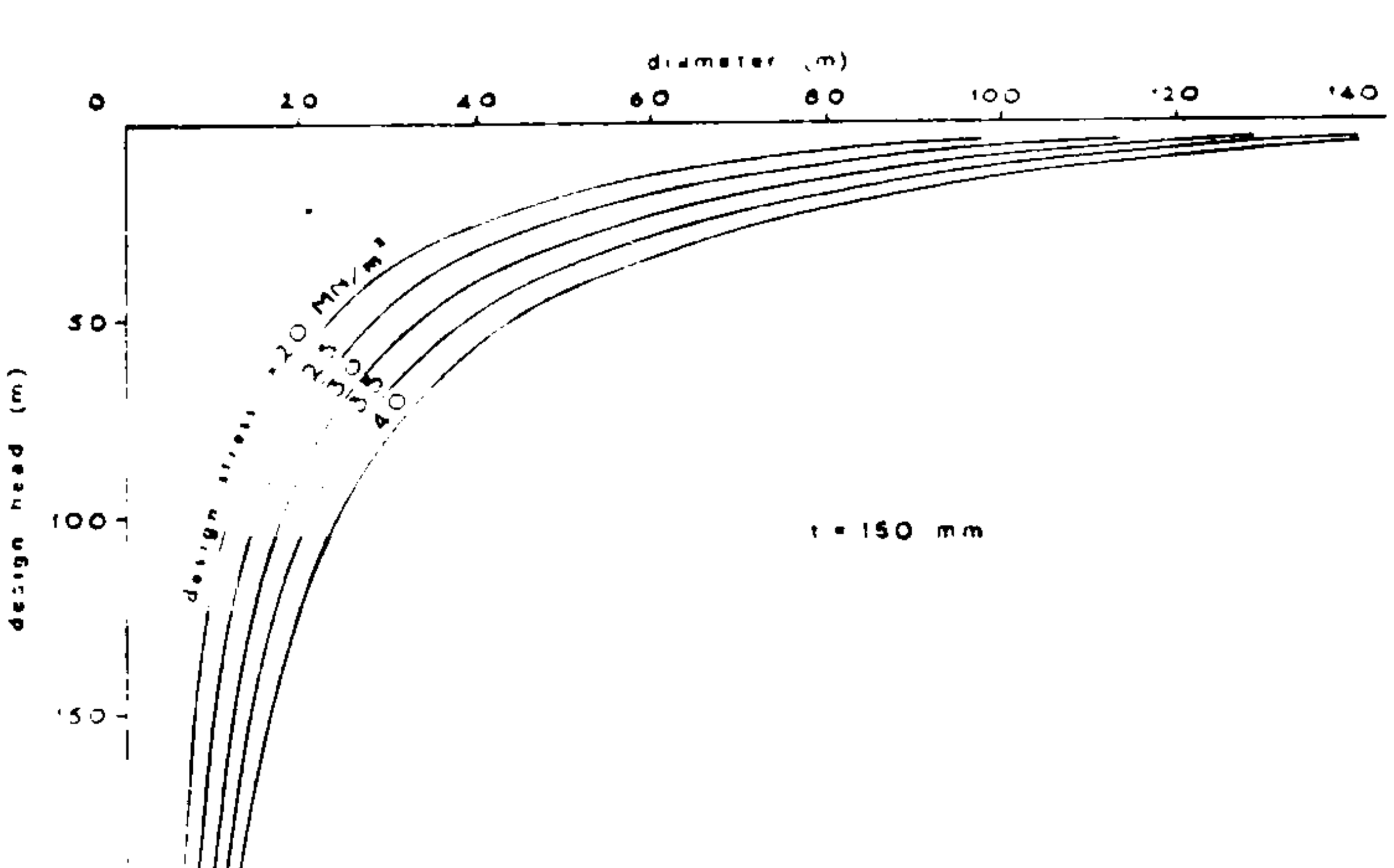

Fig 10(c) Maximum diameter *v*. design head.

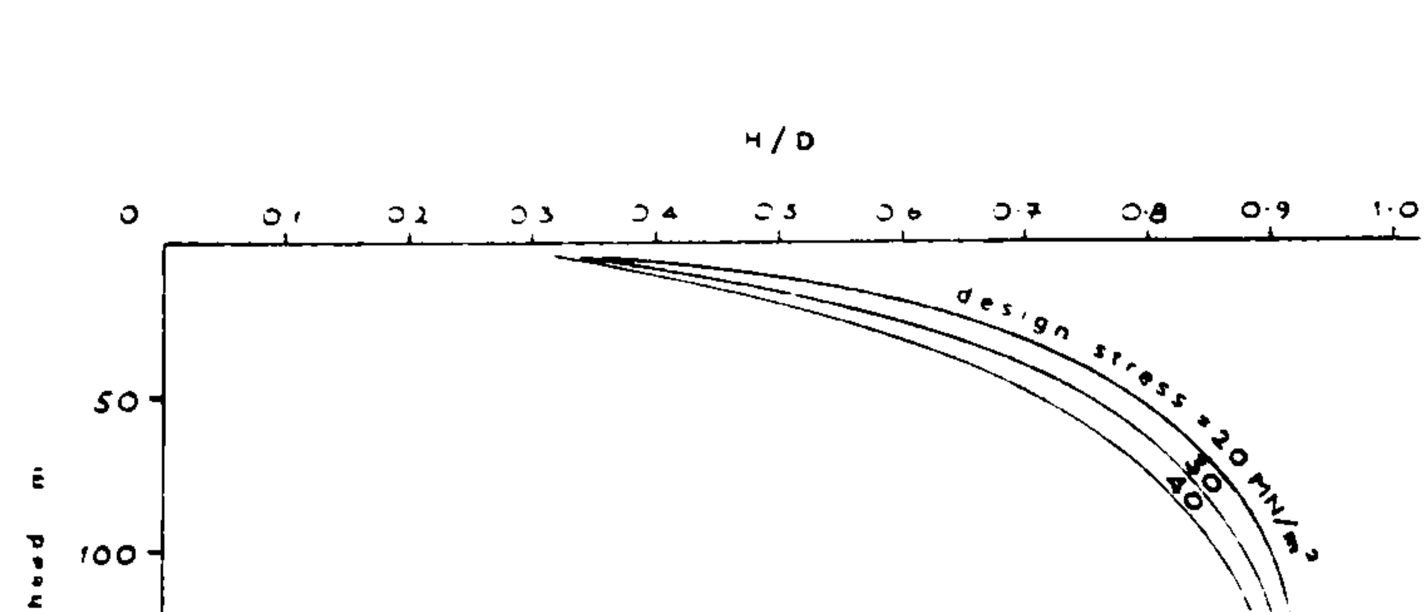

Fig 10(d) H/D *v*. design head.

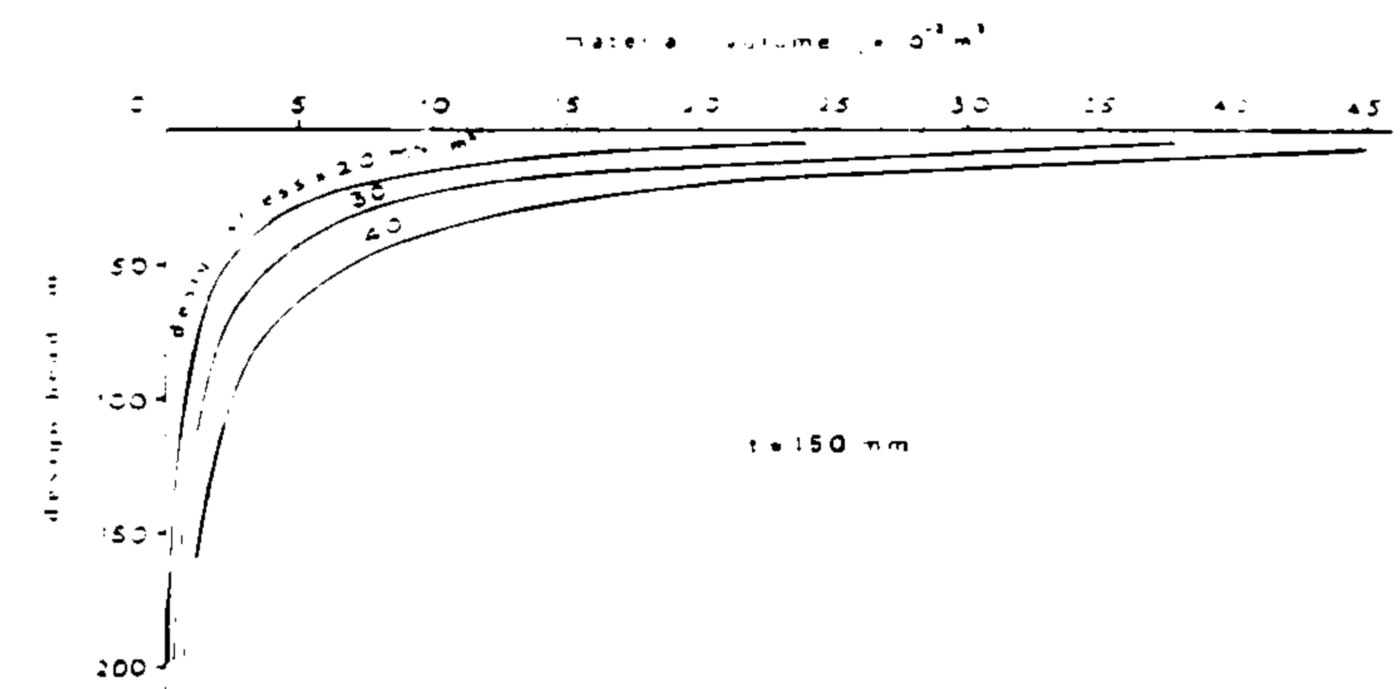

Fig 10(e) Material volume *v*. design head.

Ultimately the data for the design curves, e.g. Fig 10, could be stored in a computer ready for recall on to a graphic display and or printer at the preliminary design/selection stage.

All types of loading have not been considered and only the general shape has been examined. Any practical structure requires means of entry and exit for personnel and services in the case of a habitat or operations structure, and for services in the case of a storage vessel.

The effects of earthquakes, explosions, impact, launching effects, and tidal waves etc. on this type of structure will need detailed study along with the demands of access and suitability of materials. A combination of experimental and numerical methods will be required in these tasks. As a spur to further work it is encouraging that a sea animal appears to have chosen for its body a similar structural form to that of constant strength.

Finally it would seem appropriate that the generic term for structures of this type should be 'Echinodome'.

Acknowledgements

This work is being carried out in the Department of Civil Engineering and Building Science, University of Edinburgh and grateful thanks are expressed to Professor Hendry for his encouragement and support and to the Photographic, Secretarial, and Workshop Staff for their indispensible assistance. Also the guidance of Mr John Mistry, University of Liverpool, Department of Mechanical Engineering, with the finite element approach is appreciated greatly together with the kindness of Messrs. Ambersil Ltd., Basingstoke.

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Appendix 1

Derivation of the shape of the shell of constant strength

The membrane forces on an element of an axi-symmetrically loaded shell of revolution are as shown in Fig 11 and two general equations of equilibrium can be written as,

$$2\pi r' N_\varphi \sin\varphi + R = 0 \qquad (1)$$

and
$$\frac{N_\varphi}{r_1} + \frac{N_\varphi}{r_2} + Z = 0 \qquad (2)$$

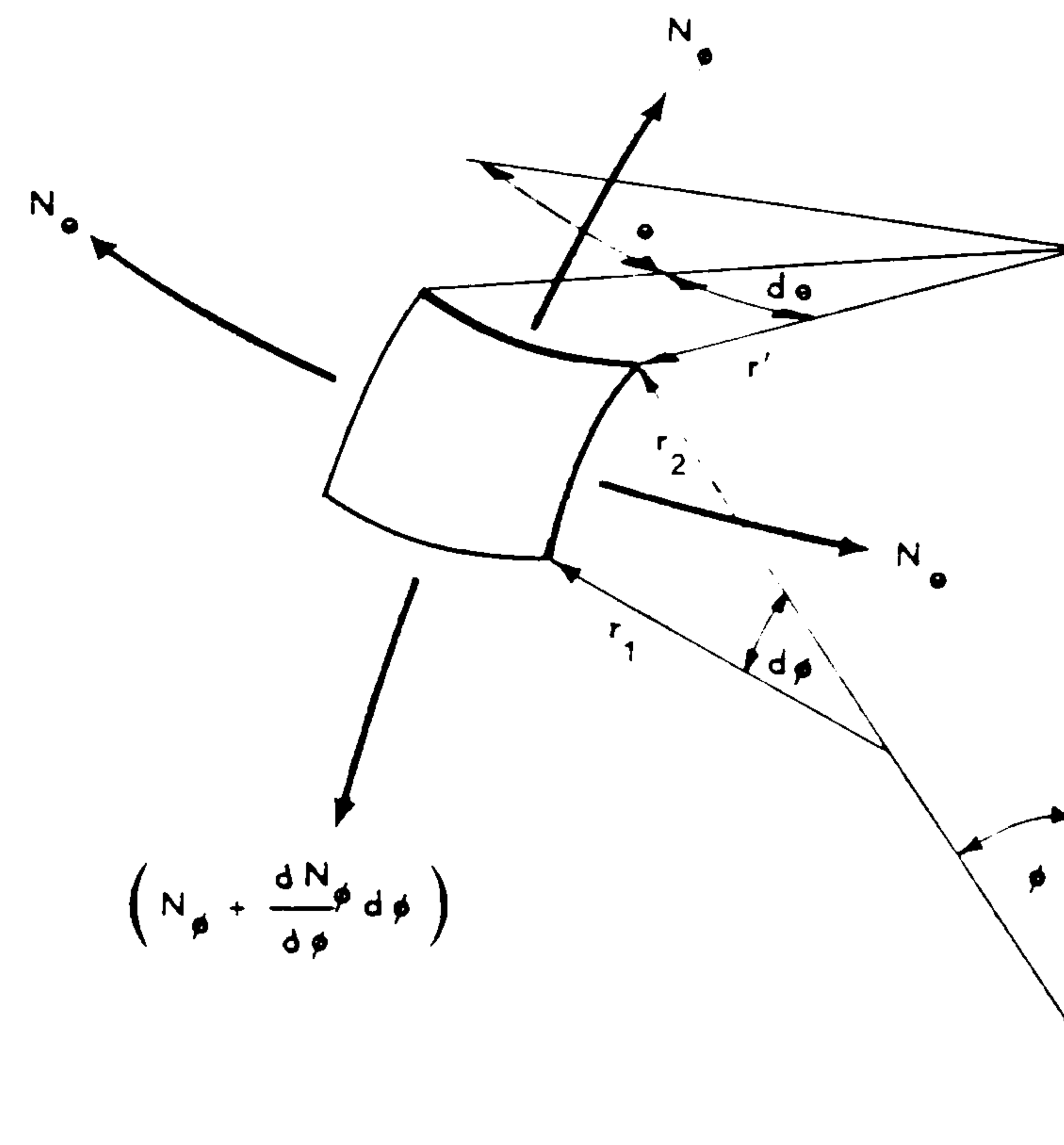


Fig 11. Membrane forces in a shell element.

Referring now to Fig 1, for the shell of constant strength of uniform thickness the shape of the meridian under a linearly varying external pressure is such that $N_\varphi = N_\theta = N$ (say), a constant. Consequently at some depth z below the datum, (2) gives,

$$N\left(\frac{1}{r_1} + \frac{1}{r_2}\right) = \gamma z \qquad (3)$$

where N is negative, i.e. compressive.

Considering the differential geometry of the element of shell in Fig 1.

$$r_1 d\varphi = ds = \frac{dx}{\cos\varphi}, \text{ and } r_2 = \frac{x}{\sin\varphi}$$

thus $\frac{1}{r_1} = \frac{\cos\varphi \, d\varphi}{dx} = \frac{d(\sin\varphi)}{dx}$ and (3) becomes

$$\frac{d(\sin\varphi)}{dx} + \frac{\sin\varphi}{x} = \frac{\gamma z}{N} \qquad (4)$$

but $\tan\varphi = \frac{dz}{dx}$ and letting $u = \sin\varphi$, (4) can be expressed in the form,

$$\frac{du}{dx} + \frac{u}{x} = \frac{\gamma z}{N} = \frac{z}{K} \qquad (5)$$

where

$$K = \frac{N}{\gamma},$$

and
$$\frac{dz}{dx} = \frac{u}{(1-u^2)^{\frac{1}{2}}} \qquad (6)$$

These are the basic differential equations of the shape.

Because of symmetry at the apex A of the shell, $r_1 = r_2$, and (3) gives $r_1 = \frac{2K}{z_0}$ at this point.

The co-ordinates of the apex are $x = x_0 = 0$, $z = z_0$, with $u = u_0 = 0$ rendering (5) indeterminate.

In order to overcome this difficulty it was suggested[4] that a small spherical cap be assumed of radius $\frac{2K}{z_0}$ and defined by a half chord Δx. Consequently at the first point on the profile from A,

$$x=x_1=\Delta x, \quad z=z_1=z_0\left[1+\frac{(\Delta x)^2}{4K}\right],$$

$$\text{and } u=u_1=z_0\,\frac{(\Delta x)}{2K}$$

Then these values are used as the initial condition point for integrating (5) and (6).

Since the solution of a differential equation is very sensitive to its initial conditions a bound must be set on the value of Δx so that for any value less than or equal to this bound or limit the subsequent solution of (5) and (6) will be consistent. This can be achieved by choosing $\Delta x={}^1\Delta x$, ${}^2\Delta x$ . . . etc., and evaluating.

$${}^1z_1=z_0\left[1+\frac{({}^1\Delta x)^2}{4K}\right], \quad {}^1u_1=z_0\frac{({}^1\Delta x)}{2K}$$

$${}^2z_1=z_0\left[1+\frac{({}^2\Delta x)^2}{4K}\right], \quad {}^2u_1=z_0\frac{({}^2\Delta x)}{2K} \text{ etc.}$$

In this way it is possible to obtain $\frac{dz}{dx}$ at $x={}^1\Delta x$, $x={}^2\Delta x$ from (6), and to recompute the z co-ordinate of the apex as,

$${}^1z_0={}^1z_1-{}^1\Delta x\left(\frac{dz}{dx}\right)_{x={}^1\Delta x}$$

or $${}^2z_0={}^2z_1-{}^2\Delta x\left(\frac{dz}{dx}\right)_{x={}^2\Delta x}$$

The associated errors in these values are

$$E_1={}^1z_0-z_0 \quad \text{and} \quad E_2={}^2z_0-z_0.$$

Plotting error E against Δx and extrapolating to find Δx at $E=0$ an upper limit, Δx_1 on Δx is found.

Thus at the first point on the profile from the apex,

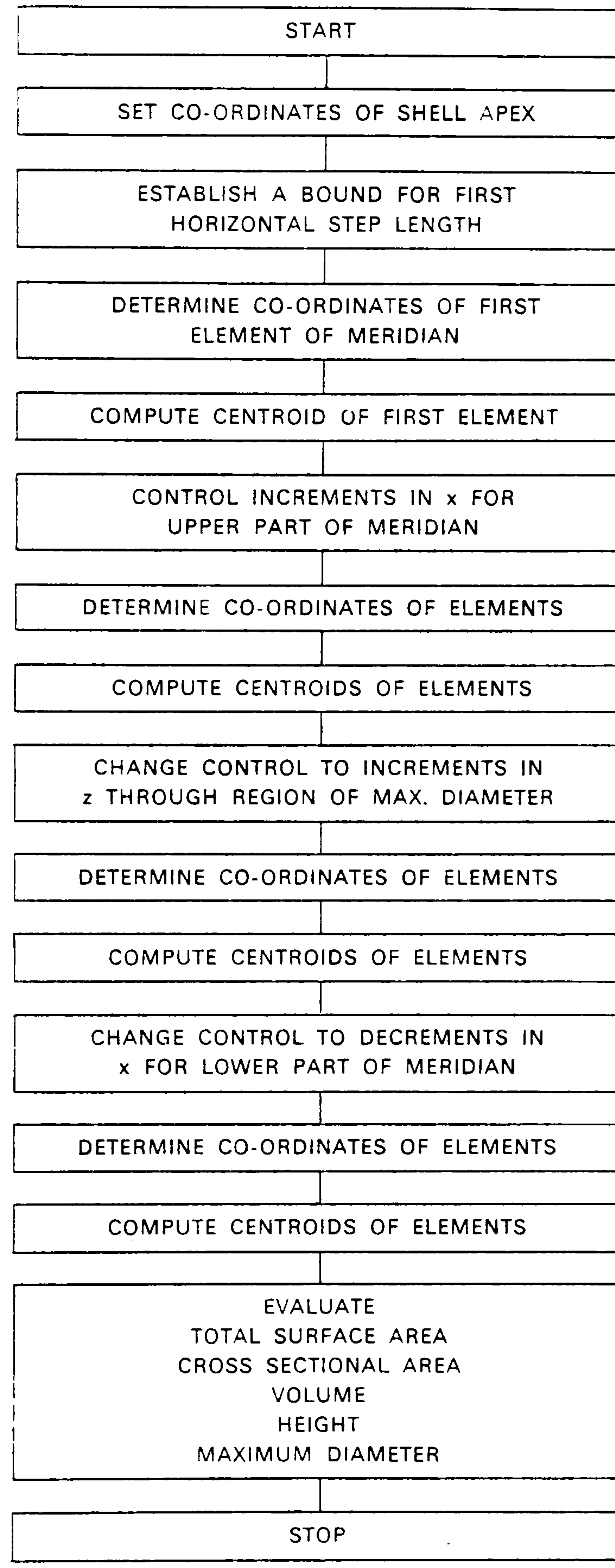
$$x_1=\Delta x_1, \; z_1=z_0\left[1+\frac{(\Delta x_1)^2}{4K}\right], \text{ and } u_1=z_0\left(\frac{\Delta x_1}{2K}\right)$$

This fixes firmly initial conditions for (5) and (6) and either step dx or dz can be varied in controlled increments or decrements to obtain a converging solution of this system of equations depending on the method employed e.g. the Euler method recommended by Timoshenko and Woinowsky Krieger[4].

Appendix 3

On recent evolutionary theory, Bonner[16] (1966) wrote: 'In some instances there is a clearly direct effect of the environment which causes, by mechanical or physical forces, the form of a living structure. In the case of these direct adaptations it may be assumed that such responsiveness to the environment is adaptively advantageous and therefore the gene complement which favours responsiveness or reactivity to these environmental conditions is favoured and maintained by natural selection'.

Appendix 2



Appendix A 2.0.1

Derivation of the Differential Equations System of the Drop Shaped Shell

An equilibrium equation for the membrane forces in an element of a thin shell can be written as (see Section 2.0),

$$\frac{N_\phi}{r_1} + \frac{N_\theta}{r_2} = - Z \quad . \tag{A 2.0.1}$$

Consider a tank which contains liquid at a pressure γd at the upper point A (Fig. 1.1.1) where γ is the unit weight of the liquid. The meridional shape of this tank, having a uniform thickness, is such that an internal pressure equal to γz will produce equal stresses at all points on the shell. That is,

$$N_\phi = N_\theta = N \text{ , say.} \tag{A 2.0.2}$$

Substituting the last result in equation (A 2.0.1) gives

$$N\left(\frac{1}{r_1} + \frac{1}{r_2}\right) = \gamma z \ . \tag{A 2.0.3}$$

If the orthogonal coordinates are as shown in Fig. 1.1.1, by considering the enlarged diagrams of the small arc ds of the tank (see Fig.A2.0.1) one obtains:

$$\frac{1}{r_2} = \frac{\sin \phi}{x} \tag{A 2.0.4}$$

and

$$\frac{1}{r_1} = \frac{\cos \phi \ d \phi}{dx} = \frac{d(\sin \phi)}{dx} \quad . \tag{A 2.0.5}$$

Substituting (A 2.0.4) and (A 2.0.5) in (A 2.0.3) gives

$$\frac{d(\sin\phi)}{dx} + \frac{\sin\phi}{x} = \frac{\gamma z}{N} \quad . \tag{A 2.0.6}$$

Using the relation

$$\frac{dz}{dx} = \tan\phi = \frac{\sin\phi}{\sqrt{(1-\sin^2\phi)}} \tag{A 2.0.7}$$

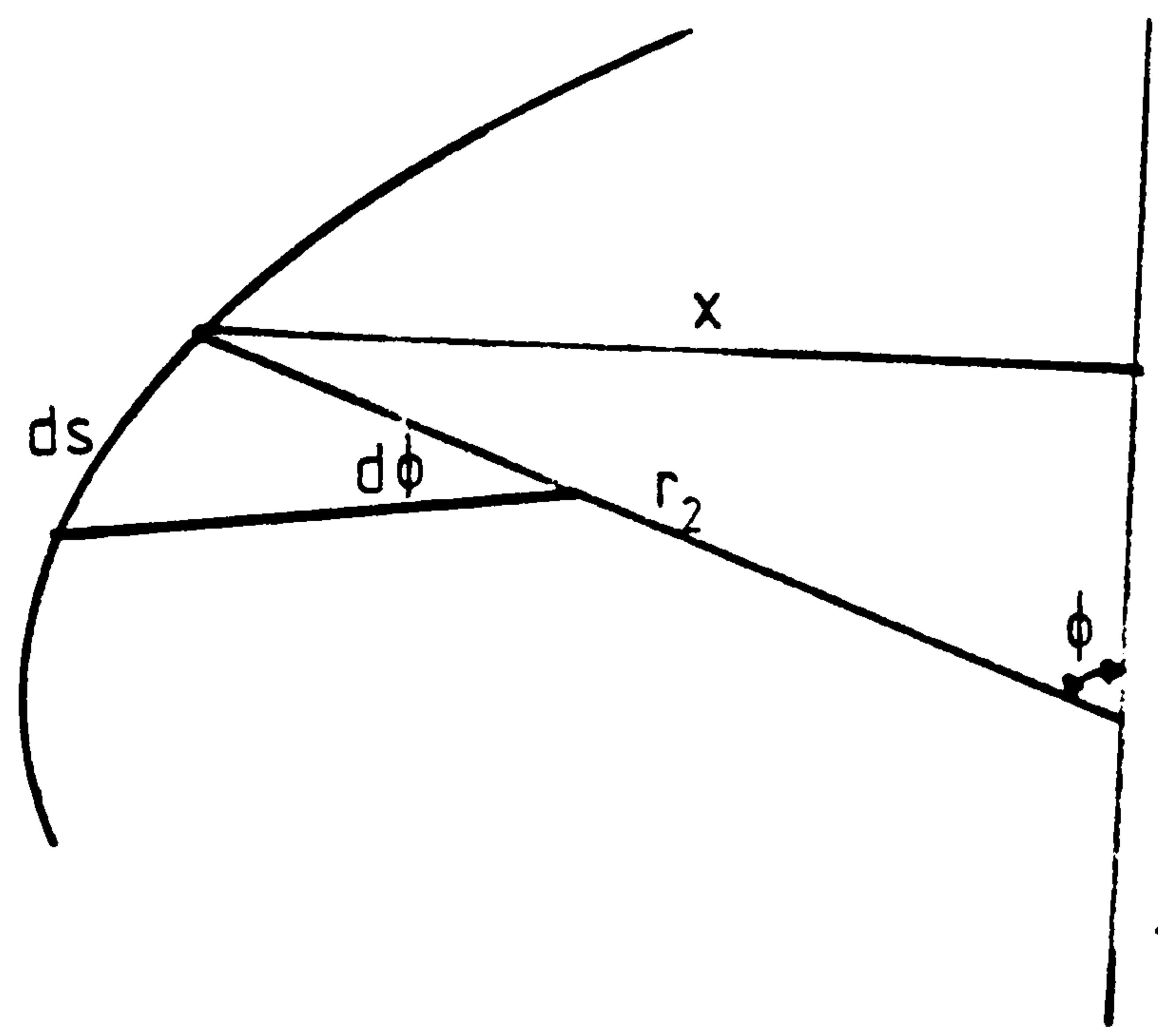
which is the gradient property of a tangent to an arbitrary point on the meridional curve of the tank and noting that the apex of the tank has coordinates given by

$$z = d, \quad \sin\phi = 0 \quad \text{when} \quad x = 0 \tag{A 2.0.8}$$

the system of ordinary differential equations

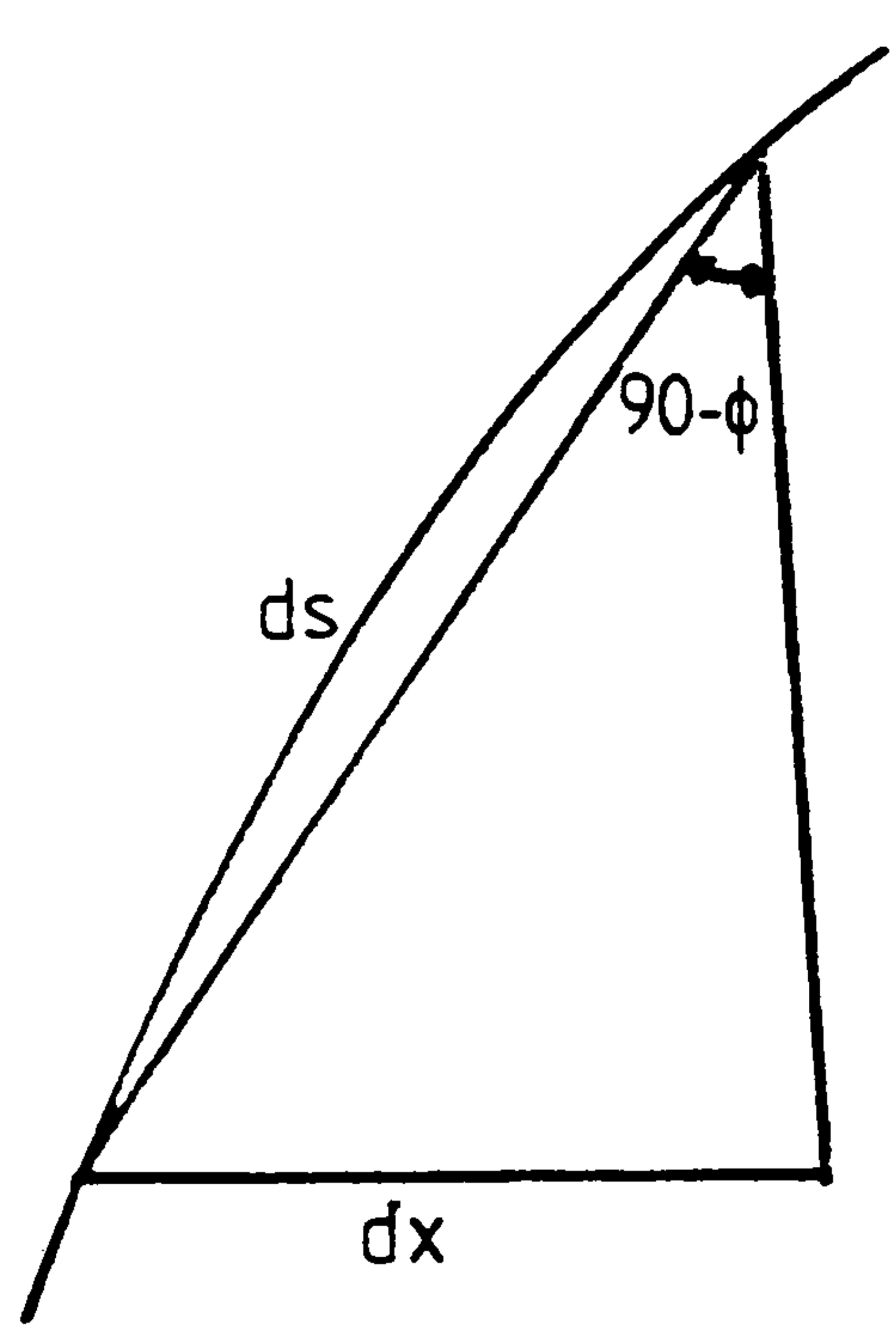
$$\left.\begin{aligned} &\frac{d(\sin\phi)}{dx} + \frac{\sin\phi}{x} = \frac{\gamma z}{N} && \text{(A 2.0.9a)} \\ &\frac{dz}{dx} = \frac{\sin\phi}{\sqrt{(1-\sin^2\phi)}} && \text{(A 2.0.9b)} \\ &\text{with the initial condition} && \\ &z = d, \quad \sin\phi = 0 \quad \text{when} \quad x = 0 && \text{(A 2.0.9c)} \end{aligned}\right\} \text{(A 2.0.9)}$$

is obtained. This system determines the meridian of the drop shape and may be referred to as "The differential equations system of the drop shaped shell".



ϕ is an angle which defines the position of the parallel circle.

(a) Section of shell



(b) Element of surface enlarged

Fig. A. 2.0.1 Enlarged section of shell.

APPENDIX A.2.3.1

Table of digital computer output showing variation in the shape of a tank due to first step-length of integration

x values	z values for first step-length 8.0	4.0	2.0	1.0
8	.3096	.2906	.2809	.2761
16	1.2062	1.1872	1.1776	1.1727
24	2.7457	2.7267	2.7171	2.7122
32	4.9599	4.9409	4.9312	4.9263
40	7.8983	7.8793	7.8695	7.8647
48	11.6346	11.6156	11.6058	11.6009
56	16.2782	16.2591	16.2492	16.2443
64	21.9955	21.9762	21.9663	21.9613
72	29.0542	29.0346	29.0246	29.0196

(x and z values in mm)

Coordinates of first part of tank only given

Problem parameters:

Design head (ZO) = 1000.0 mm

Thickness (T) = 4.0 mm

Design stress (DS) = 0.15 MN/m^2

Unit weight of fluid (G) = 11.61 KN/m^3

Subsequent step-length used after the first one is 1.0 mm

Method of integration is explicit Euler (appendix A.2.4.2)

APPENDICES A2.4.1 → A2.4.6

The Fortran computer programs in these Appendices have the same logical flow diagram (refer to Appendix 2 of Ref. 9). The main difference is the method of integration employed for the differential equations. To reduce the size and cost of the thesis, only two of the programs (Appendices A2.4.3 and A2.4.5) are listed.

APPENDIX A2.4.3

EDINBURGH FORTRAN(G) COMPILER VERSION 50.14

```
C
C
C
C     A FORTRAN COMPUTER PROGRAM USED IN OBTAINING THE
C     COORDINATES OF THE MERIDIAN OF THE DROP SHAPED TANK/SHELL.
C     METHOD OF INTEGRATION USED IS EXPLICIT IMPROVED OR MODIFIED
C     EULER.
C     PROGRAM WITH D.P. ACCURACY
C     DX1 IS INTRODUCED TO PROGRAM.
C     ***************************************************************
C     ***************************************************************
C     **************************************************************
C     **************************************************************
C
C       PARAMETERS
C
C     ZO=DEPTH OVER SHELL, DESIGN HEAD
C     T=SHELL THICKNESS
C     DS= DESIGN STRESS
C     G=GAMMA, UNIT WEIGHT OF EXTERNAL FLUID
C     DX1=FIRST STEP LENGTH IN X COORDS
C     DX= ELEMENTAL INCREMENT IN X COORDS
C     DZ= ELEMENTAL INCREMENT IN Z COORDS
C     IND= INDICATOR FOR WHETHER COORDS ARE WRITTEN OR NOT, (0 OR 1)
C     **************************************************************
C
      REAL*8 X(3000),XB(3000),Z(3000),U(3000),SA(3000),
     1  DUDX(3000),DZDX(3000),DWDZ(3000),DXDZ(3000),CSA(3000),
     1  AMZ(3000),W(3000),SMX(3000),DFX(3000),DFZ(3000),
     2  T,DS,G,DX,DZ,ANF,A,V,H,D,HRD,ZT,DX1,VSM,
     2  ADE,BOYE,SOF
          COMMON/AAA/X,XB,Z,U,SA
          COMMON/AAB/DUDX,DZDX,DWDZ,DXDZ,CSA
          COMMON/AAC/AMZ,W,SMX,DFX,DFZ
C
C     *********************
C     NUMBER OF PROBLEMS
C     *********************
C
       CALL CPUTIM(ADE)
       READ(5,80)NP,DX1
   80 FORMAT(I4,2X,F6.3)
      DO 500 N=1,NP
C
C     *************************************
C     PARAMETERS, WITH ZO,T AND DS VARYING
C     *************************************
C
      READ(5,90)ZO,T,DS,G,DX,DZ,IND
   90 FORMAT(F8.1,2X,F5.1,2X,F6.2,2X,E9.3,2X,F6.3,2X,F6.3,2X,I1)
       ANF=DS*T
       A=ANF/G
C
C     ************************************
C     SETTING X COORDINATES AT 0
C     ZO REPRESENTS THE APEX OF THE SHELL
C     ************************************
C
      XO=0.
        IAC1=0
        IAC2=0
        IAC3=0
C
C     ********************************************
C     COORDINATES AT THE END OF THE FIRST ELEMENT
C       OF THE MERIDIAN CURVE
C     ********************************************
C
```

```
      X(1)=DX1
      Z(1)=Z0*(1.+DX1*DX1/(4.*A))
      U(1)=Z0*DX1/(2.*A)
      DUDX(1)=Z(1)/A-U(1)/X(1)
      DZDX(1)=U(1)/DSQRT(1.-U(1)*U(1))
C
C     **************************
C     CENTROID OF FIRST ELEMENT
C     **************************
C
      XB(1)=X(1)/3.
      DO 100 I=2,1000
C
C     ***************************************
C     GENERATION OF COORDINATES UP TO END OF
C       ELEMENTAL INCREMENTS ON X AXIS
C     ***************************************
C
      M1=I
      X(I)=X(1)+(I-1)*DX
      U(I)=U(I-1)+DUDX(I-1)*DX
      Z(I)=Z(I-1)+DZDX(I-1)*DX
      DUDX(I)=Z(I)/A-U(I)/X(I)
      DZDX(I)=U(I)/DSQRT(1.-U(I)*U(I))
      U(I)=U(I-1)+(DUDX(I-1)+DUDX(I))/2.*DX
       Z(I)=Z(I-1)+(DZDX(I-1)+DZDX(I))/2.*DX
       DUDX(I)=Z(I)/A-U(I)/X(I)
       DZDX(I)=U(I)/DSQRT(1.-U(I)*U(I))
       XB(I)=(X(I)*X(I)+X(I)*X(I-1)+X(I-1)*X(I-1))/(3.*(X(I)+X(I-1)))
      P=0.707
      IF(U(I).GE.P)GO TO 101
  100 CONTINUE
C
C     *********************************************
C     CHANGE OVER TO ELEMENTAL INCREMENTS ON Z AXIS
C     *********************************************
C
  101 W(M1)=DSQRT(1.-U(M1)*U(M1))
          IAC1=IAC1+I-1
      DWDZ(M1)=DSQRT(1.-W(M1)*W(M1))/X(M1)-Z(M1)/A
      DXDZ(M1)=W(M1)/DSQRT(1.-W(M1)*W(M1))
      L1=M1+1
      DO 200 J=L1,2000
      M2=J
C
C     ***********************************
C     GENERATION OF COORDINATES ON Z AXIS
C     ***********************************
C
      Z(J)=Z(J-1)+DZ
      W(J)=W(J-1)+DWDZ(J-1)*DZ
      X(J)=X(J-1)+DXDZ(J-1)*DZ
      DWDZ(J)=DSQRT(1.-W(J)*W(J))/X(J)-Z(J)/A
      DXDZ(J)=W(J)/DSQRT(1.-W(J)*W(J))
       W(J)=W(J-1)+(DWDZ(J-1)+DWDZ(J))/2.*DZ
       X(J)=X(J-1)+(DXDZ(J-1)+DXDZ(J))/2.*DZ
C
       DWDZ(J)=DSQRT(1.-W(J)*W(J))/X(J)-Z(J)/A
       DXDZ(J)=W(J)/DSQRT(1.-W(J)*W(J))
C     ***********************
C     CENTROID OF EACH ELEMENT
C     ***********************
C
      XB(J)=(X(J)*X(J)+X(J)*X(J-1)+X(J-1)*X(J-1))/(3.*(X(J)+X(J-1)))
      IF(X(J).LT.X(J-1))GO TO 190
      D=2.*X(J)
  190 Q=-0.707
      IF(W(J).LE.Q)GO TO 201
  200 CONTINUE
```

```
C
C     ********************************************
C     RETURN TO GENERATION OF COORDINATES ON X AXIS
C     ********************************************
C
  201 U(M2)=DSQRT(1.-W(M2)*W(M2))
          IAC2=IAC2+M2-M1
      DUDX(M2)=Z(M2)/A-U(M2)/X(M2)
      DZDX(M2)=-U(M2)/DSQRT(1.-U(M2)*U(M2))
      L2=M2+1
      DO 300 K=L2,3000
      M3=K
      X(K)=X(K-1)-DX
      U(K)=U(K-1)-DUDX(K-1)*DX
      Z(K)=Z(K-1)-DZDX(K-1)*DX
      DUDX(K)=Z(K)/A-U(K)/X(K)
      DZDX(K)=-U(K)/DSQRT(1.-U(K)*U(K))
       U(K)=U(K-1)-(DUDX(K-1)+DUDX(K))/2.*DX
       Z(K)=Z(K-1)-(DZDX(K-1)+DZDX(K))/2.*DX
       DUDX(K)=Z(K)/A-U(K)/X(K)
       DZDX(K)=-U(K)/DSQRT(1.-U(K)*U(K))
          IF(Z(K).LT.Z(K-1))GO TO 217
C
C     ************************
C     CENTROID OF EACH ELEMENT
C     ************************
C
      XB(K)=(X(K)*X(K)+X(K)*X(K-1)+X(K-1)*X(K-1))/(3.*(X(K)+X(K-1)))
      R=-0.00001
      IF(DZDX(K).GE.R)GO TO 301
  300 CONTINUE
  217     K=K-1
          M3=K
C
C     *******************************************
C     SURFACE AREA OF FIRST ELEMENT OF REVOLUTION
C     *******************************************
C
  301 SA(1)=3.142*X(1)*DSQRT((Z(1)-Z0)**2+(X(1)-X0)**2)
          IAC3=IAC3+M3-M2
          IACT=IAC1+IAC2+IAC3
C
C     ***************************************************
C     CROSS SECTIONAL AREA OF FIRST ELEMENT OF REVOLUTION
C     ***************************************************
C
      CSA(1)=X(1)*(Z(1)-Z0)
C
C     ****************************
C     VOLUME=3.142*(SUM OF AMZ(I))
C     ****************************
C
      AMZ(1)=(X(1)/3.)*(X(1))*(Z(1)-Z0)
      DFZ(1)=Z(1)-Z0
      DFX(1)=X(1)-X0
      SMX(1)=X(1)
      DO 400 I=2,M3
      SMX(I)=(X(I)+X(I-1))
      DFX(I)=(X(I)-X(I-1))
      DFZ(I)=(Z(I)-Z(I-1))
C
C     ******************
C     TOTAL SURFACE AREA
C     ******************
C
      SA(I)=SA(I-1)+3.142*SMX(I)*DSQRT(DFX(I)*DFX(I)+
     1            DFZ(I)*DFZ(I))
```

```
C
C     **************************
C     TOTAL CROSS SECTIONAL AREA
C     **************************
C
      CSA(I)=CSA(I-1)+SMX(I)*DFZ(I)
      AMZ(I)=AMZ(I-1)+SMX(I)*DFZ(I)*XB(I)
  400 CONTINUE
C     ********************
C     VOL OF SHELL MATERIAL
C     ********************
        VSM=SA(M3)*T
C
C     **************************
C     VOLUME ENCLOSED BY SHELL
C     **************************
C
      V=3.142*AMZ(M3)-VSM/2.
C
C     ***************
C     HEIGHT OF SHELL
C     ***************
C
      H=Z(M3)-Z0
       HRD=H/D
C
C     ****************************************
C     HEIGHT, FROM MEAN SEA LEVEL TO SEA BED
C     ****************************************
      ZT=Z(M3)
      WRITE(6,399)N
  399 FORMAT(1X,'PROBLEM NO.:',I3,//)
      WRITE(6,401)
  401 FORMAT(3X,'T',6X,'DS',8X,'Z0',8X,'ZT',7X,'DX1',7X,'DX',
     1              7X,'DZ',7X,'G',/)
      WRITE(6,402)T,DS,Z0,ZT,DX1,DX,DZ,G
  402 FORMAT(1X,F5.1,2X,F6.2,2X,F8.1,2X,F9.3,2X,F6.3,2X,F6.3,
     1       4X,F6.3,4X,E9.3,//)
      WRITE(6,403)
  403 FORMAT(4X,'H(MM)',8X,'D(MM)',8X,'SA(MM**2)',8X,
     1    'CSA(MM**2)',8X,'VSM(MM**3)',8X,'V(MM**3)',8X,'HRD',/)
      WRITE(6,404)H,D,SA(M3),CSA(M3),VSM,V,HRD
  404 FORMAT(1X,E11.6,2X,E11.6,2X,E14.9,2X,E15.10,2X,
     1              E15.10,2X,E15.10,2X,F10.6,//)
       WRITE(6,878)IAC1,IAC2,IAC3,IACT
  878  FORMAT(4X,'IAC1=',I6,4X,'IAC2=',I6,4X,'IAC3=',I6,
     1 4X,'IACT=',I8,/)
          CALL CPUTIM(BOYE)
          SOF=BOYE-ADE
          WRITE(6,963)SOF
  963     FORMAT(8X,'SOF=',F14.8,/)
      IF(IND.LT.1)GO TO 500
      WRITE(6,405)
  405 FORMAT(5X,'X(MM)',6X,'Z(MM)',/)
      WRITE(6,406)X0,Z0
  406 FORMAT(1X,F10.4,2X,F10.4,/)
      WRITE(6,407)(X(I),Z(I),I=1,M3)
  407 FORMAT(1X,F10.4,2X,F10.4,/)
      WRITE(6,499)
  499 FORMAT(1H1)
  500 CONTINUE
  408 STOP
      END
```

```
CODE    10866 BYTES      PLT + DATA     1488 BYTES
STACK     792 BYTES      DIAG TABLES     524 BYTES       TOTAL  13670 BYTES
COMPILATION SUCCESSFUL
```

```
     EDINBURGH FORTRAN(G) COMPILER VERSION 50.16
C******** APPENDIX A 2.4.5 ************************

C
C
C    A FORTRAN COMPUTER PROGRAM USED IN OBTAINING THE
C    COORDINATES OF THE MERIDIAN OF THE DROP-SHAPED TANK.
C    RUNGE-KUTTA METHOD USED IN INTEGRATION.
C    DX1 IS INTRODUCED TO PROGRAM.
C    PROBLEM WITH D.P. ACCURACY.
C
C      ********************************************
C      ********************************************
C
C        PARAMETERS
C
C      ZO=DEPTH OVER SHELL, DESIGN HEAD
C      T=SHELL THICKNESS
C      DS= DESIGN STRESS
C      G=GAMMA, UNIT WEIGHT OF EXTERNAL FLUID , WATER
C      DX1=INITIAL INCREMENT FROM DESIGN HEAD
C      DX= ELEMENTAL INCREMENT IN X COORDS
C      DZ= ELEMENTAL INCREMENT IN Z COORDS
C      IND= INDICATOR FOR WHETHER COORDS ARE WRITTEN OR NOT, (0 OR 1)
C      ********************************************
C
       REAL*8 X(3000),XB(3000),Z(3000),U(3000),SA(3000),
     1  CSA(3000),XI(3000),UI(3000),WI(3000),ZI(3000),
     1  AMZ(3000),W(3000),SMX(3000),DFX(3000),DFZ(3000),
     1  T,DS,G,DX,DZ,ANF,A,V,H,ZT,K0,K1,K2,K3,M0,M1,M2,M3,DERSUM,DX1,
     2  D,HRD,VSM
          COMMON/AAA/X,XB,Z,U,SA
          COMMON/AAB/DUDX,DZDX,DWDZ,DXDZ,CSA
          COMMON/AAC/AMZ,W,SMX,DFX,DFZ
C
C      ********************
C      NUMBER OF PROBLEMS
C      ********************
C
       READ(5,80)NP,DX1
    80 FORMAT(I4,2X,F6.3)
       DO 500 N=1,NP
C
C      ************************************
C      PARAMETERS, WITH ZO,T AND DS VARYING
C      ************************************
C
       READ(5,90)ZO,T,DS,G,DX,DZ,IND
    90 FORMAT(F8.1,2X,F5.1,2X,F6.2,2X,E9.3,2X,F6.3,2X,F6.3,2X,I1)
       ANF=DS*T
       A=ANF/G
C
C      ***********************************
C      SETTING X COORDINATES AT 0
C      ZO REPRESENTS THE APEX OF THE SHELL
C      ***********************************
C
       XO=0.
C
C      ********************************************
C      COORDINATES AT THE END OF THE FIRST ELEMENT
C        OF THE MERIDIAN CURVE
C      ********************************************
C
       X(1)=DX1
       Z(1)=ZO*(1.+DX1*DX1/(4.*A))
       U(1)=ZO*DX1/(2.*A)
        XI(1)=X(1)
        UI(1)=U(1)
        ZI(1)=Z(1)
        K0=ZI(1)/A-UI(1)/XI(1)
```

APPENDIX A2.4.5 (contd.)

```
           M0=UI(1)/DSQRT(1.-UI(1)*UI(1))
           XI(1)=X(1)+DX/2.
           UI(1)=U(1)+K0/2.
           ZI(1)=Z(1)+M0/2.
           K1=ZI(1)/A-UI(1)/XI(1)
           M1=UI(1)/DSQRT(1.-UI(1)*UI(1))
           UI(1)=U(1)+K1/2.
           ZI(1)=Z(1)+M1/2.
           K2=ZI(1)/A-UI(1)/XI(1)
           M2=UI(1)/DSQRT(1.-UI(1)*UI(1))
           XI(1)=X(1)+DX
           UI(1)=U(1)+K2
           ZI(1)=Z(1)+M2
           K3=ZI(1)/A-UI(1)/XI(1)
           M3=UI(1)/DSQRT(1.-UI(1)*UI(1))
C
C         ***************************
C         CENTROID OF FIRST ELEMENT
C         ***************************
C
          XB(1)=X(1)/3.
          DO 100 I=2,1000
C
C         ****************************************
C         GENERATION OF COORDINATES UP TO END OF
C           ELEMENTAL INCREMENTS ON X AXIS
C         ****************************************
C
          N1=I
          X(I)=X(1)+(I-1)*DX
          U(I)=U(I-1)+DX*(K0+K1+K1+K2+K2+K3)/6.
          Z(I)=Z(I-1)+DX*(M0+M1+M1+M2+M2+M3)/6.
           XI(I)=X(I)
           UI(I)=U(I)
           ZI(I)=Z(I)
           K0=ZI(I)/A-UI(I)/XI(I)
           M0=UI(I)/DSQRT(1.-UI(I)*UI(I))
           XI(I)=X(I)+DX/2.
           UI(I)=U(I)+K0/2.
           ZI(I)=Z(I)+M0/2.
           K1=ZI(I)/A-UI(I)/XI(I)
           M1=UI(I)/DSQRT(1.-UI(I)*UI(I))
           UI(I)=U(I)+K1/2.
           ZI(I)=Z(I)+M1/2.
           K2=ZI(I)/A-UI(I)/XI(I)
           M2=UI(I)/DSQRT(1.-UI(I)*UI(I))
           XI(I)=X(I)+DX
           UI(I)=U(I)+K2
           ZI(I)=Z(I)+M2
           K3=ZI(I)/A-UI(I)/XI(I)
           M3=UI(I)/DSQRT(1.-UI(I)*UI(I))
           XB(I)=(X(I)*X(I)+X(I)*X(I-1)+X(I-1)*X(I-1))/(3.*(X(I)+X(I-1)))
           P=0.707
           IF(U(I).GE.P)GO TO 101
     100 CONTINUE
C
C        **********************************************
C        CHANGE OVER TO ELEMENTAL INCREMENTS ON Z AXIS
C        **********************************************
C
     101 W(N1)=DSQRT(1.-U(N1)*U(N1))
           ZI(N1)=Z(N1)
           WI(N1)=W(N1)
           XI(N1)=X(N1)
           K0=DSQRT(1.-WI(N1)*WI(N1))/XI(N1)-ZI(N1)/A
           M0=WI(N1)/DSQRT(1.-WI(N1)*WI(N1))
           ZI(N1)=Z(N1)+DZ/2.
           WI(N1)=W(N1)+K0/2.
           XI(N1)=X(N1)+M0/2.
```

```
       K1=DSQRT(1.-WI(N1)*WI(N1))/XI(N1)-ZI(N1)/A
       M1=WI(N1)/DSQRT(1.-WI(N1)*WI(N1))
       WI(N1)=W(N1)+K1/2.
       XI(N1)=X(N1)+M1/2.
       K2=DSQRT(1.-WI(N1)*WI(N1))/XI(N1)-ZI(N1)/A
       M2=WI(N1)/DSQRT(1.-WI(N1)*WI(N1))
       ZI(N1)=Z(N1)+DZ
       WI(N1)=W(N1)+K2
       XI(N1)=X(N1)+M2
       K3=DSQRT(1.-WI(N1)*WI(N1))/XI(N1)-ZI(N1)/A
       M3=WI(N1)/DSQRT(1.-WI(N1)*WI(N1))
      L1=N1+1
      DO 200 J=L1,2000
      N2=J
C
C     ***********************************
C     GENERATION OF COORDINATES ON Z AXIS
C     ***********************************
C
      Z(J)=Z(J-1)+DZ
      W(J)=W(J-1)+DZ*(K0+K1+K1+K2+K2+K3)/6.
      X(J)=X(J-1)+DZ*(M0+M1+M1+M2+M2+M3)/6.
       ZI(J)=Z(J)
       WI(J)=W(J)
       XI(J)=X(J)
       K0=DSQRT(1.-WI(J)*WI(J))/XI(J)-ZI(J)/A
       M0=WI(J)/DSQRT(1.-WI(J)*WI(J))
       ZI(J)=Z(J)+DZ/2.
       WI(J)=W(J)+K0/2.
       XI(J)=X(J)+M0/2.
       K1=DSQRT(1.-WI(J)*WI(J))/XI(J)-ZI(J)/A
       M1=WI(J)/DSQRT(1.-WI(J)*WI(J))
       WI(J)=W(J)+K1/2.
       XI(J)=X(J)+M1/2.
       K2=DSQRT(1.-WI(J)*WI(J))/XI(J)-ZI(J)/A
       M2=WI(J)/DSQRT(1.-WI(J)*WI(J))
       ZI(J)=Z(J)+DZ
       WI(J)=W(J)+K2
       XI(J)=X(J)+M2
       K3=DSQRT(1.-WI(J)*WI(J))/XI(J)-ZI(J)/A
       M3=WI(J)/DSQRT(1.-WI(J)*WI(J))
C
C     ************************
C     CENTROID OF EACH ELEMENT
C     ************************
C
      XB(J)=(X(J)*X(J)+X(J)*X(J-1)+X(J-1)*X(J-1))/(3.*(X(J)+X(J-1)))
      IF(X(J).LT.X(J-1))GO TO 190
      D=2.*X(J)
  190 Q=-0.707
      IF(W(J).LE.Q)GO TO 201
  200 CONTINUE
C
C     ********************************************
C     RETURN TO GENERATION OF COORDINATES ON X AXIS
C     ********************************************
C
  201 U(N2)=DSQRT(1.-W(N2)*W(N2))
       XI(N2)=X(N2)
       UI(N2)=U(N2)
       ZI(N2)=Z(N2)
       K0=ZI(N2)/A-UI(N2)/XI(N2)
       M0=-UI(N2)/DSQRT(1.-UI(N2)*UI(N2))
       XI(N2)=X(N2)+DX/2.
       UI(N2)=U(N2)+K0/2.
       ZI(N2)=Z(N2)+M0/2.
       K1=ZI(N2)/A-UI(N2)/XI(N2)
```

```
       M1=-UI(N2)/DSQRT(1.-UI(N2)*UI(N2))
       UI(N2)=U(N2)+K1/2.
       ZI(N2)=Z(N2)+M1/2.
       K2=ZI(N2)/A-UI(N2)/XI(N2)
       M2=-UI(N2)/DSQRT(1.-UI(N2)*UI(N2))
       XI(N2)=X(N2)+DX
       UI(N2)=U(N2)+K2
       ZI(N2)=Z(N2)+M2
       K3=ZI(N2)/A-UI(N2)/XI(N2)
       M3=-UI(N2)/DSQRT(1.-UI(N2)*UI(N2))
      L2=N2+1
      DO 300 K=L2,3000
      N3=K
      X(K)=X(K-1)-DX
      U(K)=U(K-1)-DX*(K0+K1+K1+K2+K2+K3)/6.
      Z(K)=Z(K-1)-DX*(M0+M1+M1+M2+M2+M3)/6.
      IF(Z(K).LT.Z(K-1))GO TO 217
       XI(K)=X(K)
       UI(K)=U(K)
       ZI(K)=Z(K)
       K0=ZI(K)/A-UI(K)/XI(K)
       M0=-UI(K)/DSQRT(1.-UI(K)*UI(K))
       XI(K)=X(K)+DX/2.
       UI(K)=U(K)+K0/2.
       ZI(K)=Z(K)+M0/2.
       K1=ZI(K)/A-UI(K)/XI(K)
       M1=-UI(K)/DSQRT(1.-UI(K)*UI(K))
       UI(K)=U(K)+K1/2.
       ZI(K)=Z(K)+M1/2.
       K2=ZI(K)/A-UI(K)/XI(K)
       M2=-UI(K)/DSQRT(1.-UI(K)*UI(K))
       XI(K)=X(K)+DX
       UI(K)=U(K)+K2
       ZI(K)=Z(K)+M2
       K3=ZI(K)/A-UI(K)/XI(K)
       M3=-UI(K)/DSQRT(1.-UI(K)*UI(K))
       DERSUM=(M0+M1+M1+M2+M2+M3)/6.
C
C     ************************
C     CENTROID OF EACH ELEMENT
C     ************************
C
      XB(K)=(X(K)*X(K)+X(K)*X(K-1)+X(K-1)*X(K-1))/(3.*(X(K)+X(K-1)))
      R=-0.00001
      IF(DERSUM.GE.R)GO TO 301
  300 CONTINUE
  217     K=K-1
          N3=K
C
C     ********************************************
C     SURFACE AREA OF FIRST ELEMENT OF REVOLUTION
C     ********************************************
C
  301 SA(1)=3.142*X(1)*DSQRT((Z(1)-Z0)**2+(X(1)-X0)**2)
C
C     ****************************************************
C     CROSS SECTIONAL AREA OF FIRST ELEMENT OF REVOLUTION
C     ****************************************************
C
      CSA(1)=X(1)*(Z(1)-Z0)
C
C     ***************************
C     VOLUME=3.142*(SUM OF AMZ(I))
C     ***************************
C
      AMZ(1)=(X(1)/3.)*(X(1))*(Z(1)-Z0)
      DFZ(1)=Z(1)-Z0
      DFX(1)=X(1)-X0
```

```
          SMX(1)=X(1)
          DO 400 I=2,N3
          SMX(I)=(X(I)+X(I-1))
          DFX(I)=(X(I)-X(I-1))
          DFZ(I)=(Z(I)-Z(I-1))
C
C         ******************
C         TOTAL SURFACE AREA
C         ******************
C
          SA(I)=SA(I-1)+3.142*SMX(I)*DSQRT(DFX(I)*DFX(I)+
         1              DFZ(I)*DFZ(I))
C
C         **************************
C         TOTAL CROSS SECTIONAL AREA
C         **************************
C
          CSA(I)=CSA(I-1)+SMX(I)*DFZ(I)
          AMZ(I)=AMZ(I-1)+SMX(I)*DFZ(I)*XB(I)
      400 CONTINUE
C         ********************
C         VOL OF SHELL MATERIAL
C         ********************
            VSM=SA(N3)*T
C
C         **************************
C         VOLUME ENCLOSED BY SHELL
C         **************************
C
          V=3.142*AMZ(N3)-VSM/2.
C
C         ***************
C         HEIGHT OF SHELL
C         ***************
C
          H=Z(N3)-Z0
           HRD=H/D
C
C         ****************************************
C         HEIGHT, FROM MEAN SEA LEVEL TO SEA BED
C         ****************************************
C
          ZT=Z(N3)
          WRITE(6,399)N
      399 FORMAT(1X,'PROBLEM NO.:',I3,//)
          WRITE(6,401)
      401 FORMAT(3X,'T',6X,'DS',8X,'Z0',8X,'ZT',7X,'DX1',7X,'DX',
         1               6X,'DZ',7X,'G',/)
          WRITE(6,402)T,DS,Z0,ZT,DX1,DX,DZ,G
      402 FORMAT(1X,F5.1,2X,F6.2,2X,F8.1,2X,F9.3,2X,F6.3,2X,F6.3,
         1        4X,F6.3,4X,E9.3,//)
          WRITE(6,403)
      403 FORMAT(4X,'H(MM)',8X,'D(MM)',8X,'SA(MM**2)',8X,
         1     'CSA(MM**2)',8X,'VSM(MM**3)',8X,'V(MM**3)',8X,'HRD',/)
          WRITE(6,404)H,D,SA(N3),CSA(N3),VSM,V,HRD
      404 FORMAT(1X,E11.6,2X,E11.6,2X,E14.9,2X,E15.10,2X,
         1               E15.10,2X,E15.10,2X,F10.6,//)
          IF(IND.LT.1)GO TO 500
          WRITE(6,405)
      405 FORMAT(5X,'X(MM)',6X,'Z(MM)',/)
          WRITE(6,406)X0,Z0
      406 FORMAT(1X,F9.1,2X,F9.1,/)
          WRITE(6,407)(X(I),Z(I),I=1,N3)
      407 FORMAT(1X,F12.6,2X,F12.6,/)
          WRITE(6,499)
      499 FORMAT(1H1)
      500 CONTINUE
      408 STOP
          END
```

```
CODE    17646 BYTES      PLT + DATA   97256 BYTES
STACK     928 BYTES      DIAG TABLES    556 BYTES      TOTAL  116386 BYTES
COMPILATION SUCCESSFUL
```

APPENDICES A2.5.1 → A2.5.5

The computer programs of these Appendices are similar. Their main difference is the number of the binomial terms of the derivative $\frac{dz}{dx}$ expression utilised. For example, in A2.5.1, $\frac{dz}{dx} = u$ while in A2.5.2, $\frac{dz}{dx} = u + \frac{1}{2}u^3$. Due to this, only Appendix A2.5.1 is listed.

APPENDIX A2.5.1

```
     EDINBURGH FORTRAN(G) COMPILER VERSION 50.16
C******** APPENDIX A 2.5.1 *******************

C ****************************************************
C  EXCURSION IN BINOMIAL-ECHIDOME
C  ENTIRE COORDS. OF MERIDIAN NOT GENERATED
C ************************************************************
C ************************************************************
C  DX1 INTRODUCED TO PROGRAM
C   MODIFIED-EULER OR TANGENT METHOD USED IN INTEGRATION
C    ******************************************
C    ******************************************
C      PARAMETERS
C
C    ZO=DEPTH OVER SHELL, DESIGN HEAD
C    T=SHELL THICKNESS
C    DS= DESIGN STRESS
C    G=GAMMA, UNIT WEIGHT OF EXTERNAL FLUID
C    DX= ELEMENTAL INCREMENT IN X COORDS
C    IND= INDICATOR FOR WHETHER COORDS ARE WRITTEN OR NOT, (0 OR 1)
C    ******************************************
C
      REAL*8 X(3000),Z(3000),U(3000),
     1 DUDX(3000),DZDX(3000),
     2 T,DS,G,DX,ANF,A,V,H,HRD,ZT,DX1
C
C    *********************
C    NUMBER OF PROBLEMS
C    *********************
C
      READ(5,80)NP,DX1
   80 FORMAT(I4,2X,F6.3)
      DO 500 N=1,NP
C
C    *************************************
C    PARAMETERS, WITH ZO,T AND DS VARYING
C    *************************************
C
      READ(5,90)ZO,T,DS,G,DX,IND
   90 FORMAT(F8.1,2X,F5.1,2X,F6.2,2X,E9.3,2X,F6.3,2X,I1)
       ANF=DS*T
       A=ANF/G
C
C    ************************************
C    SETTING X COORDINATES AT 0
C    ZO REPRESENTS THE APEX OF THE SHELL
C    ************************************
C
      XO=0.
C
C    *******************************************
C    COORDINATES AT THE END OF THE FIRST ELEMENT
C      OF THE MERIDIAN CURVE
C    *******************************************
      X(1)=DX1
      Z(1)=ZO*(1.+DX1*DX1/(4.*A))
      U(1)=ZO*DX1/(2.*A)
      DUDX(1)=Z(1)/A-U(1)/X(1)
      DZDX(1)=U(1)
      DO 100 I=2,1000
      M1=I
      X(I)=(I-1)*DX+X(1)
      U(I)=U(I-1)+DUDX(I-1)*DX
```

```
      Z(I)=Z(I-1)+DZDX(I-1)*DX
      DUDX(I)=Z(I)/A-U(I)/X(I)
      DZDX(I)=U(I)
      Z(I)=Z(I-1)+(DZDX(I-1)+DZDX(I))/2.0*DX
      U(I)=U(I-1)+(DUDX(I-1)+DUDX(I))/2.0*DX
       DUDX(I)=Z(I)/A-U(I)/X(I)
       DZDX(I)=U(I)
      P=0.900
      IF(U(I).GE.P)GO TO 101
  100 CONTINUE
  101  H=Z(M1)-Z0
       D=2.0*X(M1)
       ZT=Z(M1)
       WRITE(6,399)N
  399 FORMAT(1X,'PROBLEM NO.:',I3,//)
      WRITE(6,401)
  401 FORMAT(3X,'T',6X,'DS',8X,'Z0',8X,'ZT',7X,'DX1',7X,'DX',
     1             7X,'G',/)
      WRITE(6,402)T,DS,Z0,ZT,DX1,DX,G
  402 FORMAT(1X,F5.1,2X,F6.2,2X,F8.1,2X,F9.3,2X,F6.3,2X,F6.3,
     1       4X,E9.3,//)
      WRITE(6,403)
  403 FORMAT(4X,'H(MM)',8X,'D(MM)',/)
      WRITE(6,404)H,D
  404 FORMAT(1X,E11.6,2X,E11.6,//)
      IF(IND.LT.1)GO TO 500
      WRITE(6,405)
  405 FORMAT(5X,'X(MM)',6X,'Z(MM)',7X,'U(VALUE)',/)
      WRITE(6,406)X0,Z0
  406 FORMAT(1X,F9.1,2X,F9.1,/)
      WRITE(6,407)(X(I),Z(I),U(I),I=1,M1)
  407 FORMAT(1X,F10.4,2X,F10.4,3X,F11.8,/)
      WRITE(6,499)
  499 FORMAT(1H1)
  500 CONTINUE
  408 STOP
      END
```

```
CODE     2864 BYTES      PLT + DATA  121056 BYTES
STACK     296 BYTES      DIAG TABLES    228 BYTES       TOTAL  124444 BYTES
COMPILATION SUCCESSFUL
```

Appendix A 2.7.1

Local Truncation Error of Explicit Euler Method

For the initial value problem

$$y' = \frac{dy}{dx} = F(x,y) \quad , \quad y(x_0) = y_0 \qquad \text{(A 2.7.1)}$$

where prime denotes differentiation with respect to the independent variable x, the local truncation error for explicit Euler method is given by

$$\tfrac{1}{2}h^2 F'(x,y)\Big|_{x=\bar{x}_n} \qquad \text{(A 2.7.2)}$$

where h is the step-length of integration,

$x_n < \bar{x}_n < x_n+h$, and

$x_n \leqslant x \leqslant x_n+h$ is an interval being considered.

For a fixed interval $a \leqslant x \leqslant b$, the absolute value of the local truncation error for any h is bounded by $M\frac{h^2}{2}$ where M is the maximum of $|F'(x,y)|$ in this interval.

The primary difficulty in estimating the local truncation error is that of obtaining a "good" value of M . Consider the differential equations system of the drop shaped shell:

$$\frac{du}{dx} = -\frac{u}{x} + \frac{z}{A} = F_1(x,u,z) \quad , \qquad \text{(A 2.7.3a)}$$

$$\frac{dz}{dx} = \frac{u}{\sqrt{(1-u^2)}} = F_2(x,u,z) \quad , \qquad \text{(A 2.7.3b)}$$

it follows that

$$\frac{d^2u}{dx^2} = F_1'(x,u,z) = \frac{u}{x^2} - \frac{1}{x}\frac{du}{dx} = \frac{1}{x}\left(\frac{2u}{x} - \frac{z}{A}\right), \qquad \text{(A 2.7.4)}$$

and

$$\frac{d^2z}{dx^2} = F_2'(x,u,z) = \frac{d}{du}\left\{\frac{u}{\sqrt{(1-u^2)}}\right\}\frac{du}{dx} = \frac{1}{(1-u^2)^{3/2}}\cdot\left\{\frac{z}{A} - \frac{u}{x}\right\}. \qquad \text{(A 2.7.5)}$$

Restricting interest to the local truncation error in z, this is given by

$$\frac{1}{2}(DX)^2 \cdot \left.\frac{d^2z}{dx^2}\right|_{x=\bar{x}_n}, \qquad \text{(A 2.7.6)}$$

where interval $x_n \leqslant x \leqslant x_n + DX$ is being considered, and $\left.\frac{d^2z}{dx^2}\right|_{x=\bar{x}_n}$ is the maximum value of $\frac{d^2z}{dx^2}$ evaluated in this interval. As $\frac{d^2z}{dx^2} = \frac{1}{(1-u^2)^{3/2}}\left\{\frac{z}{A} - \frac{u}{x}\right\}$ from equation (A 2.7.5) one then substitutes this in equation (A 2.7.6).

Then M = maximum value of $\left|\frac{1}{(1-u^2)^{3/2}}\left\{\frac{z}{A} - \frac{u}{x}\right\}\right|$ is the interval under consideration. For an estimate of M one has to consider the following:

$\frac{1}{(1-u^2)^{3/2}}$ is a maximun when u is a maximum and where $0 < u < 1$;

$\left\{\frac{z}{A} - \frac{u}{x}\right\}$ is a maximum for a maximum value of z, minimum value of u and maximum value of x.

With this in mind, the computer program written for explicit Euler method (see appendix A 2.4.2) is suitably modified and used in evaluating the local truncation error of the coordinates of the first part of shell. In the example considered, the parameter-values used are Design head = 1000.0 mm, Thickness = 4.0 mm, Design stress = 0.15 MN/m^2, Unit weight of fluid = 11.61 KN/m^3 with step-lengths DX, DZ = 8.0 mm, 4.0 mm and 2.0 mm respectively. The results of this exercise are as in Table (A 2.7.1). From this table it is observed that the local truncation errors decrease as the step-lengths get smaller. This is in support of the expectation that the numerical results get better as step-lengths decrease, thus implying smaller local truncation errors. Unfortunately the errors are still large enough to cast doubt even on the second decimal value of the z values computed. Possibly the situation can be improved upon by considering smaller step-lengths values. Finally, notice that the evaluation of the truncation errors for z values can be extended quite easily to the remaining part of the shell and the errors of u values will also have to be considered in a more complete investigation of this method of integration.

a)

x value (mm)	z value (mm)	local truncation error of z (mm)
8	0.0001	0.62477
16	0.6212	0.48200
24	1.8748	0.44988
32	3.7855	0.45293
40	6.3937	0.47914
48	9.7601	0.52946
56	13.9745	0.61297
64	19.1708	0.75146
72	25.5568	0.99725

DX = DZ = 8.0 mm

b)

x value (mm)	z value (mm)	local truncation error of z (mm)
8	0.1550	0.11717
16	0.9332	0.10048
24	2.3496	0.09850
32	4.4327	0.10204
40	7.2281	0.10999
48	10.8037	0.12321
56	15.2599	0.14416
64	20.7483	0.17834
72	27.5088	0.23876

DX = DZ = 4.0 mm

c)

x value (mm)	z value (mm)	local truncation error of z (mm)
8	0.2325	0.02441
16	1.0897	0.02261
24	2.5881	0.02288
32	4.7584	0.02411
40	7.6489	0.02627
48	11.3313	0.02965
56	15.9118	0.03490
64	21.5517	0.04338
72	28.5091	0.05835

DX = DZ = 2.0 mm

TABLE A 2.7.1

APPENDIX A3.4.1

This is omitted since the computer program is a slightly modified form of the one listed in Appendix A2.4.3. Some extra statements are introduced which allowed the evaluation and listing of the other symmetric half of the shell.

APPENDIX A3.4.2

```
EDINBURGH FORTRAN(G) COMPILER VERSION 50.16
 ******** APPENDIX A 3.4.2    ******************

      C       A FORTRAN COMPUTER PROGRAM USED FOR PLOTTING THE VARYING
      C       SIZES OF THE DROP SHAPE DUE TO VARYING DESIGN HEAD.
      C       X AND Y ARE LOCATIONS IN WHICH COORDS. ARE STORED.
      C       PLOTS,SCALE,AXIS AND LINE ARE NAMES OF SUBROUTINES IN
      C       CALCOMP PACKAGE.
      C
      C
              DIMENSION X(1000),Y(1000)
              DO 100 I=1,283
              READ(5,10)X(I),Y(I)
        100   CONTINUE
        10    FORMAT(1X,F10.4,2X,F10.4)
              CALL PLOTS('A.B SOFOLUWE J.C.M.B',20,50)
              CALL SCALE(Y,10.,283,1)
              CALL SCALE(X,10.,283,1)
              CALL AXIS(0.,0.,'X-AXIS',-6,10.,0.,-5.0,1.)
              CALL AXIS(5.,0.,'Z-AXIS',+6,10.,90.,Y(1),1.)
              CALL LINE(X,Y,283,1,1,243)
              STOP
              END


CODE      818 BYTES       PLT + DATA    8224 BYTES
STACK     208 BYTES       DIAG TABLES     60 BYTES       TOTAL    9310 BYTES
COMPILATION SUCCESSFUL
```

APPENDIX A 4.2.1

Membrane theory can be used to derive the equilibrium equations of a shell of revolution[17]. By considering the equilibrium of a portion of a shell above a parallel circle defined by an angle ϕ, see Figure 4.2.1, if the resultant of the external load on that part of the shell is R unit of force (acting vertically downwards), then for equilibrium

$$2\pi r_0 N_\phi \sin\phi + R = 0 \qquad \text{(A 4.2.1a)},$$

resolving the forces vertically. From this equation

$$N_\phi = \frac{-R}{2\pi r_0 \sin\phi} \qquad \text{(A 4.2.1b)} .$$

Substituting this for N_ϕ in the equation

$$\frac{N_\phi}{r_1} + \frac{N_\theta}{r_2} + Z = 0 \qquad \text{(A 4.2.2)}$$

where N_ϕ and N_θ are the stress resultants in the meridional and parallel circle directions respectively at the level defined by angle ϕ, and Z is the total external normal force intensity acting positive inwards, one can obtain N_θ . Hence the two forces N_ϕ and N_θ can be obtained successively using equations (A 4.2.1) and (A 4.2.2).

APPENDICES A4.3.1 → A4.3.2

The Fortran computer programs of these Appendices are almost identical. The difference is in the field of evaluated and listed stress resultants. In one, the resultants are given correct to one decimal place, and in the other they are given correct to eight decimal places. For this reason, only Appendix A4.3.1 is listed.

APPENDIX A4.3.1

```
     EDINBURGH FORTRAN(G) COMPILER VERSION 50.16
C******** APPENDIX A 4.3.1 *******************

C
C
C       A FORTRAN COMPUTER PROGRAM PERFORMING
C       MEMBRANE EVALUATION OF STRESS RESULTANTS FOR
C       THE CASE OF  LOADING EQUAL  OR DIFFERENT FROM THE   DESIGN HEAD
C ***********************************************************************
C ***********************************************************************
C ***********************************************************************
C    PROGRAM WITH  D.P. ACCURACY
C    DX1 IS INTRODUCED TO PROGRAM
C    MODIFIED EULER METHOD USED IN INTEGRATION
C     ***********************************************
C     ***********************************************
C
C       PARAMETERS
C
C     ZO=DEPTH OVER SHELL, DESIGN HEAD
C     T=SHELL THICKNESS
C     DS= DESIGN STRESS
C     G=GAMMA, UNIT WEIGHT OF EXTERNAL FLUID
C     DX= ELEMENTAL INCREMENT IN X COORDS
C     DZ= ELEMENTAL INCREMENT IN Z COORDS
C     DXX=SECOND ELEMENTAL INCREMENT IN X COORDS
C     IND= INDICATOR FOR WHETHER COORDS ARE WRITTEN OR NOT, (0 OR 1)
C     *****************************************************************
C
      REAL*8 X(3000),XB(3000),Z(3000),U(3000),SA(3000),
     1  DUDX(3000),DZDX(3000),DWDZ(3000),DXDZ(3000),CSA(3000),
     1  AMZ(3000),W(3000),SMX(3000),DFX(3000),DFZ(3000),
     1  R1(3000),R2(3000),PHI(3000),THETA(3000),SUM(3000),
     2  T,DS,G,DX,DZ,ANF,A,V,H,D,HRD,ZT,DX1,DXX,CONST,VSM
       COMMON/AAA/X,XB,Z,U,SA
       COMMON/AAB/DUDX,DZDX,DWDZ,DXDZ,CSA
       COMMON/AAC/AMZ,W,SMX,DFX,DFZ
       COMMON/AAD/R1,R2,PHI,THETA,SUM
C     *********************
C     NUMBER OF PROBLEMS
C     *********************
C
      READ(5,80)NP,DX1,CONST
   80 FORMAT(I4,2X,F6.3,F8.1)
      DO 500 N=1,NP
C
C     *************************************
C     PARAMETERS, WITH ZO,T AND DS VARYING
C     *************************************
C
      READ(5,90)ZO,T,DS,G,DX,DZ,DXX,IND
   90 FORMAT(F8.1,2X,F5.1,2X,F6.2,2X,E9.3,2X,F6.3,2X,F6.3,2X,F6.3,2X,I1)
       ANF=DS*T
       A=ANF/G
C
C     ************************************
C     SETTING X COORDINATES AT 0
C     ZO REPRESENTS THE APEX OF THE SHELL
C     ************************************
C
      XO=0.
       R10=2.*A/ZO
       R20=R10
C
C     *******************************************
C     COORDINATES AT THE END OF THE FIRST ELEMENT
C       OF THE MERIDIAN CURVE
C     *******************************************
C
      X(1)=DX1
      Z(1)=ZO*(1.+DX1*DX1/(4.*A))
```

```
      U(1)=ZO*DX1/(2.*A)
      DUDX(1)=Z(1)/A-U(1)/X(1)
      DZDX(1)=U(1)/DSQRT(1.-U(1)*U(1))
       R1(1)=1.0/(DUDX(1))
       R2(1)=X(1)/(U(1))
       SUM(1)=-G*(Z(1)+CONST)*X(1)*DX1
       PHI(1)=SUM(1)/(X(1)*U(1))
       THETA(1)=-(G*(Z(1)+CONST)+PHI(1)/R1(1))*R2(1)
C
C     **************************
C     CENTROID OF FIRST ELEMENT
C     **************************
C
      XB(1)=X(1)/3.
      DO 100 I=2,1000
C
C     ***************************************
C     GENERATION OF COORDINATES UP TO END OF
C       ELEMENTAL INCREMENTS ON X AXIS
C     ***************************************
C
      M1=I
      X(I)=X(1)+(I-1)*DX
      U(I)=U(I-1)+DUDX(I-1)*DX
      Z(I)=Z(I-1)+DZDX(I-1)*DX
      DUDX(I)=Z(I)/A-U(I)/X(I)
      DZDX(I)=U(I)/DSQRT(1.-U(I)*U(I))
       U(I)=U(I-1)+(DUDX(I-1)+DUDX(I))/2.*DX
       Z(I)=Z(I-1)+(DZDX(I-1)+DZDX(I))/2.*DX
       DUDX(I)=Z(I)/A-U(I)/X(I)
       DZDX(I)=U(I)/DSQRT(1.-U(I)*U(I))
       R1(I)=1.0/(DUDX(I))
       R2(I)=X(I)/(U(I))
       SUM(I)=SUM(I-1)-G*(Z(I)+CONST)*X(I)*DX
       PHI(I)=SUM(I)/(X(I)*U(I))
       THETA(I)=-(G*(Z(I)+CONST)+PHI(I)/R1(I))*R2(I)
       XB(I)=(X(I)*X(I)+X(I)*X(I-1)+X(I-1)*X(I-1))/(3.*(X(I)+X(I-1)))
      P=0.707
      IF(U(I).GE.P)GO TO 101
  100 CONTINUE
C
C     *********************************************
C     CHANGE OVER TO ELEMENTAL INCREMENTS ON Z AXIS
C     *********************************************
C
  101 W(M1)=DSQRT(1.-U(M1)*U(M1))
      DWDZ(M1)=DSQRT(1.-W(M1)*W(M1))/X(M1)-Z(M1)/A
      DXDZ(M1)=W(M1)/DSQRT(1.-W(M1)*W(M1))
      L1=M1+1
      DO 200 J=L1,2000
      M2=J
C
C     ***********************************
C     GENERATION OF COORDINATES ON Z AXIS
C     ***********************************
C
      Z(J)=Z(J-1)+DZ
      W(J)=W(J-1)+DWDZ(J-1)*DZ
      X(J)=X(J-1)+DXDZ(J-1)*DZ
      DWDZ(J)=DSQRT(1.-W(J)*W(J))/X(J)-Z(J)/A
      DXDZ(J)=W(J)/DSQRT(1.-W(J)*W(J))
       W(J)=W(J-1)+(DWDZ(J-1)+DWDZ(J))/2.*DZ
       U(J)=DSQRT(1.-W(J)*W(J))
       X(J)=X(J-1)+(DXDZ(J-1)+DXDZ(J))/2.*DZ
       DWDZ(J)=DSQRT(1.-W(J)*W(J))/X(J)-Z(J)/A
       DXDZ(J)=W(J)/DSQRT(1.-W(J)*W(J))
       R1(J)=-1.0/(DWDZ(J))
       R2(J)=X(J)/DSQRT(1.-W(J)*W(J))
       SUM(J)=SUM(J-1)-G*(Z(J)+CONST)*X(J)*(X(J)-X(J-1))
```

```
       PHI(J)=SUM(J)/(X(J)*U(J))
       THETA(J)=-(G*(Z(J)+CONST)+PHI(J)/R1(J))*R2(J)
C
C     ***********************
C     CENTROID OF EACH ELEMENT
C     ***********************
C
      XB(J)=(X(J)*X(J)+X(J)*X(J-1)+X(J-1)*X(J-1))/(3.*(X(J)+X(J-1)))
      IF(X(J).LT.X(J-1))GO TO 190
      D=2.*X(J)
  190 Q=-0.707
      IF(W(J).LE.Q)GO TO 201
  200 CONTINUE
C
C     *********************************************
C     RETURN TO GENERATION OF COORDINATES ON X AXIS
C     *********************************************
C
  201 U(M2)=DSQRT(1.-W(M2)*W(M2))
      DUDX(M2)=Z(M2)/A-U(M2)/X(M2)
      DZDX(M2)=-U(M2)/DSQRT(1.-U(M2)*U(M2))
      L2=M2+1
      DO 300 K=L2,3000
      M3=K
      X(K)=X(K-1)-DXX
      U(K)=U(K-1)-DUDX(K-1)*DXX
      Z(K)=Z(K-1)-DZDX(K-1)*DXX
      IF(Z(K).LT.Z(K-1))GO TO 217
      DUDX(K)=Z(K)/A-U(K)/X(K)
      DZDX(K)=-U(K)/DSQRT(1.-U(K)*U(K))
       U(K)=U(K-1)-(DUDX(K-1)+DUDX(K))/2.*DXX
       Z(K)=Z(K-1)-(DZDX(K-1)+DZDX(K))/2.*DXX
       DUDX(K)=Z(K)/A-U(K)/X(K)
       DZDX(K)=-U(K)/DSQRT(1.-U(K)*U(K))
       R1(K)=1.0/(DUDX(K))
       R2(K)=X(K)/(U(K))
       SUM(K)=SUM(K-1)+G*(Z(K)+CONST)*X(K)*DXX
       PHI(K)=SUM(K)/(X(K)*U(K))
       THETA(K)=-(G*(Z(K)+CONST)+PHI(K)/R1(K))*R2(K)
C
C     ***********************
C     CENTROID OF EACH ELEMENT
C     ***********************
C
      XB(K)=(X(K)*X(K)+X(K)*X(K-1)+X(K-1)*X(K-1))/(3.*(X(K)+X(K-1)))
      R=-0.00001
      IF(DZDX(K).GE.R)GO TO 301
  300 CONTINUE
  217     K=K-1
           M3=K
C
C     ******************************************
C     SURFACE AREA OF FIRST ELEMENT OF REVOLUTION
C     ******************************************
C
  301 SA(1)=3.142*X(1)*DSQRT((Z(1)-Z0)**2+(X(1)-X0)**2)
C
C     ************************************************
C     CROSS SECTIONAL AREA OF FIRST ELEMENT OF REVOLUTION
C     ************************************************
C
      CSA(1)=X(1)*(Z(1)-Z0)
C
C     ***************************
C     VOLUME=3.142*(SUM OF AMZ(I))
C     ***************************
C
      AMZ(1)=(X(1)/3.)*(X(1))*(Z(1)-Z0)
      DFZ(1)=Z(1)-Z0
      DFX(1)=X(1)-X0
      SMX(1)=X(1)
```

```
      DO 400 I=2,M3
      SMX(I)=(X(I)+X(I-1))
      DFX(I)=(X(I)-X(I-1))
      DFZ(I)=(Z(I)-Z(I-1))
C
C     ******************
C     TOTAL SURFACE AREA
C     ******************
C
      SA(I)=SA(I-1)+3.142*SMX(I)*DSQRT(DFX(I)*DFX(I)+
     1             DFZ(I)*DFZ(I))
C
C     **************************
C     TOTAL CROSS SECTIONAL AREA
C     **************************
C
      CSA(I)=CSA(I-1)+SMX(I)*DFZ(I)
      AMZ(I)=AMZ(I-1)+SMX(I)*DFZ(I)*XB(I)
  400 CONTINUE
C     *********************
C     VOL OF SHELL MATERIAL
C     *************************
          VSM=SA(M3)*T
C
C     **************************
C     VOLUME ENCLOSED BY SHELL
C     **************************
C
      V=3.142*AMZ(M3)-VSM/2.
C
C     ***************
C     HEIGHT OF SHELL
C     ***************
C
      H=Z(M3)-Z0
       HRD=H/D
C
C     ***************************************
C     HEIGHT, FROM MEAN SEA LEVEL TO SEA BED
C     ***************************************
C
      ZT=Z(M3)
      WRITE(6,399)N
  399 FORMAT(1X,'PROBLEM NO.:',I3,//)
      WRITE(6,401)
  401 FORMAT(3X,'T',6X,'DS',8X,'Z0',8X,'ZT',7X,'DX1',7X,'DX',
     1             7X,'DZ',7X,'DXX',7X,'G',/)
      WRITE(6,402)T,DS,Z0,ZT,DX1,DX,DZ,DXX,G
  402 FORMAT(1X,F5.1,2X,F6.2,2X,F8.1,2X,F9.3,2X,F6.3,2X,F6.3,
     1       4X,F6.3,4X,F6.3,4X,E9.3,//)
      WRITE(6,403)
  403 FORMAT(4X,'H(MM)',8X,'D(MM)',8X,'SA(MM**2)',8X,
     1    'CSA(MM**2)',8X,'VSM(MM**3)',8X,'V(MM**3)',8X,'HRD',/)
      WRITE(6,404)H,D,SA(M3),CSA(M3),VSM,V,HRD
  404 FORMAT(1X,E11.6,2X,E11.6,2X,E14.9,2X,E15.10,2X,
     1             E15.10,2X,E15.10,2X,F10.6,//)
       WRITE(6,440)CONST
 440   FORMAT(8X,'CONST VALUE =',3X,F8.1,//)
      IF(IND.LT.1)GO TO 500
      WRITE(6,405)
  405 FORMAT(5X,'X(MM)',6X,'Z(MM)',10X,'R1',9X,'R2',
     1 10X,'PHI',10X,'THETA',/)
      WRITE(6,406)X0,Z0,R10,R20
  406 FORMAT(1X,F10.4,2X,F10.4,2X,F9.1,2X,F9.1,/)
      WRITE(6,407)(X(I),Z(I),R1(I),R2(I),PHI(I),THETA(I),I=1,M3)
  407 FORMAT(1X,F10.4,2X,F10.4,2X,F9.1,2X,F9.1,2X,F12.1,2X,F12.1,/)
      WRITE(6,499)
  499 FORMAT(1H1)
  500 CONTINUE
  408 STOP
      END
```

```
CODE   13398 BYTES      PLT + DATA    1560 BYTES
                            TABLES     540 BYTES      TOTAL  16378 BYTES
```

Appendix A.4.3.3. (a)

z (mm)

A 0.0

Range of z-values for which $N_\phi \neq 0.60$ and $N_\theta \neq 0.60$

B 0.8211

Range of z-values for which $N_\phi = N_\theta = 0.60$

C 161.0191

Range of z-values for which $N_\phi \neq 0.60$ and $N_\theta \neq 0.60$

D 174.9909

N_ϕ, N_θ in N/mm

Step-lengths DX = 1.000 mm
DZ = 4.000 mm
DXX = 1.000 mm

Overall percentage of z-values in which $N_\phi \neq 0.60$ and $N_\theta \neq 0.60$

= 9.23

Appendix A.4.3.3. (b)

z (mm)

A 0.0

Range of z-values for which $N_\phi \neq 0.60$ and $N_\theta \neq 0.60$

B 0.1744

Range of z-values for which $N_\phi = N_\theta = 0.60$

C 163.8830

Range of z-values for which $N_\phi \neq 0.60$ and $N_\theta \neq 0.60$

D 174.9858

N_ϕ, N_θ in N/mm

Step-lengths DX = 0.500 mm
DZ = 4.000 mm
DXX = 0.500 mm

Overall percentage of z-values in which $N_\phi \neq 0.60$ and $N_\theta \neq 0.60$

= 6.8886

Appendix A.4.3.3. (c)

z (mm)

A 0.0

Range of z-values for which $N_\phi \neq 0.60$ and $N_\theta \neq 0.60$

B 0.0436

Range of z-values for which $N_\phi = N_\theta = 0.60$

C 165.1747

Range of z-values for which $N_\phi \neq 0.60$ and $N_\theta \neq 0.60$

D 174.9854

N_ϕ, N_θ in N/mm

Step-lengths DX = 0.250 mm
DZ = 4.000 mm
DXX = 0.250 mm

Overall percentage of z-values in which $N_\phi \neq 0.60$ and $N_\theta \neq 0.60$

= 5.9633

Appendix A.4.3.3. (d)

z (mm)

A 0.0

Range of z-values for which $N_\phi \neq 0.60$ and $N_\theta \neq 0.60$

B 0.0109

Range of z-values for which $N_\phi = N_\theta = 0.60$

C 174.1061

Range of z-values for which $N_\phi \neq 0.60$ and $N_\theta \neq 0.60$

D 175.1244

N_ϕ, N_θ in N/mm

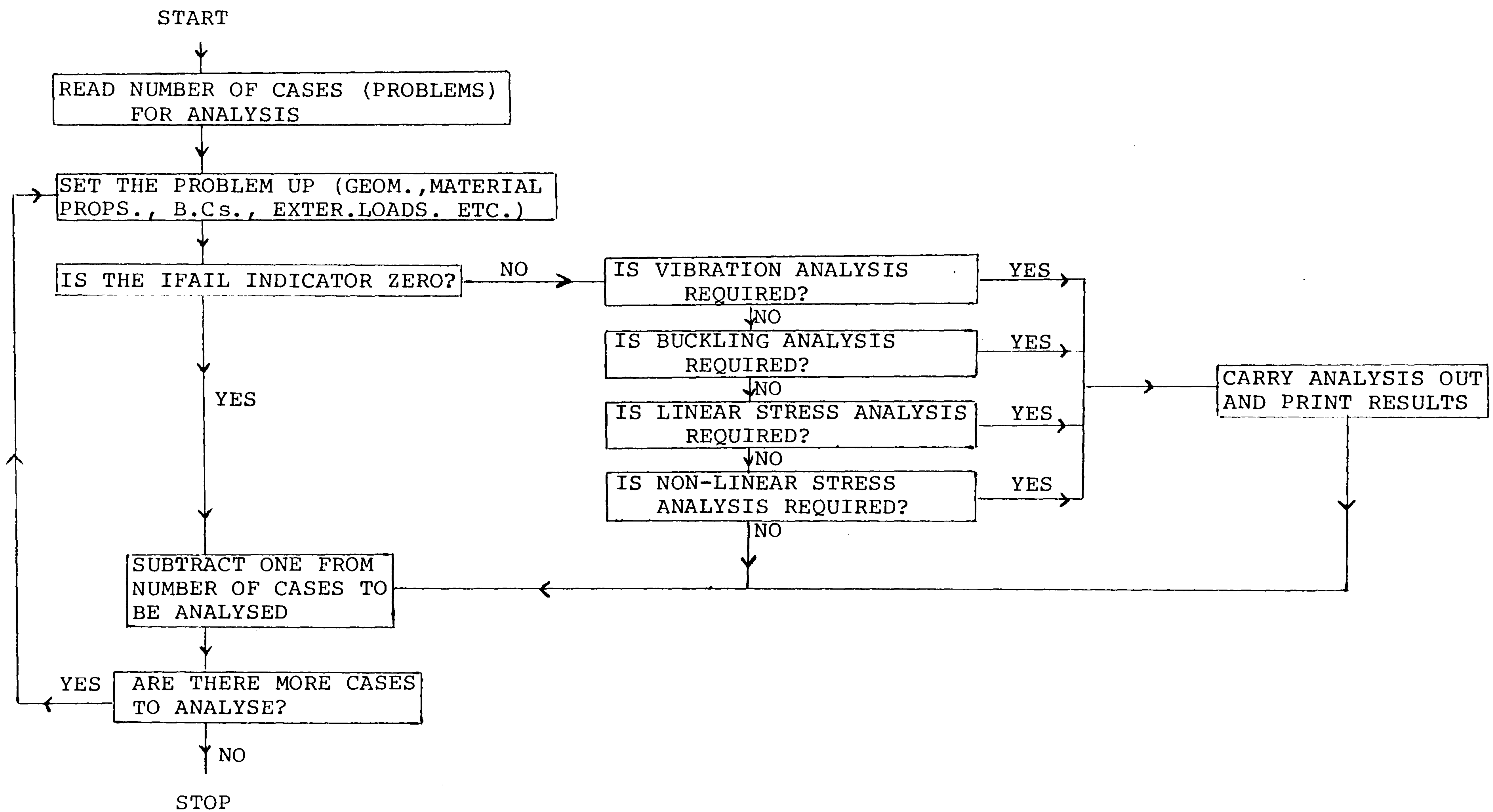
Step-lengths DX = 0.125 mm
DZ = 0.125 mm
DXX = 0.125 mm

Overall percentage of z-values in which $N_\phi \neq 0.60$ and $N_\theta \neq 0.60$

= 0.5912

<u>APPENDIX A5.3.1</u>

<u>A FLOW DIAGRAM OF MISTRY'S COMPUTER PROGRAM</u>



APPENDIX A5.3.2

The flow diagram of the computer program is given in Appendix A5.3.1. Due to the size of the program it is omitted. Any interested person should obtain the permission of Mr. John Mistry, Department of Mechanical Engineering, Applied Mechanics Division, University of Liverpool, who will also supply relevant information.

APPENDIX A5.3.3

A LIST OF INPUT DATA REQUIRED BY MISTRY'S COMPUTER PROGRAM

NCASES	Number of cases for analyses
TJOB	Job title
ICASE	Type of analysis: 1 for linear vibration frequency analysis 2 for linear buckling pressure 3 for non-linear buckling pressure by method A 4 for non-linear buckling pressure by method B 5 for non-linear stress analysis 6 for linear stress analysis
NSEGS	Number of shell segments in the structure, maximum number allowed is 10
E	Young's modulus
ANU	Poisson's Ratio
RO	Shell wall density
P	Initial uniform pressure, positive for internal pressure
DP	Pressure increment (used for ICASE = 3, 4, 5)
PMAX	Maximum pressure (used for ICASE = 3, 4, 5)
PATM	Atmospheric pressure
ROG	Liquid density × gravitational acceleration
DZA	Depth of end A of shell structure
DZ	Increment in depth
DZAMAX	Maximum depth of end A (the two extreme ends of the shell structure are labelled ends A and B)
T(I)	Shell wall thickness of I^{th} segment
NMIN	Minimum value of n
NMAX	Maximum value of n. (Note: above two parameters give a range of N(wave number) for which the analysis is to be carried out. Both should be zero for ICASE = 5 or 6).

IBCA(I) / IBCB(I)	Control integers for boundary conditions at ends A and B for variables u(axial), v(circumferential), w(radial) and β(rotation). 0 for fixed and 1 for free.
ENDLOD(I,J)	External loads on shell ends A and B corresponding to deflections/displacements u,v,w and β. Loads are required for the free (constraint free) boundary displacements if loaded externally in the respective directions.
ITYPE	Segment type: 1 for cones or cylinders, 2 for constant curvature, e.g. spherical caps and toroids, 3 for a general shaped shell
IELS	Number of elements in the segment. Maximum allowed in the whole structure is 125.
R	equals 1 for equal length elements, less than 1 for decreasing length elements and greater than 1 for increasing length elements. NOT required for ITYPE = 3.
X,R	end coordinates (axial and radial) of the segment
XC,RC	coordinates of the centre of curvature (NOT required for ITYPE = 3).
R1	Radius of curvature (NOT required for ITYPE = 3).
	REPEAT (X,R) as often as required

APPENDIX A5.4.1

```
C ********* APPENDIX A 5.4.1 *************************
   1
 LINEAR STRESS ANALYSIS OF AN ECHIDOME MARCH 1979
   6    1
00.30000E+1000.38000E+0000.10000E+05-0.98100E+04-0.98100E+04-0.39240E+06
 0.00000E+00-0.98100E+04 0.10000E+01 0.10000E+01 0.10000E+02
  .00040E+01
   0    0
   1    1    1    1    0    0    0    0
00.00000E+0000.00000E+0000.00000E+0000.00000E+00
00.00000E+0000.00000E+0000.00000E+0000.00000E+00
  .00000E+00  .00000E+00
   3   60
  .00030E+00  .00800E+00
  .00070E+00  .01200E+00
  .00120E+00  .01600E+00
  .00190E+00  .02000E+00
  .00280E+00  .02400E+00
  .00380E+00  .02800E+00
  .00510E+00  .03200E+00
  .00650E+00  .03600E+00
  .00800E+00  .04000E+00
  .00980E+00  .04400E+00
  .01180E+00  .04800E+00
  .01410E+00  .05200E+00
  .01650E+00  .05600E+00
  .01930E+00  .06000E+00
  .02230E+00  .06400E+00
  .02570E+00  .06800E+00
  .02940E+00  .07200E+00
  .03340E+00  .07580E+00
  .03740E+00  .07920E+00
  .04140E+00  .08230E+00
  .04540E+00  .08500E+00
  .04940E+00  .08750E+00
  .05340E+00  .08970E+00
  .05740E+00  .09170E+00
  .06140E+00  .09350E+00
  .06540E+00  .09500E+00
  .06940E+00  .09640E+00
  .07340E+00  .09750E+00
  .07740E+00  .09850E+00
  .08140E+00  .09920E+00
  .08540E+00  .09980E+00
  .08940E+00  .10020E+00
  .09340E+00  .10040E+00
  .09740E+00  .10050E+00
  .10140E+00  .10030E+00
  .10540E+00  .10000E+00
  .10940E+00  .09950E+00
  .11340E+00  .09880E+00
  .11740E+00  .09790E+00
  .12140E+00  .09690E+00
  .12540E+00  .09550E+00
  .12940E+00  .09400E+00
  .13340E+00  .09220E+00
  .13740E+00  .09020E+00
  .14140E+00  .08780E+00
  .14540E+00  .08520E+00
  .14940E+00  .08210E+00
  .15340E+00  .07860E+00
  .15740E+00  .07460E+00
  .16100E+00  .07060E+00
  .16400E+00  .06660E+00
  .16670E+00  .06260E+00
  .16890E+00  .05860E+00
  .17080E+00  .05460E+00
  .17240E+00  .05060E+00
  .17370E+00  .04660E+00
  .17470E+00  .04260E+00
  .17540E+00  .03860E+00
  .17580E+00  .03460E+00
  .17590E+00  .03060E+00
```

APPENDIX A 6.2.2.1

Basic Computer Program used by PET and FLUKE for scanning, storing and printing measured voltages

```
5     K1 = 27
10    DIM A(K1), B(K1)
20    SW = 59457
30    DD = 59459
40    SA = 16
50    OPEN 6,6
60    PRINT #6, "%VROS5FOT2,"
70    POKE DD, 255
75    T = TI
80    FOR  I = 0  TO  27
90    POKE SW, SA+I
100   PRINT  #6, "?"
110   INPUT #6, A$ : IF ST < > 0  THEN 110
120   A(I) = VAL(A$)
125   B(I) = (4* A(I))/(2.15*2.5)
130   PRINT I, A(I), B(I)
140   NEXT I
142   T = (TI-T)/60
143   PRINT "TIME FOR SCAN ..."; T; "SECS"
200   INPUT "CASSETTE (C) OR PRINTER (P)"; A$
210   CH = -(4*(A$="P")+(A$="C"))
220   OPEN 4, CH, 1
230   FOR I = 0  TO  K1
```

```
240     PRINT # 4, I, A(I), B(I)
250     NEXT I
252     PRINT # 4, "TIME FOR SCAN ..."; T; "SECS"
255     PRINT # 4, "END OF SCAN ..."; "TIME IS", TI$;
        CHR$(010); CHR$(010); CHR$(010)
260     CLOSE 3
```

APPENDIX A6.2.4.1

EDINBURGH FORTRAN(G) COMPILER VERSION 50.16

```
C ********APPENDIX A 6.2.4.1  ******************************
      C        A FORTRAN COMPUTER PROGRAM FOR CALCULATING
      C        PRINCIPAL STRAINS AND STRESSES FROM EXPERIMENTAL DATA.
      C          PROGRAM ALSO EVALUATES MERIDIONAL AND PARALLEL
      C          CIRCLE STRESSES
      C
      C
      C       YM=YOUNG'S MODULUS
      C       PR=POISSON RATIO
      C       EPS( )=MEASURED VALUE OF STRAIN
      C       PSTP=PRINCIPAL STRAIN P
      C       PSTQ=PRINCIPAL STRAIN Q
      C       ANG=ANGLE
      C       PSRP=PRINCIPAL STRESS P
      C       PSRQ=PRINCIPAL STRESS Q
      C       XGAM=STRESS IN PARALLEL CIRCLE DIRECTION
      C       YGAM=STRESS IN MERIDIONAL DIRECTION
      C
      C
      C
                REAL*8 EPS(3),YM,PR,SUM1,SUM2,SUM3,SUM4,SUM5,
              1 A(27),B(27),C(27),D(27),AVC(27),AVD(27),
              2 XEPS,YEPS,XGAM,YGAM
                READ(5,1)YM,PR
           1    FORMAT(F10.2,2X,F8.3)
                WRITE(6,20)YM,PR
                DO 111 I=1,25,3
                READ(5,112)A(I),A(I+1),A(I+2)
         112    FORMAT(E15.3,3X,E15.3,3X,E15.3)
         111    CONTINUE
                DO 113 I=1,25,3
                READ(5,112)B(I),B(I+1),B(I+2)
         113    CONTINUE
                DO 114 I=1,27
                C(I)=B(I)-A(I)
                D(I)=C(I)*10.**6
                WRITE(6,115)A(I),B(I),C(I)
         115    FORMAT(2X,E15.3,3X,E15.3,5X,E15.3,/)
         114    CONTINUE
                DO 120 I=3,27,3
                EPS(1)=D(I)
                EPS(2)=D(I-1)
                EPS(3)=D(I-2)
                XEPS=C(I)
                YEPS=C(I-2)
                SUM1=(EPS(1)-EPS(2))**2
                SUM2=(EPS(2)-EPS(3))**2
                SUM3=DSQRT(SUM1+SUM2)
                SUM4=1.4142*SUM3/2.0
                PSTP=(EPS(1)+EPS(3))/2.0+SUM4
                PSTQ=(EPS(1)+EPS(3))/2.0-SUM4
                ANG=DATAN((2.0*EPS(2)-EPS(1)-EPS(3))/(EPS(1)-EPS(3)))
          20    FORMAT(2X,16HYOUNG'S MODULUS=,F10.2,4X,
              1 14HPOISSON RATIO=,F12.8,/)
                WRITE(6,25)EPS(1),EPS(2),EPS(3)
          25    FORMAT(2X,5HEPS1=,2X,F14.6,2X,5HEPS2=,2X,F14.6,2X,5HEPS3=,
              1 2X,F14.6,//)
                WRITE(6,21)PSTP,PSTQ,ANG
          21    FORMAT(2X,5HPSTP=,2X,F12.4,2X,5HPSTQ=,2X,F12.4,
              1 2X,7H2ANGLE=,2X,F8.4,/)
                SUM5=1.0-PR*PR
                PSTP=PSTP/1.0E+06
                PSTQ=PSTQ/1.0E+06
                PSRP=YM*(PSTP+PR*PSTQ)/SUM5
                PSRQ=YM*(PSTQ+PR*PSTP)/SUM5
                WRITE(6,30)PSRP,PSRQ
          30    FORMAT(2X,5HPSRP=,4X,F12.4,2X,5HPSRQ=,4X,F12.4,//)
                XGAM=YM*(XEPS+PR*YEPS)/SUM5
                YGAM=YM*(YEPS+PR*XEPS)/SUM5
                WRITE(6,70)XEPS,YEPS
          70    FORMAT(10X,5HXEPS=,4X,F18.10,4X,
              1 5HYEPS=,4X,F18.10,/)
                WRITE(6,71)XGAM,YGAM
          71    FORMAT(10X,5HXGAM=,4X,F12.4,4X,5HYGAM=,
              1 4X,F12.4,///)
         120    CONTINUE
      C
      C       ********************************************************
```

```
C       AVERAGED STRAIN-STRESS   EVALUATIONS
C     ***********************************************************
C
C
C          AVC( )=AVERAGED STRAIN VALUE ALONG A PARALLEL CIRCLE
C          EPS( )=AVERAGED MEASURED STRAIN VALUE
C
C
        DO 150 I=1,3
        AVC(I)=(C(I)+C(I+3)+C(I+6))/3.
        AVD(I)=AVC(I)*10.**6
  150   CONTINUE
        DO 160 I=10,12
        AVC(I)=(C(I)+C(I+3)+C(I+6))/3.
        AVD(I)=AVC(I)*10.**6
  160   CONTINUE
        DO 170 I=19,21
        AVC(I)=(C(I)+C(I+3)+C(I+6))/3.
        AVD(I)=AVC(I)*10.**6
  170   CONTINUE
        WRITE(6,200)
  200   FORMAT(10X,'AVERAGED CALCULATIONS RESULTS',//)
        DO 202 I=1,19,9
        WRITE(6,201)AVC(I),AVC(I+1),AVC(I+2)
  201   FORMAT(10X,E15.3,/,10X,E15.3,/,10X,E15.3,//)
  202   CONTINUE
        DO 220 I=3,21,9
        EPS(1)=AVD(I)
        EPS(2)=AVD(I-1)
        EPS(3)=AVD(I-2)
        XEPS=AVC(I)
        YEPS=AVC(I-2)
        SUM1=(EPS(1)-EPS(2))**2
        SUM2=(EPS(2)-EPS(3))**2
        SUM3=DSQRT(SUM1+SUM2)
        SUM4=1.4142*SUM3/2.0
        PSTP=(EPS(1)+EPS(3))/2.0+SUM4
        PSTQ=(EPS(1)+EPS(3))/2.0-SUM4
        ANG=DATAN((2.0*EPS(2)-EPS(1)-EPS(3))/(EPS(1)-EPS(3)))
        WRITE(6,25)EPS(1),EPS(2),EPS(3)
        WRITE(6,21)PSTP,PSTQ,ANG
C
C
C
        SUM5=1.0-PR*PR
        PSTP=PSTP/1.0E+06
        PSTQ=PSTQ/1.0E+06
        PSRP=YM*(PSTP+PR*PSTQ)/SUM5
        PSRQ=YM*(PSTQ+PR*PSTP)/SUM5
        WRITE(6,30)PSRP,PSRQ
        XGAM=YM*(XEPS+PR*YEPS)/SUM5
        YGAM=YM*(YEPS+PR*XEPS)/SUM5
        WRITE(6,70)XEPS,YEPS
        WRITE(6,71)XGAM,YGAM
  220   CONTINUE
      STOP
      END
```

```
CODE     5198 BYTES      PLT + DATA    2864 BYTES
STACK     376 BYTES      DIAG TABLES    280 BYTES      TOTAL   8718 BYTES
COMPILATION SUCCESSFUL
```

APPENDIX A6.2.4.2

```
C***********  APPENDIX A 6.2.4.2  *****************************

        LINEAR STRESS ANALYSIS  OF  BAIG- ECHIDOME APRIL 1979

    TYPE OF ANALYSIS    6

    NUMBER OF SEGMENTS IN THE STRUCTURE    1

    E= 0.80000D 10  NU= 0.36000D 00  RO= 0.10000D 05

      ATMOSPHERIC PRESSURE 0.00000D 00
      LIQUID DENSITY * G =-0.98100D 04
      DEPTH OF END A 0.13725D 02
      INCREMENT IN DEPTH 0.00000D 00
      MAXIMUM DEPTH OF END A 0.13725D 02

         SEGMENT THICKNESSES
         0.25000D-02

         ANALYSIS FOR N VARYING FROM     0  TO     0

    BOUNDARY CONDITIONS
             1    1    1    1
             0    0    0    0

    EXTERNAL LOADS AT ENDS 0.00000D 00 0.00000D 00 0.00000D 00 0.00000D 00
                           0.00000D 00 0.00000D 00 0.00000D 00 0.00000D 00

                  NODE DATA FOR THE SEGMENTS IN THE STRUCTURE

    SEGMENT    1  IS A GENERAL CURVE

    NO. OF ELEMENTS IN THE SEGMENT   65
```

NODE	X	R
1	0.000000000D 00	0.000000000D 00
2	0.100000000D-03	0.800000000D-02
3	0.500000000D-03	0.160000000D-01
4	0.120000000D-02	0.240000000D-01
5	0.220000000D-02	0.320000000D-01
6	0.340000000D-02	0.400000000D-01
7	0.500000000D-02	0.480000000D-01
8	0.680000000D-02	0.560000000D-01
9	0.890000000D-02	0.640000000D-01
10	0.114000000D-01	0.720000000D-01
11	0.141000000D-01	0.800000000D-01
12	0.172000000D-01	0.880000000D-01
13	0.207000000D-01	0.960000000D-01
14	0.245000000D-01	0.104000000D 00
15	0.287000000D-01	0.112000000D 00
16	0.333000000D-01	0.120000000D 00
17	0.384000000D-01	0.128000000D 00
18	0.440000000D-01	0.136000000D 00
19	0.501000000D-01	0.144000000D 00
20	0.569000000D-01	0.152000000D 00

21	0.64300000D-01	0.16000000D 00
22	0.72500000D-01	0.16800000D 00
23	0.80500000D-01	0.17500000D 00
24	0.88500000D-01	0.18150000D 00
25	0.96500000D-01	0.18730000D 00
26	0.10450000D 00	0.19250000D 00
27	0.11250000D 00	0.19730000D 00
28	0.12050000D 00	0.20160000D 00
29	0.12850000D 00	0.20540000D 00
30	0.13650000D 00	0.20890000D 00
31	0.14450000D 00	0.21190000D 00
32	0.15250000D 00	0.21460000D 00
33	0.16050000D 00	0.21690000D 00
34	0.16850000D 00	0.21890000D 00
35	0.17650000D 00	0.22050000D 00
36	0.18450000D 00	0.22180000D 00
37	0.19250000D 00	0.22270000D 00
38	0.20050000D 00	0.22330000D 00
39	0.20850000D 00	0.22360000D 00
40	0.21650000D 00	0.22350000D 00
41	0.22450000D 00	0.22310000D 00
42	0.23250000D 00	0.22230000D 00
43	0.24050000D 00	0.22120000D 00
44	0.24850000D 00	0.21970000D 00
45	0.25650000D 00	0.21790000D 00
46	0.26450000D 00	0.21560000D 00
47	0.27250000D 00	0.21300000D 00
48	0.28050000D 00	0.20990000D 00
49	0.28850000D 00	0.20640000D 00
50	0.29650000D 00	0.20230000D 00
51	0.30450000D 00	0.19770000D 00
52	0.31250000D 00	0.19250000D 00
53	0.32050000D 00	0.18660000D 00
54	0.32850000D 00	0.17980000D 00
55	0.33650000D 00	0.17210000D 00
56	0.34370000D 00	0.16410000D 00
57	0.35000000D 00	0.15610000D 00
58	0.35550000D 00	0.14810000D 00
59	0.36020000D 00	0.14010000D 00
60	0.36430000D 00	0.13210000D 00
61	0.36770000D 00	0.12410000D 00
62	0.37050000D 00	0.11610000D 00
63	0.37270000D 00	0.10810000D 00
64	0.37430000D 00	0.10010000D 00
65	0.37530000D 00	0.92100000D-01
66	0.37570000D 00	0.84100000D-01

SEGMENT NO. 1

ELEMENT	R(1)	R(2)	PHI(1)	PHI(2)	SL
1	0.0000000D 00	0.8000000D-02	0.8928384D 02	0.8928384D 02	0.8000625D-02
2	0.8000000D-02	0.1600000D-01	0.8713759D 02	0.8713759D 02	0.8009994D-02
3	0.1600000D-01	0.2400000D-01	0.8499936D 02	0.8499936D 02	0.8030567D-02
4	0.2400000D-01	0.3200000D-01	0.8287498D 02	0.8287498D 02	0.8062258D-02
5	0.3200000D-01	0.4000000D-01	0.8146923D 02	0.8146923D 02	0.8089499D-02
6	0.4000000D-01	0.4800000D-01	0.7869007D 02	0.7869007D 02	0.8158431D-02
7	0.4800000D-01	0.5600000D-01	0.7731962D 02	0.7731962D 02	0.8200000D-02
8	0.5600000D-01	0.6400000D-01	0.7529170D 02	0.7529170D 02	0.8271034D-02
9	0.6400000D-01	0.7200000D-01	0.7264598D 02	0.7264598D 02	0.8381527D-02
10	0.7200000D-01	0.8000000D-01	0.7135046D 02	0.7135046D 02	0.8443341D-02
11	0.8000000D-01	0.8800000D-01	0.6881865D 02	0.6881865D 02	0.8579627D-02
12	0.8800000D-01	0.9600000D-01	0.6637062D 02	0.6637062D 02	0.8732125D-02
13	0.9600000D-01	0.1040000D 00	0.6459228D 02	0.6459228D 02	0.8856636D-02
14	0.1040000D 00	0.1120000D 00	0.6230053D 02	0.6230053D 02	0.9035486D-02
15	0.1120000D 00	0.1200000D 00	0.6010110D 02	0.6010110D 02	0.9228218D-02

16	0.1200000D 00	0.1280000D 00	0.5748249D 02	0.5748249D 02	0.9487360D-02
17	0.1280000D 00	0.1360000D 00	0.5500798D 02	0.5500798D 02	0.9765244D-02
18	0.1360000D 00	0.1440000D 00	0.5267448D 02	0.5267448D 02	0.1006032D-01
19	0.1440000D 00	0.1520000D 00	0.4963546D 02	0.4963546D 02	0.1049952D-01
20	0.1520000D 00	0.1600000D 00	0.4723117D 02	0.4723117D 02	0.1089771D-01
21	0.1600000D 00	0.1680000D 00	0.4429268D 02	0.4429268D 02	0.1145600D-01
22	0.1680000D 00	0.1750000D 00	0.4118593D 02	0.4118593D 02	0.1063015D-01
23	0.1750000D 00	0.1815000D 00	0.3909386D 02	0.3909386D 02	0.1030776D-01
24	0.1815000D 00	0.1873000D 00	0.3594211D 02	0.3594211D 02	0.9881295D-02
25	0.1873000D 00	0.1925000D 00	0.3302387D 02	0.3302387D 02	0.9541488D-02
26	0.1925000D 00	0.1973000D 00	0.3096376D 02	0.3096376D 02	0.9329523D-02
27	0.1973000D 00	0.2016000D 00	0.2825803D 02	0.2825803D 02	0.9082401D-02
28	0.2016000D 00	0.2054000D 00	0.2540772D 02	0.2540772D 02	0.8856636D-02
29	0.2054000D 00	0.2089000D 00	0.2362938D 02	0.2362938D 02	0.8732125D-02
30	0.2089000D 00	0.2119000D 00	0.2055605D 02	0.2055605D 02	0.8544004D-02
31	0.2119000D 00	0.2146000D 00	0.1864954D 02	0.1864954D 02	0.8443341D-02
32	0.2146000D 00	0.2169000D 00	0.1603994D 02	0.1603994D 02	0.8324062D-02
33	0.2169000D 00	0.2189000D 00	0.1403624D 02	0.1403624D 02	0.8246211D-02
34	0.2189000D 00	0.2205000D 00	0.1130993D 02	0.1130993D 02	0.8158431D-02
35	0.2205000D 00	0.2218000D 00	0.9229886D 01	0.9229886D 01	0.8104937D-02
36	0.2218000D 00	0.2227000D 00	0.6418787D 01	0.6418787D 01	0.8050466D-02
37	0.2227000D 00	0.2233000D 00	0.4289153D 01	0.4289153D 01	0.8022468D-02
38	0.2233000D 00	0.2236000D 00	0.2147585D 01	0.2147585D 01	0.8005623D-02
39	0.2236000D 00	0.2235000D 00	-0.7161599D 00	-0.7161599D 00	0.8000625D-02
40	0.2235000D 00	0.2231000D 00	-0.2862405D 01	-0.2862405D 01	0.8009994D-02
41	0.2231000D 00	0.2223000D 00	-0.5710593D 01	-0.5710593D 01	0.8039900D-02
42	0.2223000D 00	0.2212000D 00	-0.7829077D 01	-0.7829077D 01	0.8075271D-02
43	0.2212000D 00	0.2197000D 00	-0.1061966D 02	-0.1061966D 02	0.8139410D-02
44	0.2197000D 00	0.2179000D 00	-0.1268038D 02	-0.1268038D 02	0.8200000D-02
45	0.2179000D 00	0.2156000D 00	-0.1603994D 02	-0.1603994D 02	0.8324062D-02
46	0.2156000D 00	0.2130000D 00	-0.1800416D 02	-0.1800416D 02	0.8411896D-02
47	0.2130000D 00	0.2099000D 00	-0.2118135D 02	-0.2118135D 02	0.8579627D-02
48	0.2099000D 00	0.2064000D 00	-0.2362938D 02	-0.2362938D 02	0.8732125D-02
49	0.2064000D 00	0.2023000D 00	-0.2713514D 02	-0.2713514D 02	0.8989438D-02
50	0.2023000D 00	0.1977000D 00	-0.2989890D 02	-0.2989890D 02	0.9228218D-02
51	0.1977000D 00	0.1925000D 00	-0.3302387D 02	-0.3302387D 02	0.9541488D-02
52	0.1925000D 00	0.1866000D 00	-0.3640877D 02	-0.3640877D 02	0.9940322D-02
53	0.1866000D 00	0.1798000D 00	-0.4036454D 02	-0.4036454D 02	0.1049952D-01
54	0.1798000D 00	0.1721000D 00	-0.4390531D 02	-0.4390531D 02	0.1110360D-01
55	0.1721000D 00	0.1641000D 00	-0.4801279D 02	-0.4801279D 02	0.1076290D-01
56	0.1641000D 00	0.1561000D 00	-0.5177957D 02	-0.5177957D 02	0.1018283D-01
57	0.1561000D 00	0.1481000D 00	-0.5549148D 02	-0.5549148D 02	0.9708244D-02
58	0.1481000D 00	0.1401000D 00	-0.5956576D 02	-0.5956576D 02	0.9278470D-02
59	0.1401000D 00	0.1321000D 00	-0.6286486D 02	-0.6286486D 02	0.8989438D-02
60	0.1321000D 00	0.1241000D 00	-0.6697451D 02	-0.6697451D 02	0.8692526D-02
61	0.1241000D 00	0.1161000D 00	-0.7070995D 02	-0.7070995D 02	0.8475848D-02
62	0.1161000D 00	0.1081000D 00	-0.7462375D 02	-0.7462375D 02	0.8296987D-02
63	0.1081000D 00	0.1001000D 00	-0.7869007D 02	-0.7869007D 02	0.8158431D-02
64	0.1001000D 00	0.9210000D-01	-0.8287498D 02	-0.8287498D 02	0.8062258D-02
65	0.9210000D-01	0.8410000D-01	-0.8713759D 02	-0.8713759D 02	0.8009994D-02

DEGREES OF FREEDOM, IDF= 194 MAXIMUM BAND SIZE, IBAND= 6

DISPLACEMENT NUMBERS ASSIGNED TO THE NODES

NODE	U	V	W	BETA
1	1	0	2	0
2	3	0	4	5
3	6	0	7	8
4	9	0	10	11
5	12	0	13	14
6	15	0	16	17
7	18	0	19	20

8	21	0	22	23
9	24	0	25	26
10	27	0	28	29
11	30	0	31	32
12	33	0	34	35
13	36	0	37	38
14	39	0	40	41
15	42	0	43	44
16	45	0	46	47
17	48	0	49	50
18	51	0	52	53
19	54	0	55	56
20	57	0	58	59
21	60	0	61	62
22	63	0	64	65
23	66	0	67	68
24	69	0	70	71
25	72	0	73	74
26	75	0	76	77
27	78	0	79	80
28	81	0	82	83
29	84	0	85	86
30	87	0	88	89
31	90	0	91	92
32	93	0	94	95
33	96	0	97	98
34	99	0	100	101
35	102	0	103	104
36	105	0	106	107
37	108	0	109	110
38	111	0	112	113
39	114	0	115	116
40	117	0	118	119
41	120	0	121	122
42	123	0	124	125
43	126	0	127	128
44	129	0	130	131
45	132	0	133	134
46	135	0	136	137
47	138	0	139	140
48	141	0	142	143
49	144	0	145	146
50	147	0	148	149
51	150	0	151	152
52	153	0	154	155
53	156	0	157	158
54	159	0	160	161
55	162	0	163	164
56	165	0	166	167
57	168	0	169	170
58	171	0	172	173
59	174	0	175	176
60	177	0	178	179
61	180	0	181	182
62	183	0	184	185
63	186	0	187	188
64	189	0	190	191
65	192	0	193	194
66	0	0	0	0

DISPLACEMENT AND STRESS RESULTANTS OF 'SEGMENT 1 SHELL TYPE 3 DZA= 0.13725D 02

ELM	S	U	V	W	β	N_ϕ	N_θ	$N_{\phi\theta}$	M_ϕ	M_θ	$M_{\phi\theta}$
1	0.400D-02	0.908D-05	0.000D 00	-0.886D-03	-0.794D-04	-0.157D 05	-0.156D 05	0.000D 00	0.153D 00	0.262D 00	0.000D 00
2	0.120D-01	0.382D-04	0.000D 00	-0.885D-03	-0.135D-04	-0.156D 05	-0.156D 05	0.000D 00	-0.151D 00	-0.426D-01	0.000D 00
3	0.200D-01	0.672D-04	0.000D 00	-0.883D-03	0.106D-03	-0.156D 05	-0.157D 05	0.000D 00	-0.145D 00	-0.107D 00	0.000D 00
4	0.281D-01	0.959D-04	0.000D 00	-0.881D-03	0.114D-03	-0.157D 05	-0.157D 05	0.000D 00	0.147D 00	0.108D-01	0.000D 00
5	0.361D-01	0.113D-03	0.000D 00	-0.879D-03	-0.251D-04	-0.157D 05	-0.157D 05	0.000D 00	-0.432D-01	-0.838D-02	0.000D 00
6	0.443D-01	0.152D-03	0.000D 00	-0.873D-03	0.116D-03	-0.157D 05	-0.158D 05	0.000D 00	0.526D-01	-0.806D-02	0.000D 00
7	0.525D-01	0.169D-03	0.000D 00	-0.869D-03	-0.121D-03	-0.157D 05	-0.158D 05	0.000D 00	0.282D 00	0.125D 00	0.000D 00
8	0.607D-01	0.195D-03	0.000D 00	-0.862D-03	-0.149D-03	-0.157D 05	-0.157D 05	0.000D 00	-0.160D 00	-0.327D-01	0.000D 00
9	0.690D-01	0.230D-03	0.000D 00	-0.851D-03	-0.714D-05	-0.157D 05	-0.156D 05	0.000D 00	0.909D-01	0.338D-01	0.000D 00
10	0.774D-01	0.245D-03	0.000D 00	-0.845D-03	-0.222D-03	-0.157D 05	-0.155D 05	0.000D 00	0.817D-01	0.583D-01	0.000D 00
11	0.859D-01	0.278D-03	0.000D 00	-0.832D-03	-0.417D-04	-0.157D 05	-0.155D 05	0.000D 00	-0.327D 00	-0.113D 00	0.000D 00
12	0.946D-01	0.309D-03	0.000D 00	-0.820D-03	0.105D-03	-0.157D 05	-0.155D 05	0.000D 00	0.776D-01	0.170D-01	0.000D 00
13	0.103D 00	0.330D-03	0.000D 00	-0.810D-03	-0.129D-03	-0.156D 05	-0.155D 05	0.000D 00	0.311D 00	0.124D 00	0.000D 00
14	0.112D 00	0.358D-03	0.000D 00	-0.794D-03	-0.275D-03	-0.156D 05	-0.154D 05	0.000D 00	0.132D 00	0.709D-01	0.000D 00
15	0.121D 00	0.383D-03	0.000D 00	-0.777D-03	-0.300D-03	-0.156D 05	-0.152D 05	0.000D 00	-0.112D 00	-0.168D-01	0.000D 00
16	0.131D 00	0.413D-03	0.000D 00	-0.757D-03	-0.117D-03	-0.156D 05	-0.150D 05	0.000D 00	-0.203D 00	-0.650D-01	0.000D 00
17	0.140D 00	0.441D-03	0.000D 00	-0.738D-03	-0.260D-04	-0.155D 05	-0.150D 05	0.000D 00	-0.254D-01	-0.747D-02	0.000D 00
18	0.150D 00	0.466D-03	0.000D 00	-0.719D-03	-0.131D-04	-0.155D 05	-0.150D 05	0.000D 00	-0.152D 00	-0.540D-01	0.000D 00
19	0.161D 00	0.498D-03	0.000D 00	-0.695D-03	0.171D-03	-0.155D 05	-0.151D 05	0.000D 00	-0.963D-02	-0.126D-01	0.000D 00
20	0.171D 00	0.521D-03	0.000D 00	-0.674D-03	-0.227D-04	-0.155D 05	-0.152D 05	0.000D 00	0.200D 00	0.731D-01	0.000D 00
21	0.183D 00	0.550D-03	0.000D 00	-0.646D-03	-0.128D-03	-0.155D 05	-0.152D 05	0.000D 00	0.916D-01	0.387D-01	0.000D 00
22	0.194D 00	0.578D-03	0.000D 00	-0.614D-03	-0.223D-03	-0.154D 05	-0.150D 05	0.000D 00	0.313D 00	0.122D 00	0.000D 00
23	0.204D 00	0.595D-03	0.000D 00	-0.588D-03	-0.587D-03	-0.154D 05	-0.147D 05	0.000D 00	0.812D-01	0.509D-01	0.000D 00
24	0.214D 00	0.621D-03	0.000D 00	-0.550D-03	-0.300D-03	-0.154D 05	-0.143D 05	0.000D 00	-0.445D 00	-0.150D 00	0.000D 00
25	0.224D 00	0.643D-03	0.000D 00	-0.516D-03	-0.421D-04	-0.154D 05	-0.142D 05	0.000D 00	-0.156D-01	-0.435D-02	0.000D 00
26	0.233D 00	0.656D-03	0.000D 00	-0.492D-03	-0.205D-03	-0.153D 05	-0.141D 05	0.000D 00	0.101D 00	0.418D-01	0.000D 00
27	0.242D 00	0.674D-03	0.000D 00	-0.458D-03	-0.111D-03	-0.153D 05	-0.140D 05	0.000D 00	-0.238D 00	-0.828D-01	0.000D 00
28	0.251D 00	0.691D-03	0.000D 00	-0.424D-03	0.354D-04	-0.153D 05	-0.140D 05	0.000D 00	0.631D-01	0.219D-01	0.000D 00
29	0.260D 00	0.700D-03	0.000D 00	-0.402D-03	-0.140D-03	-0.152D 05	-0.140D 05	0.000D 00	0.298D-01	0.135D-01	0.000D 00
30	0.269D 00	0.716D-03	0.000D 00	-0.364D-03	-0.767D-05	-0.152D 05	-0.139D 05	0.000D 00	-0.124D-01	-0.433D-02	0.000D 00
31	0.277D 00	0.723D-03	0.000D 00	-0.339D-03	-0.165D-03	-0.152D 05	-0.139D 05	0.000D 00	0.195D 00	0.729D-01	0.000D 00
32	0.286D 00	0.734D-03	0.000D 00	-0.304D-03	-0.240D-03	-0.152D 05	-0.138D 05	0.000D 00	0.161D 00	0.611D-01	0.000D 00
33	0.294D 00	0.740D-03	0.000D 00	-0.276D-03	-0.371D-03	-0.152D 05	-0.136D 05	0.000D 00	0.302D-01	0.152D-01	0.000D 00
34	0.302D 00	0.748D-03	0.000D 00	-0.238D-03	-0.282D-03	-0.152D 05	-0.133D 05	0.000D 00	-0.929D-01	-0.308D-01	0.000D 00
35	0.310D 00	0.751D-03	0.000D 00	-0.208D-03	-0.256D-03	-0.151D 05	-0.131D 05	0.000D 00	-0.140D 00	-0.485D-01	0.000D 00
36	0.318D 00	0.756D-03	0.000D 00	-0.170D-03	-0.114D-03	-0.151D 05	-0.130D 05	0.000D 00	-0.522D-01	-0.182D-01	0.000D 00
37	0.326D 00	0.758D-03	0.000D 00	-0.140D-03	-0.215D-03	-0.151D 05	-0.129D 05	0.000D 00	0.217D 00	0.789D-01	0.000D 00
38	0.335D 00	0.758D-03	0.000D 00	-0.110D-03	-0.313D-03	-0.151D 05	-0.127D 05	0.000D 00	-0.641D-01	-0.225D-01	0.000D 00
39	0.343D 00	0.759D-03	0.000D 00	-0.700D-04	-0.160D-03	-0.151D 05	-0.126D 05	0.000D 00	-0.164D 00	-0.590D-01	0.000D 00
40	0.351D 00	0.757D-03	0.000D 00	-0.405D-04	-0.116D-03	-0.151D 05	-0.125D 05	0.000D 00	-0.127D 00	-0.460D-01	0.000D 00
41	0.359D 00	0.753D-03	0.000D 00	-0.252D-05	-0.329D-05	-0.151D 05	-0.124D 05	0.000D 00	-0.961D-02	-0.348D-02	0.000D 00
42	0.367D 00	0.749D-03	0.000D 00	0.256D-04	-0.106D-03	-0.151D 05	-0.124D 05	0.000D 00	0.119D 00	0.423D-01	0.000D 00
43	0.375D 00	0.742D-03	0.000D 00	0.630D-04	-0.150D-03	-0.152D 05	-0.123D 05	0.000D 00	0.157D 00	0.554D-01	0.000D 00
44	0.383D 00	0.735D-03	0.000D 00	0.914D-04	-0.281D-03	-0.152D 05	-0.121D 05	0.000D 00	-0.425D-01	-0.182D-01	0.000D 00
45	0.391D 00	0.724D-03	0.000D 00	0.136D-03	-0.142D-03	-0.152D 05	-0.119D 05	0.000D 00	0.472D-01	0.151D-01	0.000D 00
46	0.400D 00	0.714D-03	0.000D 00	0.163D-03	-0.356D-03	-0.153D 05	-0.117D 05	0.000D 00	0.168D 00	0.553D-01	0.000D 00
47	0.408D 00	0.699D-03	0.000D 00	0.205D-03	-0.329D-03	-0.153D 05	-0.113D 05	0.000D 00	-0.844D-02	-0.890D-02	0.000D 00
48	0.417D 00	0.685D-03	0.000D 00	0.237D-03	-0.350D-03	-0.154D 05	-0.110D 05	0.000D 00	-0.174D 00	-0.696D-01	0.000D 00
49	0.426D 00	0.664D-03	0.000D 00	0.281D-03	-0.113D-03	-0.155D 05	-0.108D 05	0.000D 00	-0.143D 00	-0.542D-01	0.000D 00
50	0.435D 00	0.644D-03	0.000D 00	0.313D-03	-0.196D-03	-0.156D 05	-0.105D 05	0.000D 00	0.171D 00	0.566D-01	0.000D 00
51	0.444D 00	0.621D-03	0.000D 00	0.350D-03	-0.298D-03	-0.157D 05	-0.102D 05	0.000D 00	0.393D-01	0.548D-02	0.000D 00
52	0.454D 00	0.593D-03	0.000D 00	0.389D-03	-0.253D-03	-0.159D 05	-0.981D 04	0.000D 00	-0.144D 00	-0.600D-01	0.000D 00
53	0.464D 00	0.558D-03	0.000D 00	0.431D-03	-0.201D-03	-0.161D 05	-0.942D 04	0.000D 00	0.293D 00	0.982D-01	0.000D 00
54	0.475D 00	0.523D-03	0.000D 00	0.469D-03	-0.853D-03	-0.164D 05	-0.872D 04	0.000D 00	0.928D 00	0.299D 00	0.000D 00
55	0.486D 00	0.480D-03	0.000D 00	0.519D-03	-0.180D-02	-0.168D 05	-0.718D 04	0.000D 00	0.119D 01	0.345D 00	0.000D 00
56	0.496D 00	0.437D-03	0.000D 00	0.575D-03	-0.297D-02	-0.173D 05	-0.468D 04	0.000D 00	0.119D 01	0.275D 00	0.000D 00
57	0.506D 00	0.389D-03	0.000D 00	0.635D-03	-0.374D-02	-0.180D 05	-0.135D 04	0.000D 00	0.208D 00	-0.136D 00	0.000D 00
58	0.516D 00	0.333D-03	0.000D 00	0.696D-03	-0.335D-02	-0.191D 05	0.220D 04	0.000D 00	-0.144D 01	-0.727D 00	0.000D 00
59	0.525D 00	0.282D-03	0.000D 00	0.737D-03	-0.143D-02	-0.204D 05	0.517D 04	0.000D 00	-0.434D 01	-0.166D 01	0.000D 00
60	0.534D 00	0.218D-03	0.000D 00	0.749D-03	0.324D-02	-0.221D 05	0.648D 04	0.000D 00	-0.807D 01	-0.266D 01	0.000D 00
61	0.542D 00	0.159D-03	0.000D 00	0.705D-03	0.102D-01	-0.239D 05	0.513D 04	0.000D 00	-0.108D 02	-0.304D 01	0.000D 00
62	0.551D 00	0.104D-03	0.000D 00	0.594D-03	0.186D-01	-0.259D 05	0.960D 03	0.000D 00	-0.110D 02	-0.229D 01	0.000D 00
63	0.559D 00	0.565D-04	0.000D 00	0.416D-03	0.255D-01	-0.277D 05	-0.495D 04	0.000D 00	-0.570D 01	0.452D 00	0.000D 00
64	0.567D 00	0.234D-04	0.000D 00	0.202D-03	0.263D-01	-0.293D 05	-0.102D 05	0.000D 00	0.826D 01	0.580D 01	0.000D 00
65	0.575D 00	0.528D-05	0.000D 00	0.298D-04	0.140D-01	-0.307D 05	-0.119D 05	0.000D 00	0.340D 02	0.139D 02	0.000D 00

APPENDIX A6.2.4.2 (contd.)

AXISYMMETRIC STRESSES AND EQUIVALENT STRESSES

SEGMENT NO. 1

ELEMENT	STATION	σ_θ (IN)	σ_θ (OUT)	σ_ϕ (IN)	σ_ϕ (OUT)	σ_{EQ} (IN)	σ_{EQ} (OUT)
1	0.4000312D-02	-0.6006564D 07	-0.6509404D 07	-0.6114275D 07	-0.6408205D 07	0.6061138D 07	0.6324769D 07
2	0.1200562D-01	-0.6293094D 07	-0.6211228D 07	-0.6402252D 07	-0.6112396D 07	0.6348377D 07	0.6027775D 07
3	0.2002590D-01	-0.6370530D 07	-0.6164167D 07	-0.6396906D 07	-0.6118381D 07	0.6383759D 07	0.6006754D 07
4	0.2807231D-01	-0.6286698D 07	-0.6307530D 07	-0.6123296D 07	-0.6406460D 07	0.6206610D 07	0.6222926D 07
5	0.3614819D-01	-0.6306159D 07	-0.6290076D 07	-0.6313734D 07	-0.6230762D 07	0.6309950D 07	0.6125965D 07
6	0.4427216D-01	-0.6316913D 07	-0.6301445D 07	-0.6227055D 07	-0.6327962D 07	0.6272467D 07	0.6180063D 07
7	0.5245137D-01	-0.6186537D 07	-0.6427035D 07	-0.6012066D 07	-0.6554412D 07	0.6101173D 07	0.6356980D 07
8	0.6068689D-01	-0.6293319D 07	-0.6230562D 07	-0.6436618D 07	-0.6128900D 07	0.6366178D 07	0.6045653D 07
9	0.6901317D-01	-0.6216198D 07	-0.6281015D 07	-0.6192282D 07	-0.6366761D 07	0.6204275D 07	0.6189592D 07
10	0.7742561D-01	-0.6163621D 07	-0.6275467D 07	-0.6196682D 07	-0.6353557D 07	0.6180218D 07	0.6180115D 07
11	0.8593709D-01	-0.6290887D 07	-0.6074111D 07	-0.6581114D 07	-0.5953243D 07	0.6440906D 07	0.5879813D 07
12	0.9459296D-01	-0.6190724D 07	-0.6223336D 07	-0.6186855D 07	-0.6335799D 07	0.6188790D 07	0.6145511D 07
13	0.1033873D 00	-0.6092702D 07	-0.6330746D 07	-0.5959743D 07	-0.6556385D 07	0.6027323D 07	0.6311727D 07
14	0.1123334D 00	-0.6083606D 07	-0.6219824D 07	-0.6125599D 07	-0.6378866D 07	0.6104710D 07	0.6165980D 07
15	0.1214653D 00	-0.6082631D 07	-0.6050350D 07	-0.6348955D 07	-0.6134660D 07	0.6220071D 07	0.5958006D 07
16	0.1308230D 00	-0.6073764D 07	-0.5949015D 07	-0.6423576D 07	-0.6032894D 07	0.6256010D 07	0.5856411D 07
17	0.1404493D 00	-0.6012993D 07	-0.5998650D 07	-0.6239671D 07	-0.6190870D 07	0.6129476D 07	0.5962038D 07
18	0.1503621D 00	-0.6060201D 07	-0.5956487D 07	-0.6349039D 07	-0.6056811D 07	0.6209660D 07	0.5872188D 07
19	0.1606421D 00	-0.6063721D 07	-0.6039452D 07	-0.6202978D 07	-0.6184498D 07	0.6134535D 07	0.5978128D 07
20	0.1713407D 00	-0.6021811D 07	-0.6162078D 07	-0.5996715D 07	-0.6380415D 07	0.6009302D 07	0.6138922D 07
21	0.1825175D 00	-0.6028553D 07	-0.6102782D 07	-0.6094952D 07	-0.6270811D 07	0.6062025D 07	0.6053233D 07
22	0.1935606D 00	-0.5892708D 07	-0.6126297D 07	-0.5876797D 07	-0.6478069D 07	0.5884769D 07	0.6174311D 07
23	0.2040295D 00	-0.5814223D 07	-0.5911927D 07	-0.6089694D 07	-0.6245669D 07	0.5956738D 07	0.5950351D 07
24	0.2141241D 00	-0.5849837D 07	-0.5561718D 07	-0.6580056D 07	-0.5726586D 07	0.6247037D 07	0.5510453D 07
25	0.2238355D 00	-0.5675308D 07	-0.5666956D 07	-0.6155918D 07	-0.6126002D 07	0.5930238D 07	0.5774551D 07
26	0.2332710D 00	-0.5604602D 07	-0.5684943D 07	-0.6032492D 07	-0.6225575D 07	0.5830335D 07	0.5838356D 07
27	0.2424769D 00	-0.5677110D 07	-0.5518151D 07	-0.6344814D 07	-0.5888594D 07	0.6038712D 07	0.5576822D 07
28	0.2514465D 00	-0.5580956D 07	-0.5623065D 07	-0.6046401D 07	-0.6167510D 07	0.5827636D 07	0.5778691D 07
29	0.2602408D 00	-0.5580774D 07	-0.5606772D 07	-0.6069419D 07	-0.6126540D 07	0.5840448D 07	0.5748365D 07
30	0.2688789D 00	-0.5582308D 07	-0.5573999D 07	-0.6101876D 07	-0.6078084D 07	0.5859394D 07	0.5706743D 07
31	0.2773726D 00	-0.5492757D 07	-0.5632681D 07	-0.5896069D 07	-0.6270976D 07	0.5705115D 07	0.5841941D 07
32	0.2857563D 00	-0.5449266D 07	-0.5566610D 07	-0.5922683D 07	-0.6231586D 07	0.5700737D 07	0.5791623D 07
33	0.2940414D 00	-0.5406890D 07	-0.5436013D 07	-0.6041920D 07	-0.6099860D 07	0.5750762D 07	0.5660949D 07
34	0.3022437D 00	-0.5359158D 07	-0.5299983D 07	-0.6154234D 07	-0.5975893D 07	0.5797728D 07	0.5532656D 07
35	0.3103754D 00	-0.5302396D 07	-0.5209209D 07	-0.6194525D 07	-0.5925361D 07	0.5800148D 07	0.5466171D 07
36	0.3184531D 00	-0.5225486D 07	-0.5190537D 07	-0.6106060D 07	-0.6005797D 07	0.5716865D 07	0.5507122D 07
37	0.3264896D 00	-0.5092412D 07	-0.5243805D 07	-0.5845155D 07	-0.6261683D 07	0.5507500D 07	0.5684933D 07
38	0.3345036D 00	-0.5111306D 07	-0.5068070D 07	-0.6112672D 07	-0.5989659D 07	0.5678598D 07	0.5450962D 07
39	0.3425068D 00	-0.5077576D 07	-0.4964237D 07	-0.6207816D 07	-0.5893483D 07	0.5726963D 07	0.5352971D 07
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43	0.3747019D 00	-0.4848164D 07	-0.4954448D 07	-0.5915440D 07	-0.6217636D 07	0.5460595D 07	0.5557729D 07
44	0.3828716D 00	-0.4843589D 07	-0.4808601D 07	-0.6116015D 07	-0.6034497D 07	0.5589502D 07	0.5390020D 07
45	0.3911337D 00	-0.4738848D 07	-0.4767826D 07	-0.6042230D 07	-0.6132787D 07	0.5507451D 07	0.5443020D 07
46	0.3995017D 00	-0.4612905D 07	-0.4719071D 07	-0.5941733D 07	-0.6265169D 07	0.5401336D 07	0.5519707D 07
47	0.4079974D 00	-0.4548237D 07	-0.4531150D 07	-0.6131612D 07	-0.6115416D 07	0.5513175D 07	0.5364352D 07
48	0.4166533D 00	-0.4473776D 07	-0.4340186D 07	-0.6316168D 07	-0.5982499D 07	0.5625969D 07	0.5221356D 07
49	0.4255141D 00	-0.4354469D 07	-0.4250355D 07	-0.6320380D 07	-0.6045198D 07	0.5602384D 07	0.5245841D 07
50	0.4346229D 00	-0.4164285D 07	-0.4272956D 07	-0.6060692D 07	-0.6389641D 07	0.5369805D 07	0.5507711D 07
51	0.4440078D 00	-0.4078934D 07	-0.4089451D 07	-0.6239157D 07	-0.6314664D 07	0.5487775D 07	0.5418645D 07
52	0.4537487D 00	-0.3979641D 07	-0.3864505D 07	-0.6480910D 07	-0.6205125D 07	0.5661099D 07	0.5300014D 07
53	0.4639686D 00	-0.3675132D 07	-0.3863749D 07	-0.6148968D 07	-0.6712310D 07	0.5358930D 07	0.5710545D 07
54	0.4747701D 00	-0.3199847D 07	-0.3773878D 07	-0.5657223D 07	-0.7438557D 07	0.4913343D 07	0.6322567D 07
55	0.4857034D 00	-0.2539245D 07	-0.3201837D 07	-0.5563589D 07	-0.7845960D 07	0.4824310D 07	0.6721908D 07
56	0.4961763D 00	-0.1608204D 07	-0.2136456D 07	-0.5780819D 07	-0.8058339D 07	0.5167925D 07	0.7134193D 07
57	0.5061218D 00	-0.6693854D 06	-0.4076099D 06	-0.7018125D 07	-0.7416867D 07	0.6708526D 07	0.7147822D 07
58	0.5156152D 00	0.1823660D 06	0.1577675D 07	-0.9009065D 07	-0.6245442D 07	0.9101618D 07	0.7121945D 07
59	0.5247491D 00	0.4736313D 06	0.3663649D 07	-0.1233268D 08	-0.3991834D 07	0.1257619D 08	0.6629894D 07
60	0.5335901D 00	0.3635478D 05	0.5147885D 07	-0.1656984D 08	-0.1079302D 07	0.1658805D 08	0.5814057D 07
61	0.5421743D 00	-0.8687485D 06	0.4970419D 07	-0.1992148D 08	0.7709439D 06	0.1950163D 08	0.4720216D 07
62	0.5505607D 00	-0.1814643D 07	0.2582419D 07	-0.2089477D 08	0.1915362D 06	0.2004914D 08	0.2571706D 07
63	0.5587884D 00	-0.1545770D 07	-0.2413506D 07	-0.1654833D 08	-0.5605762D 07	0.1583214D 08	0.4757105D 07
64	0.5668987D 00	0.1509007D 07	-0.9632881D 07	-0.3775569D 07	-0.1963286D 08	0.4714804D 07	0.1688466D 08
65	0.5749349D 00	0.8568580D 07	-0.1808694D 08	0.2036279D 08	-0.4489232D 08	0.1770829D 08	0.3901026D 08

DAMAGED TEXT IN ORIGINAL

APPENDIX A7.0.1

AN OPTIMUM FORM FOR UNDERWATER STORAGE VESSELS

by

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S U M M A R Y

An optimum form of design for large tanks suitable for underwater storage is described. Installation and operating procedures are outlined together with the relative merits of founding such structures on the sea bed or anchoring them as submerged floating vessels. The general features of loading are discussed in addition to the specific design criteria.

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presented at the I.A.S.S. symposium on "Recent developments in the field of liquified gas tanks" Delft, 16-20 June 1980.

OPTIMUM DESIGN VESSELS

A structure, in which a uniform pattern of internal resistance is set up in response to a system of applied loads, could be said to exhibit the characteristic of uniform strength under those circumstances. For a shell of revolution having uniform thickness this would be equivalent to a state of uniform stress existing at all points within the structure. Such a state of affairs in a particular elastic shell structure would exist normally for one system of applied forces only and any change in that system, either of arrangement or magnitude would result in a non-uniform pattern of stresses or internal forces being established unless the structure could adapt its shape accordingly. Shells of this type could occur in the form of complete chambers or domes and for the situation in which the uniform stress prevailing was equal to the "permissible" value for the material and structure type the design could be considered one of minimum weight and optimum form from a strength and stability point of view.

In order to contain a liquid under pressure such that uniform stressing exists in the contained wall the increase in pressure from the top or apex of the container downwards, due to the weight of the stored liquid, must be significant in relation to the liquid pressure at the apex. For these conditions it has been shown [1, 2, 3] that the shape of the container or shell must be similar to that of a drop of liquid on a plane surface. The apex pressure and membrane stress in the shell influence its shape - there being one shape for a given membrane stress, wall thickness and apex pressure.

The derivation of the drop shape can be traced back to Laplace's work [4] on capillarity with later refinements by Kelvin [5] and Milanković [6] and depends upon membrane theory (see Appendix 1 of item 10 in the list of references).

The above principles have been employed in the design of surface storage tanks for use with volatile liquids in the oil industry [7, 8, 9] and although these tanks were designed as tension skins some evidence was found that external pressures of the order of twice the apex design internal pressure could be sustained without signs of failure.

It has been suggested and supported by preliminary tests [10] that such a structural form could be appropriate to underwater operations especially as it would appear that there are precedents for this within the marine animal kingdom (namely the sea urchin of the phylum Echinodermata).

BASIC SELECTION PROCEDURE

The optimum shape corresponding to a particular apex pressure head (design head), design stress, and shell thickness, does not evolve from an exact solution of the fundamental equations and both numerical [1] and graphical [2] methods were proposed in consequence. These give comparable results [11] and a computer program [10], based on a modified form of the numerical method, has been developed which gives results for shape prediction in agreement with manual methods and computes surface area, cross-sectional area, and capacity of a shell.

The shape prediction program permits a whole range of shapes and capacities etc. to be generated for a series of design heads relating to a range of design stresses and shell thicknesses in a particular material. The information is storable in computer data banks and from it design curves can be produced easily to aid selection of a shell of optimum strength.

These curves relate

(a) capacity - operating depth,

(b) capacity - design head,

(c) maximum diameter - design head,

(d) (shell height/maximum diameter) - design head,

and (e) weight of material - design head;

all for a particular material and shell thickness.

Starting with a known operating depth and tank capacity required together with a knowledge of the mass density of the surrounding fluid a choice must be made of the material of construction, shell thickness, and design stress. Clearly, by varying these latter three parameters there exists the possibility of more than one design satisfying the capacity requirements at a particular operating depth.

The design curves (a) to (e) above could be used in the following way.

1. Consider a material, a range of design stresses, and a minimum practical thickness.
2. Determine (a) to see whether the required volume and operating depth fall within its bounds. If not, increment the material thickness using small practical steps until a new (a) is generated which embraces the desired values; the corresponding thickness represents the minimum to be employed in producing relations (b) to (e).
3. Choose an appropriate design stress from the new (a) and using it establish the design head from (b).

   Tank height = operating depth (synonymous with shell bottom) - design head
4. Obtain maximum diameter of tank from (c) and check it from (d).
5. Determine quantity of material to be used from (e).

The cost of a minimum thickness design is influenced by the amount of material used. However, other designs could be produced using thicknesses greater than the minimum with the final choice resting on overall construction costs consistent with durability and safety.

PRELIMINARY DESIGN OF A LARGE TANK

Consider an off-shore requirement for a 100 000 m^3 oil storage tank at a nominal operating depth of 85 m of sea water.

The two most likely materials of construction are some form of reinforced concrete, and steel. Purely for purposes of illustration a steel solution is examined here using a design stress of 200 MN/m^2 and a tank wall thickness of 40 mm. The curves relating volume and operating depth shown in Fig 1 encompass the specified requirements and consequently the curves of Figs 2 - 5 were established for the same material thickness.

Fig 1 gives an operating depth of 83.3 m for a tank capacity of 100 000 m^3 and design stress of 200 MN/m^2. The corresponding apex design head is found from Fig 2 as 42.0 m. The maximum height to diameter ratio is taken from Fig 4 as 0.64 which is equivalent to a maximum diameter of 64.2 m. This latter value is confirmed from Fig 3 and finally the amount of steel involved in the shell is obtained from Fig 5 as 380.0 m^3.

The co-ordinates for this tank design are generated by means of the shape prediction program. A finite element analysis program using ring elements [12] has been adapted to use these co-ordinates for predicting the behaviour of the tank under static heads.

Typical deformations of the tank predicted by the latter program are given in Fig 6 for an external apex pressure equivalent to approximately twice the design head.

INSTALLATION AND OPERATION

Growing attention is being paid to the inallation and operation of large storage tanks in the vicinity of off-shore drilling platforms.[13 -]

The tank considered above would be fabricated on shore at a coastal site, launched and towed in relatively calm weather to its location and sunk into position in a controlled manner.

The sinking procedure would involve ballasting the tank with oil until it was in a position to be connected via a universal joint at its apex to a column floating with its base tilted a little above the horizontal [15]. Further ballasting of the tank and column would sink them gradually into their final position with the column adopting a vertical attitude. The assistance of flotation barges and power winches would be required to control the descent and some equalisation of external and internal pressure on the tank could be achieved by opening the base of the tank to the sea when full of oil during the sinking stages.

The column would be of a length in excess of the design head and would carry service lines to and from the tank to the surface for onward connection to a ship. Mooring facilities for ships could be incorporated in the column, and the connection of the tank to the drilling rig would be by underwater pipeline.

The tank would operate with nominal pressure equalisation between inside and outside by means of opening it to the sea at the base and utilising some form of separating membrane between the oil and sea water [13, 14]. The oil would be stored additionally under a mean pressure of 10 m of water (say) to reduce the level of pumping power required in loading a ship. The net pressure distribution under operating conditions is indicated in Fig 7. However further forces with which the shell would have to contend are those due to pressure differences from the varying level of the oil/water interface in the tank; external current drag effects; external pressure caused by the inertia effect of the water mass against the shell arising from surface waves [16] (e.g. 100 year wave).

The tank considered here would be resting on the sea bed, the difference between the operating depth (lower level of curved shell, 83.3 m) and the mean water depth of 85 m being taken up by the base structure. The overall design would have to conform with the governing principles of codes of practice for marine structures [17, 18].

DISCUSSION

The preliminary tank design considered in this work is based on the assumption that the most critical loading condition would occur with the tank empty at the operating depth. In reality such a situation would not arise so long as the pressure equalisation technique was adopted between inside and outside. The design condition might be approached in deballasting the tank for recovery from the sea bed. It could be argued, therefore, that a preliminary design on the lines proposed here would be too conservative. Up to the present time experimental evidence is lacking on the magnitude and effect of current drag and hydrodynamic forces on such tank shapes although on a theoretical basis it would appear that the water inertia forces would exceed by far those of current drag. However, this seemingly conservative approach would provide an outline design which could be checked under the influence of hydrodynamic impact and seismic forces together with wind effects during the towing stages. Some work towards this end is planned [10], and it has been found by the authors already that shells of this type are capable of sustaining comfortably external static pressures of the order of twice the design head with theoretical predictions indicating collapse in an axi-symmetric manner at pressures in excess of nine times the design head.

Tanks of this form could be constructed in concrete as well as steel. A high quality dense concrete with low permeability and good durability in a saline enviroment would be required giving excellent resistance to marine boring animals. Shell thicknesses in concrete for a vessel comparable in size with the steel design considered above would be 100 mm or more. A steel structure would require special precautions against corrosion and the advantages and disadvantages of each material should be considered carefully.

The question of whether or not to found a storage tank on the sea bed is open to some debate. A submerged floating structure would be relatively easy to retrieve at the end of its working life. The inertia forces on the structure would have to be resisted by substantial anchor cables in order to keep a tank on station and some form of flexibility might be required in the connection of inlet and outlet pipes. The connection of anchor cables would require special provision at the base of a tank. However a floating structure would be better able to ride out any seismic effects and special provision for an uneven sea bed would be avoided. By contrast a sea bed structure could transfer horizontal forces on it to the foundation with the aid of friction and with a large gravity tank skirt piles might not be required to hold it in position.

The safety and maintenance aspects of underwater storage might dictate that it would be preferable to have several smaller tanks rather than one large vessel. In those circumstances a cluster of floating submerged drop-shaped tanks could suffice.

As well as for gas and oil storage underwater tanks of this kind could be used as fresh water reservoirs or as containers of fire fighting gas such as Halon (halogenated hydrocarbon) in support of the off-shore oil industry.

The drop-shaped tank or Echinodome [10] could be used also as a habitat or operational chamber within which work related to undersea activities could be pursued at atmospheric pressure; entry would be gained by submarine vehicle through suitable flooding chambers preferably in the base. This latter feature would be achieved more readily with a floating submerged structure than one founded on the sea bed.

The design criterion of the pressure over the apex of the shell at its operating depth would approximate more to the working conditions in the case of a habitat than a storage vessel and, for the former, fluctuations in the design head due to wave action would have to be taken into account. Present indications are that such variations could be accommodated by the preliminary design method for tanks of constant strength. Fig 6 shows that some tension can be expected near the base with the upper regions in compression.

The tank of constant strength whilst being an optimum type of design for only one set of loading circumstances could have a very useful role to play in underwater storage and other submarine functions.

An important operational feature of the storage of different liquids in a single chamber is the nature of the separating membrane. The durability of the diaphragm material in the presence of the liquids contained must be established together with the endurance of the diaphragm under repeated deformations.

ACKNOWLEDGEMENTS

The authors wish to thank Professor A.W. Hendry, Department of Civil Engineering and Building Science, University of Edinburgh, for his continuing encouragement of this work and the photographic, secretarial, and technical staff in the department for their support.

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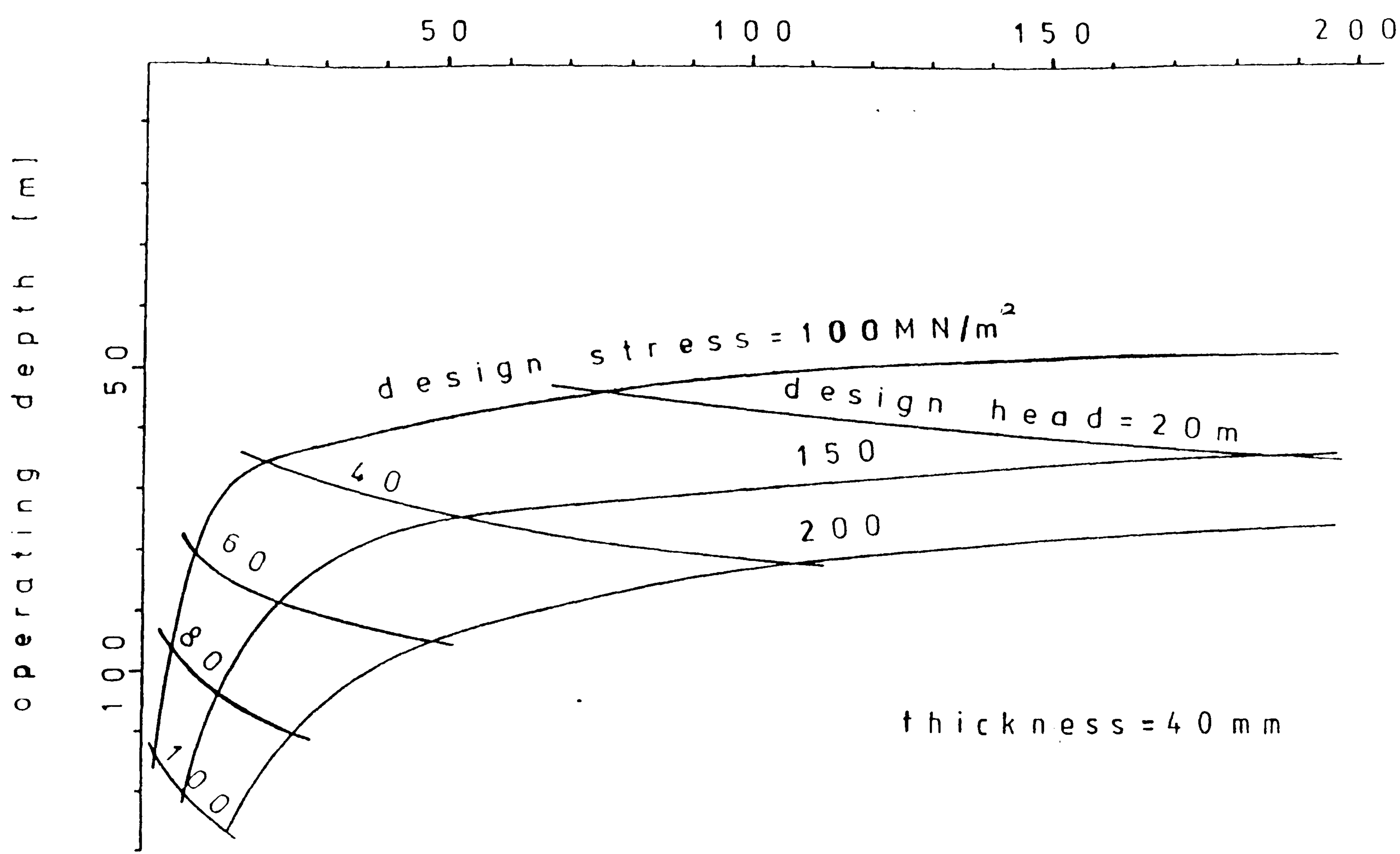

Fig.1 Capacity v operating depth

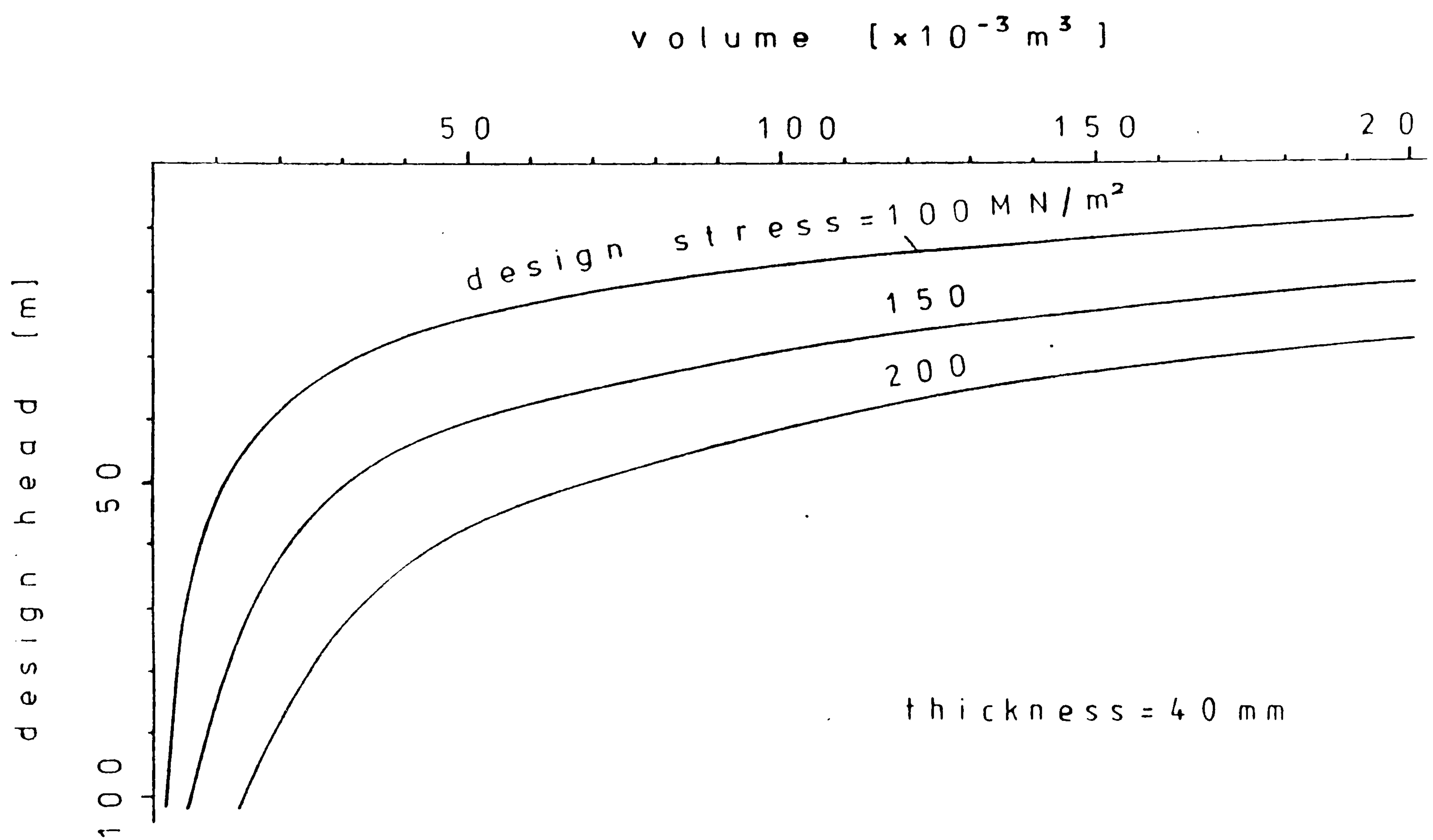

Fig.2 Capacity v design head

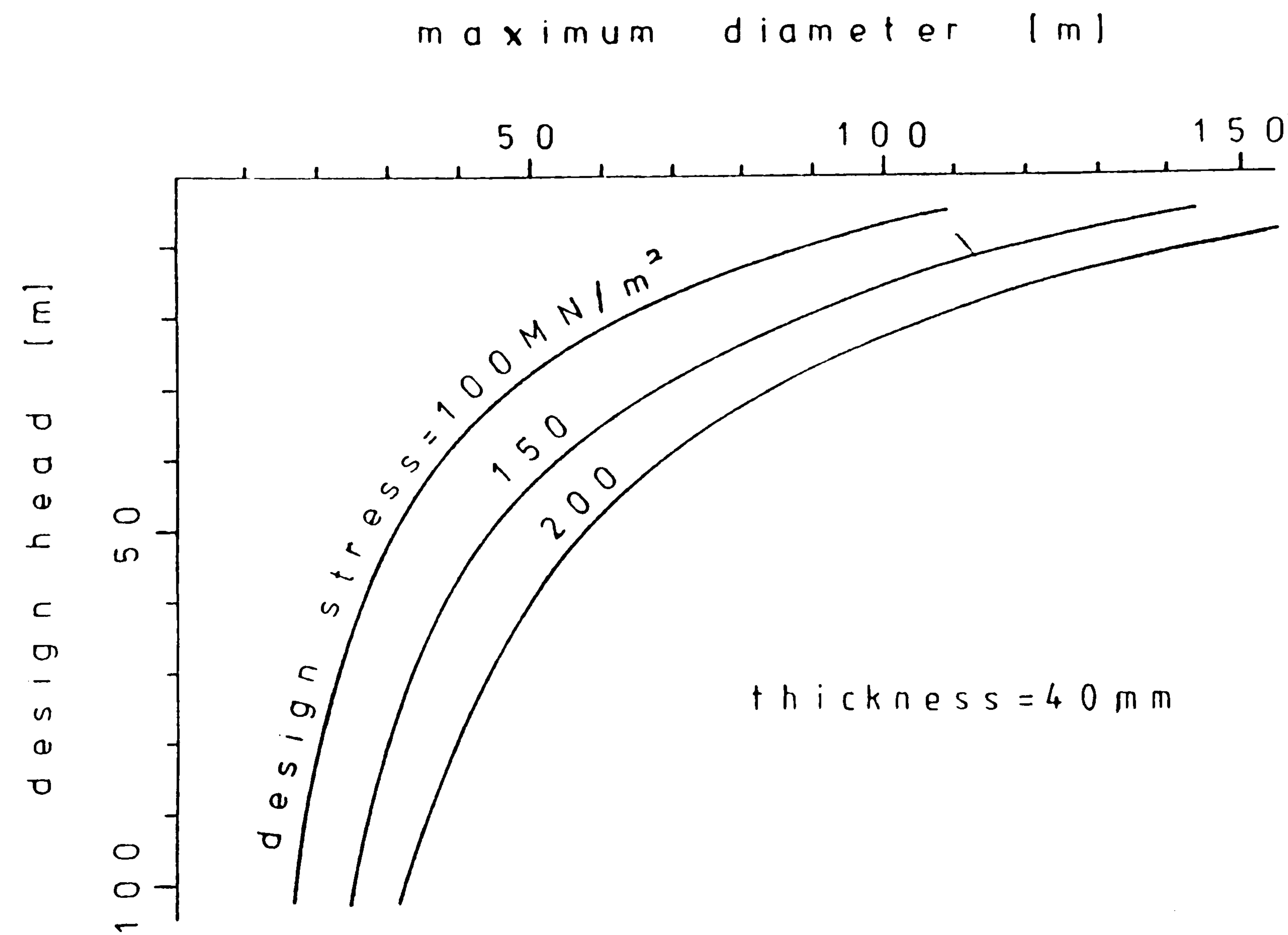

Fig.3 Maximum diameter v design head

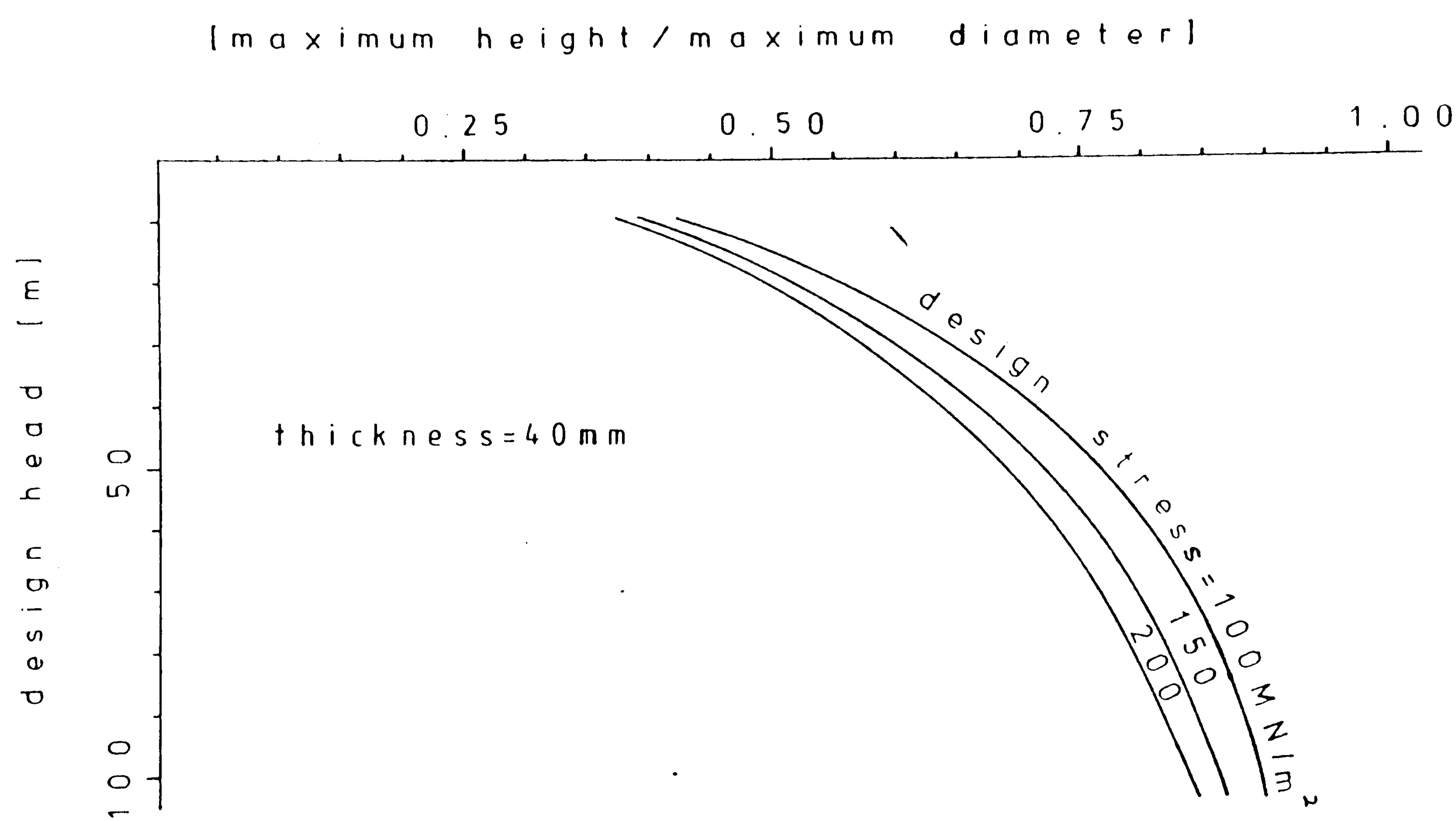

Fig.4 Max.height/max.dia.v design hd.

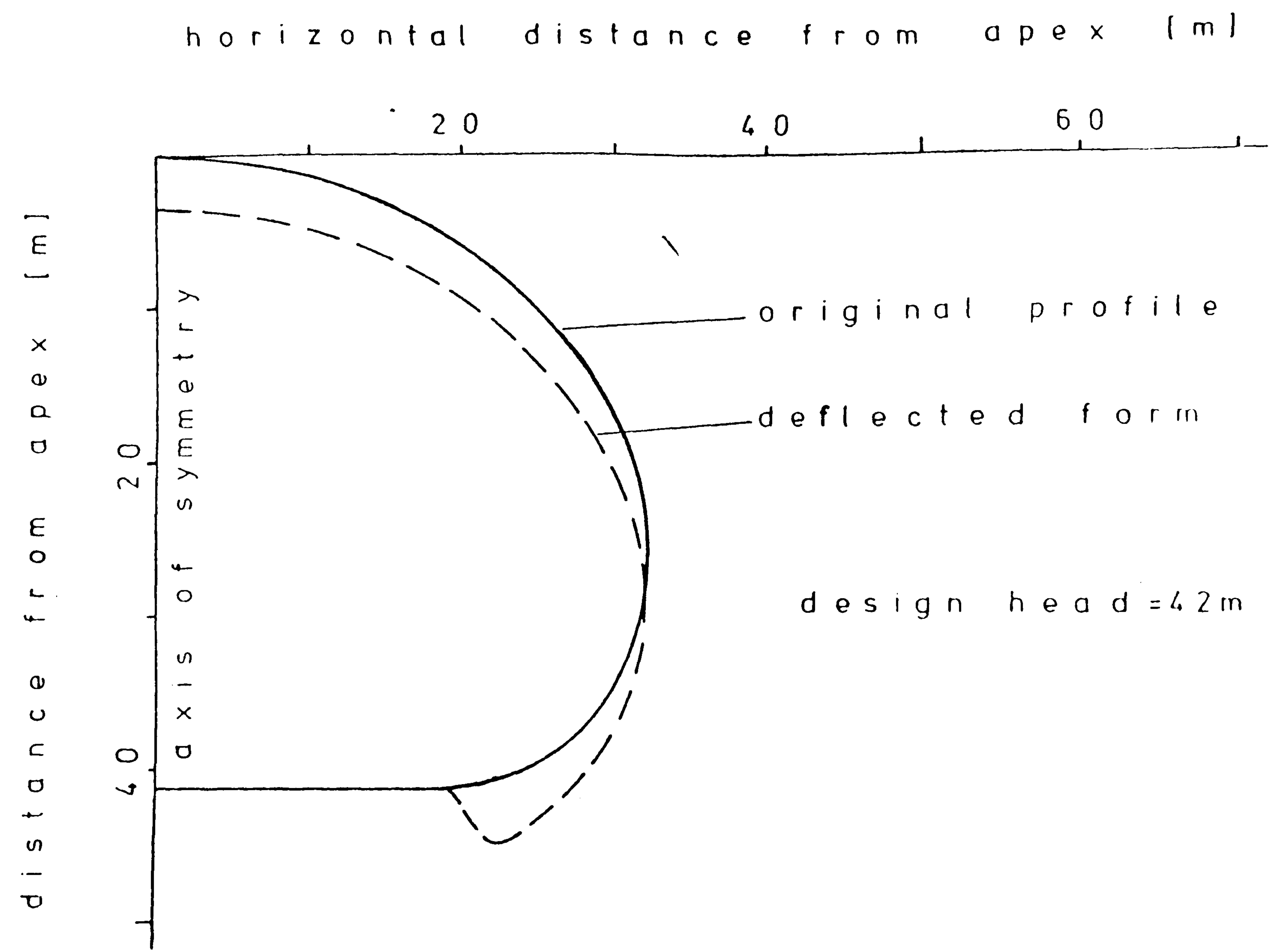

Fig. 6 Deformations at 1.95 × des. hd.

[displacements × 10]